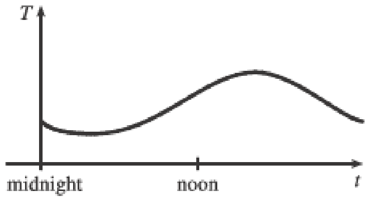
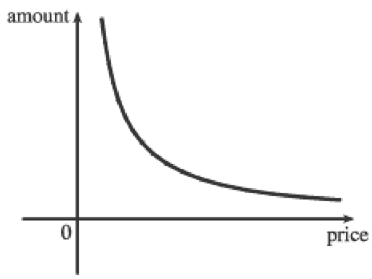
15. Of course, this graph depends strongly on the geographical location!



17. As the price increases, the amount sold decreases.



1. $y = (x^4 - 3x^2 + 5)^3 \quad \Rightarrow$

$$y' = 3(x^4 - 3x^2 + 5)^2 \frac{d}{dx}(x^4 - 3x^2 + 5) = 3(x^4 - 3x^2 + 5)^2(4x^3 - 6x) = 6x(x^4 - 3x^2 + 5)^2(2x^2 - 3)$$

3. $y = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}} = x^{1/2} + x^{-4/3} \quad \Rightarrow \quad y' = \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-7/3} = \dfrac{1}{2\sqrt{x}} - \dfrac{4}{3\sqrt[3]{x^7}}$

6. $y = \dfrac{e^x}{1 + x^2} \quad \Rightarrow \quad y' = \dfrac{(1 + x^2)e^x - e^x(2x)}{(1 + x^2)^2} = \dfrac{e^x(x^2 - 2x + 1)}{(1 + x^2)^2} = \dfrac{e^x(x - 1)^2}{(1 + x^2)^2}$

15. $y = \dfrac{\sec 2\theta}{1 + \tan 2\theta} \quad \Rightarrow$

$$y' = \frac{(1 + \tan 2\theta)(\sec 2\theta\ \tan 2\theta \cdot 2) - (\sec 2\theta)(\sec^2 2\theta \cdot 2)}{(1 + \tan 2\theta)^2} = \frac{2\sec 2\theta\,[(1 + \tan 2\theta)\tan 2\theta - \sec^2 2\theta]}{(1 + \tan 2\theta)^2}$$

$$= \frac{2\sec 2\theta\,(\tan 2\theta + \tan^2 2\theta - \sec^2 2\theta)}{(1 + \tan 2\theta)^2} = \frac{2\sec 2\theta\,(\tan 2\theta - 1)}{(1 + \tan 2\theta)^2} \qquad [1 + \tan^2 x = \sec^2 x]$$

18. $y = \ln(x^2 e^x) = \ln x^2 + \ln e^x = 2\ln x + x \quad \Rightarrow \quad y' = 2/x + 1$

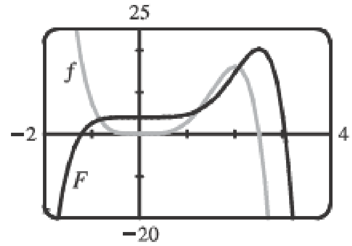
4. $f(x) = x\,(2 - x)^2 = x\left(4 - 4x + x^2\right) = 4x - 4x^2 + x^3 \quad \Rightarrow$

$$F(x) = 4\left(\tfrac{1}{2}x^2\right) - 4\left(\tfrac{1}{3}x^3\right) + \tfrac{1}{4}x^4 + C = 2x^2 - \tfrac{4}{3}x^3 + \tfrac{1}{4}x^4 + C$$

17. $f(x) = 5x^4 - 2x^5 \quad \Rightarrow \quad F(x) = 5 \cdot \dfrac{x^5}{5} - 2 \cdot \dfrac{x^6}{6} + C = x^5 - \frac{1}{3}x^6 + C.$

$F(0) = 4 \quad \Rightarrow \quad 0^5 - \frac{1}{3} \cdot 0^6 + C = 4 \quad \Rightarrow \quad C = 4$, so $F(x) = x^5 - \frac{1}{3}x^6 + 4.$

The graph confirms our answer since $f(x) = 0$ when F has a local maximum, f is positive when F is increasing, and f is negative when F is decreasing.



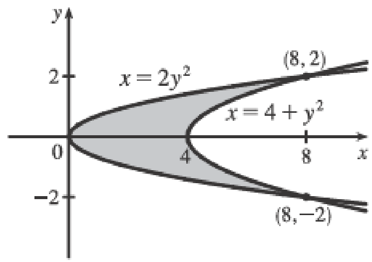
37. Given $f'(x) = 2x + 1$, we have $f(x) = x^2 + x + C$. Since f passes through $(1, 6)$, $f(1) = 6 \quad \Rightarrow \quad 1^2 + 1 + C = 6 \quad \Rightarrow$ $C = 4$. Therefore, $f(x) = x^2 + x + 4$ and $f(2) = 2^2 + 2 + 4 = 10$.

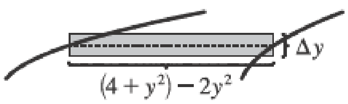
11. $2y^2 = 4 + y^2 \quad \Leftrightarrow \quad y^2 = 4 \quad \Leftrightarrow \quad y = \pm 2$, so

$$A = \int_{-2}^{2} \left[(4 + y^2) - 2y^2\right] dy$$

$$= 2\int_0^2 (4 - y^2)\, dy \qquad \text{[by symmetry]}$$

$$= 2\left[4y - \tfrac{1}{3}y^3\right]_0^2 = 2\left(8 - \tfrac{8}{3}\right) = \tfrac{32}{3}$$





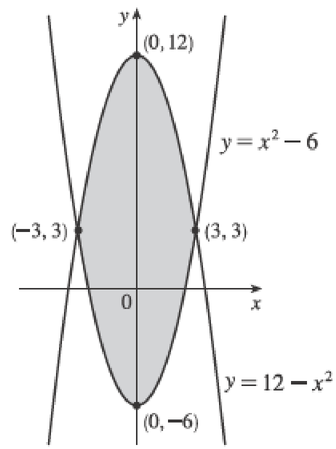
13. $12 - x^2 = x^2 - 6 \quad \Leftrightarrow \quad 2x^2 = 18 \quad \Leftrightarrow$ $x^2 = 9 \quad \Leftrightarrow \quad x = \pm 3$, so

$$A = \int_{-3}^{3} \left[(12 - x^2) - (x^2 - 6)\right] dx$$

$$= 2\int_0^3 \left(18 - 2x^2\right) dx \qquad \text{[by symmetry]}$$

$$= 2\left[18x - \tfrac{2}{3}x^3\right]_0^3 = 2\left[(54 - 18) - 0\right]$$

$$= 2(36) = 72$$

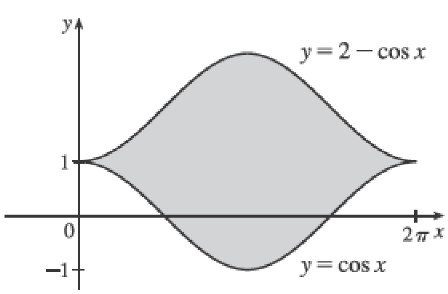


16. $A = \displaystyle\int_0^{2\pi} \left[(2 - \cos x) - \cos x\right] dx$

$$= \int_0^{2\pi} (2 - 2\cos x)\, dx$$

$$= \Big[2x - 2\sin x\Big]_0^{2\pi}$$

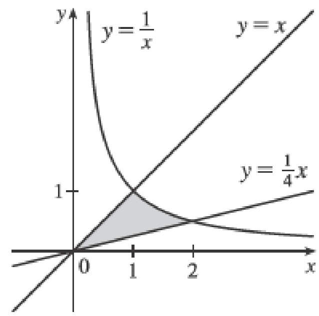
$$= (4\pi - 0) - 0 = 4\pi$$



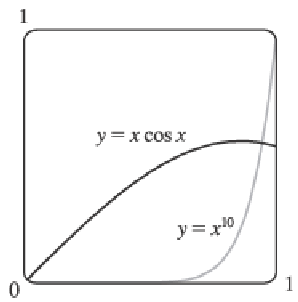
17. $1/x = x \quad \Leftrightarrow \quad 1 = x^2 \quad \Leftrightarrow \quad x = \pm 1$ and $1/x = \frac{1}{4}x \quad \Leftrightarrow$

$4 = x^2 \quad \Leftrightarrow \quad x = \pm 2$, so for $x > 0$,

$$A = \int_0^1 \left(x - \frac{1}{4}x\right) dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{4}x\right) dx$$
$$= \int_0^1 \left(\frac{3}{4}x\right) dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{4}x\right) dx$$
$$= \left[\tfrac{3}{8}x^2\right]_0^1 + \left[\ln|x| - \tfrac{1}{8}x^2\right]_1^2$$
$$= \tfrac{3}{8} + \left(\ln 2 - \tfrac{1}{2}\right) - \left(0 - \tfrac{1}{8}\right) = \ln 2$$



22.



From the graph, we see that the curves intersect at $x = 0$ and $x = a \approx 0.94$, with $x \cos x > x^{10}$ on $(0, a)$. So the area A of the region bounded by the curves is

$$A = \int_0^a (x \cos x - x^{10})\, dx$$
$$= \left[x \sin x + \cos x - \tfrac{1}{11}x^{11}\right]_0^a \qquad \begin{bmatrix} u = x, & dv = \cos x\, dx \\ du = dx, & v = \sin x \end{bmatrix}$$
$$\approx 0.30$$