Please complete the following questions by Tuesday October 4th. Show all your reasoning. Use diagrams to illustrate questions 3,4 and 5.

1. For the following sequences, find a recursive formula and a general formula

   (a) 10,7,4,1,$\cdots$ recursive: $u_n = u_{n-1} - 3$ general: $u_n = 10 - 3(n-1) = 13 - 3n$.

   (b) 3,-6,12,-24, $\cdots$ recursive: $u_n = -2 \times u_{n-1}$ general: $u_n = 3(-2)^{n-1}$.

2. A sequence is defined by the relation $u_n = u_{n-1} + 5$ with $u_1 = 3$. Find a general formula for the sequence and hence find the 30th term.
   $u_n = 3 + 5(n-1) = 5n - 2$. So $u_{30} = 5(30) - 2 = 148$

3. A ball is dropped and bounces to to 3/4 of its original height. On each successive bounce its height is reduced further by a factor of 3/4. If the ball is released from a height of 4 feet, find

   (a) the maximum height it reaches after the first, second, third, and fourth, bounce.
   $3, \frac{9}{4}, \frac{27}{16}, \frac{81}{64}$.

   (b) how many bounces it takes before the ball no longer bounces higher than 6 inches above the ground.
   After 7 bounces.

   (c) a recursive formula for the height after n bounces.
   $u_n = (\frac{3}{4})u_{n-1}$ and $u_1 = 3$.

   (d) a general formula for the height after n bounces.
   $u_n = 3(\frac{3}{4})^{n-1}$.

4. Consider the problem of stacking balls in the shape of a square pyramid. On the top there is one ball, on the next level there are 4 balls arranged in a square, on the next level there are 9 balls arrange in a square and so on. The total number of balls in a square pyramid with n levels is called a square pyramidal number.

   (a) What is the total number of balls in a pyramid of 2 levels? 3 Levels? 4 Levels?
   $1 + 4 = 5,\ 1 + 4 + 9 = 14,\ 1 + 4 + 9 + 16 = 30.$

   (b) How high a pyramid could you build with 100 balls?
   $1 + 4 + 9 + 16 + 25 + 36 = 81$. So there would be 6 levels with 19 left over.

   (c) Find a recursive formula for the nth square pyramidal number. $u_n = u_{n-1} + n^2$ and $u_1 = 1$

   (d) Challenge for the mathematically inquisitive: Find a general formula the nth square pyramidal number.
   The idea here is to realize that a square pyramidal number is the sum of two consecutive tetrahedral numbers (because a square is the some of two consecutive triangular numbers). In the worksheet solutions I explained that the general formula for the tetrahedral numbers is $t_n = \frac{1}{6}n(n+1)(n+2)$. So the general formula for the square pyramidal numbers is

   $$p_n = t_n + t_{n-1} = \frac{1}{6}n(n+1)(n+2) + \frac{1}{6}(n-1)n(n+1) = \frac{1}{6}n(n+1)(n-1+n+2) = \frac{1}{6}n(n+1)(2n+1).$$

5. A drone bee is a male bee in a hive whose sole purpose in life is to fertilize the eggs produced by the queen bee. Interestingly fertilized eggs always result in female bees. A new drone is produced from an egg which is not fertilized. Consequently a drone bee has a mother (the queen) but no father! On the other hand, as a female the queen bee has both a mother and a father.

   (a) How many grand parents does a drone have?
   2 grand parents

   (b) How many great grand parents does it have? How many great great grand parents? What sequence is this?
   3 great grand parents and 5 great great grand parents. This is the Fibonacci sequence

   (c) Find an recursive definition for the resulting sequence?
   $u_n = u_{n-1} + u_{n-2}$, with $u_1 = 2$ and $u_2 = 3$.