# WOLFSON ch 32

**11.** **INTERPRET** This problem is about double-slit interference. We are interested in the spacing between adjacent bright fringes.

**DEVELOP** We assume that the geometrical arrangement of the source, slits, and screen is that for which Equations 32.2a and 32.2b apply. The location of bright fringes is given by

$$y_{\text{bright}} = m\frac{\lambda L}{d}$$

where $m$ is the order number.

**EVALUATE** The spacing of bright fringes is

$$\Delta y = (m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d} = \frac{\lambda L}{d} = \frac{(550\text{ nm})(75\text{ cm})}{0.025\text{ mm}} = 1.65\text{ cm}$$

**ASSESS** Since $\lambda \ll d$, the spacing between bright fringes is much smaller than $L$, as it should.

**14.** The interference minima fall at angles given by Equation 32.1b; therefore $d = (4+\frac{1}{2})\lambda/\sin\theta = 4.5(546\text{ nm})/\sin 0.113° = 1.25\text{ mm}$. (Note that $m = 0$ gives the first dark fringe.)

**19.** **INTERPRET** This problem is about diffraction gratings. For a given wavelength, we are interested in the highest visible order.

**DEVELOP** The grating condition is $\sin\theta = m\lambda/d$, and, of course, for the diffracted light to be visible, $\theta < 90°$, or $m\lambda/d < 1$. Therefore, the highest order visible is the greatest integer $m$ less than $d/\lambda$.

**EVALUATE** **(a)** For this grating, $d = 1\text{ cm}/10^4 = 10^3\text{ nm}$, so for $\lambda = 450$ nm the highest visible order is less than $10^3/450 = 2.22$, or $m_{\max} = 2$.

**(b)** Similarly, for $\lambda =$ 650 nm, the highest visible order is less than $10^3/650 = 1.54$, or $m_{\max} = 1$.

**ASSESS** Increasing wavelength lowers $m_{\max}$. This can be seen from Equation 32.1a, $d\sin\theta = m\lambda$.

**27.** **INTERPRET** This problem involves a single-slit diffraction of light. We are interested in the angular width of the central peak.

**DEVELOP** The condition for destructive interference in a single-slit diffraction is given by Equation 32.8:

$$a\sin\theta = m\lambda, \quad m = \pm 1, \pm 2, \ \ldots$$

The first minima $(m = \pm 1)$ occur at

$$\sin\theta = \pm\frac{\lambda}{a} = \pm\frac{633\text{ nm}}{2.5\ \mu\text{m}} \ \rightarrow \ \theta = \pm 14.7°$$

**EVALUATE** The total angular width of the diffracted beam is $2|\theta| = 29.3°$.

**ASSESS** The case $m = 0$ is excluded in Equation 32.8 because it corresponds to the central maximum in which all waves are in phase

**33.** **INTERPRET** This is a problem about the diffraction limit with a circular aperture. We want to know the aperture diameter needed to achieve a certain resolution.

**DEVELOP** As shown in Equation 32.11b, the minimum resolvable source separation for a circular aperture is (Rayleigh criterion)

$$\theta_{\min} = \frac{1.22\lambda}{D}$$

where $D$ is the aperture diameter. The angle subtended by a human feature 5 cm across at 100 km is

$$\theta_{\min} = 5\text{ cm}/100\text{ km} = 5\times 10^{-7}\text{ (radians)}$$

**EVALUATE** The Rayleigh criterion for a diffraction-limited telescope, using light of wavelength $\lambda = 550$ nm, requires an aperture of

$$D = \frac{1.22\lambda}{\theta_{\min}} = \frac{1.22(550\text{ nm})}{5\times10^{-7}} = 1.34\text{ m}$$

**ASSESS** Atmospheric turbulence would limit the resolution to no better than $\frac{1}{2}'' = 2.4\times10^{-6}$ radians.

**37.** **INTERPRET** We have a double-slit experiment here. Given the slit spacing, we are asked to find the highest-order bright fringes.

**DEVELOP** The maximum diffraction angle for which light hits the screen is

$$\theta_{\max} = \tan^{-1}(0.5\text{ m}/2.0\text{ m}) = 14.0°$$

Bright fringes will appear on the screen in orders of interference for which

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) < 14.0° \quad \rightarrow \quad m < \frac{d\sin 14°}{633\text{ nm}} = d(3.83\times10^{5}\text{ m}^{-1})$$

**EVALUATE** **(a)** For $d = 0.10\text{ mm} = 10^{-4}\text{ m}$, $m_{\max} = 38$.

**(b)** For $d = 10^{-5}\text{ m}$, $m_{\max} = 3$.

**ASSESS** The maximum order $m_{\max}$ increases with slit spacing $d$.

**39.** **INTERPRET** Our light source for the double-slit experiment has two wavelengths. The angular position where interference is constructive for one wavelength is destructive for the other.

**DEVELOP** For a bright fringe of order $m_1$, from wavelength $\lambda_1$, to have the same angular position, in a double-slit apparatus of the type described in the text, as a dark fringe of order $m_2$, from wavelength $\lambda_2$, we must have

$$m_1\lambda_1 = (m_2 + 1/2)\lambda_2$$

For $\lambda_1 = 550$ nm and $\lambda_2 = 400$ nm, one finds

$$11m_1 = 8m_2 + 4$$

**EVALUATE** By inspection, the smallest integer values satisfying this condition are $m_1 = 4$ and $m_2 = 5$; that is, the fourth bright fringe of wavelength 550 nm coincides with the sixth dark fringe of wavelength 400 nm (recall that the first bright fringe has $m = 1$, while the first dark fringe has $m = 0$).

**ASSESS** This problem demonstrates the role played by the wavelength in determining the nature of the interference at an angular position.

**52.** The minimum thickness of the bubble, which produces interference colors, is $d_{\min} = \lambda_{\min}/4n$, where $\lambda_{\min}$ is the shortest visible wavelength, normally 400 nm violet light. (See the solution to Exercise 21.) Thus, $d_{\min} =$ 400 nm/4(1.33) = 75.2 nm.

**80.** **INTERPRET** We find the angular resolution of the Arecibo radio telescope, using the Rayleigh criterion.

**DEVELOP** The Rayleigh criterion is $\theta_{\min} = \frac{1.22\lambda}{D}$, where the wavelength is $\lambda = 0.21$ m and the diameter of the radio dish is $D = 305$ m. We find $\theta_{\min}$.

**EVALUATE** $\theta_{\min} = 0.84$ mRad.

**ASSESS** The other reason to use large radiotelescopes is collection efficiency, which is proportional to area for a dish such as this.