1. **Random Coin Toss**

   When you toss a coin many times you expect to get a random sequence of heads and tails. How do we know when we look at a sequence of coin tosses whether or not it is random?

   (a) Write down what you perceive to be a random sequence of heads and tails. Make your sequence 50 coin tosses long. (Do not use a real coin!).

   (b) Find the frequency of heads and tails in your sequence, find the expected frequencies assuming the sequence is random and then calculate χ^2. Use Excel to find the p value for this value of χ^2. If $p < 0.05$ then χ^2 is too large and the sequence is not a good fit to a random distribution. Is your sequence random by this standard? Note: we can use the χ^2 test in another way, if we want to check if someone has made a fake random sequence. If χ^2 is very small, that would mean that the observed values are very close to the expected values. When a real coin is tossed 50 times it will be quite rare that the number of heads and tails will be very close to 25 each. We would be suspicious that the data may be faked if we got a χ^2 with a p value as high as say 0.80. This would mean that there is only a 20% chance of getting such a small χ^2 from a real coin. Does your sequence have a suspiciously small χ^2 ?

   (c) A true test of randomness will not just look at the number of heads and tails, but also, all possible subsequences of heads and tails. For example if we look at groups of two in our sequence, we would expected the subsequences HH, TT, HT and TH to occur equally often. Find the frequency of these pairs in your long sequence (you should have 49 pairs). Calculate the expected frequencies, find χ^2 and the corresponding p value. Is your sequence random based on this test?

   (d) Repeat the analysis for groups of three, where there are eight possible triplets you could get with three coin tosses, and in a sequence of 50 heads and tails there are 48 different triplets to count.

2. Suppose a pitcher can pitch either a fast ball (F) or a curve ball (C). The batter tries to guess what pitch will come and prepare for it. Assume the batter has the following batting averages (fraction of time he makes a hit.)

   - 0.300 if the batter guesses a fast ball and it is a fast ball.
   - 0.200 if the batter guesses a fast ball and it is a curve ball.
   - 0.100 if the batter guesses a curve ball and it is a fast ball.
   - 0.500 if the batter guesses a curve ball and it is a curve ball.

   Setup the payoff matrix, find the optimal mixed strategy for each player. Data from 100 hits during a series is shown below. Find the expected frequencies of each of the outcomes and use a χ^2 test to test if the pitcher and batter are using an optimal mixed strategy.

		Pitcher	
		fast	curve
Batter	fast	35	43
	curve	9	13

3. The following data shows the results of a study comparing whether or not their is a link between being trustworthy and trusting. What follows is a contingency table with the results of this study. Use a χ^2 to see if there is a link. What is the p value?

	trustworthy	untrustworthy
trusting	24	5
distrustful	4	22