$$\langle E \rangle = \frac{\int (c_1^* 1s_A^* + c_2^* 1s_B^*) \hat{H} (c_1 1s_A + c_2 1s_B)\, d\tau}{\int (c_1^* 1s_A^* + c_2^* 1s_B^*)(c_1 1s_A + c_2 1s_B)\, d\tau}$$

$$= \frac{c_1^2 \int 1s_A^* \hat{H} 1s_A\, d\tau + c_1^* c_2 \int 1s_A^* \hat{H} 1s_B\, d\tau + c_2^* c_1 \int 1s_B^* \hat{H} 1s_A\, d\tau + c_2^* c_2 \int 1s_B^* \hat{H} 1s_B\, d\tau}{c_1^* c_1 \int 1s_A^* 1s_A\, d\tau + c_1^* c_2 \int 1s_A^* 1s_B\, d\tau + c_2^* c_1 \int 1s_B^* 1s_A\, d\tau + c_2^* c_1 \int 1s_B^* 1s_B\, d\tau}$$

Since c_1 and c_2 are real numbers $c_1^* = c_1$ $\quad c_2^* = c_2$

Also $\underbrace{\int 1s_A^* \hat{H}\, 1s_B\, d\tau = \int 1s_B^* \hat{H}\, 1s_A\, d\tau}_{\text{since } 1s_A \text{ and } 1s_B \text{ are } 1s \text{ orbitals}} = H_{AB}$

Also $\int 1s_A^* \hat{H}\, 1s_A\, d\tau = \int 1s_B^* \hat{H}\, 1s_B\, d\tau = H_{AA}$

$\int 1s_A^* 1s_A\, d\tau = \int 1s_B^* 1s_B\, d\tau = 1$ normalized f^{n}

$\int 1s_A^* 1s_B\, d\tau = \int 1s_B^* 1s_A\, d\tau = S$

$$\langle E \rangle = \frac{c_1^2 H_{AA} + 2c_1c_2 H_{AB} + c_2^2 H_{AA}}{c_1^2 + 2c_1c_2 S + c_2^2}$$