The following assignment is due on October 25th at 9:30 am. Please show all your work on a separate piece of paper.

1. Let $F_n$ be the $n$th Fibonacci number. That is $F_1 = 1$, $F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$. Write out the 6 terms of the following sequences.

   (a) $u_n = 5F_n$
   for reference, the Fibonacci numbers are 1,1,2,3,5,8,13,21 ..., so $u_n$ is 5,5,10,15,25,40,65

   (b) $u_n = 2F_{n+1} - F_n$
   $u_1 = 2(1) - 1 = 1$, $u_2 = 2(2) - 1 = 3$, $u_3 = 2(3) - 2 = 4$, $u_4 = 2(5) - 3 = 7$, $u_5 = 2(8) - 5 = 11$, and $u_6 = 2(13) - 8 = 18$

   Show that both the sequences above are approximately geometric when $n$ is large and evaluate the approximate growth factor.
   In both cases notice that $u_n = u_{n-1} + u_{n-2}$, just like for the Fibonacci numbers, so the ratio of neighboring terms approaches $\phi$ also.

2. Given that the golden ratio $\phi$ satisfies the equation $\phi^2 = 1 + \phi$ Show, by multiplying both sides of this equation by $\phi$ that:

   (a) $\phi^3 = 2\phi + 1$
   $phi^3 = \phi(\phi^2) = \phi(1 + \phi) = \phi + \phi^2 = \phi + (1 + \phi) = 2\phi + 1$

   (b) $\phi^4 = 3\phi + 2$
   $phi^4 = \phi(\phi^3) = \phi(2\phi + 1) = 2\phi^2 + \phi = 2(1 + \phi) + \phi = 3\phi + 2$

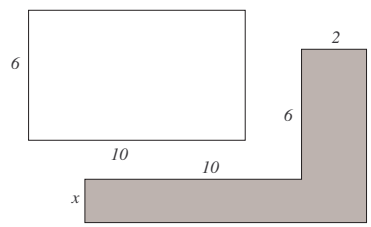
   (c) $\phi^5 = 5\phi + 3$
   $phi^5 = \phi(\phi^4) = \phi(3\phi + 2) = 3\phi^2 + 2\phi = 3(1 + \phi) + 2\phi = 5\phi + 3$

   Write down an expression for $\phi^n$ in terms of Fibonacci numbers.

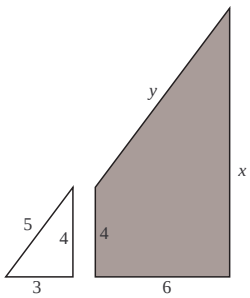
3. Find the value of $x$ so that the shaded area is a gnomon to the rectangle.
   We want growth factors to be equal, so

$$\frac{x + 6}{6} = \frac{10 + 2}{10}$$

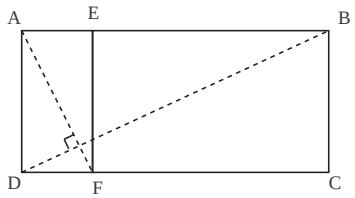
   Thus $x + 6 = 7.2$, so $x = 1.2$.




4. Find the values of $x$ and $y$ so that the shaded area is a gnomon to the white triangle.
   The growth factor for the base is $(3 + 6)/3 = 3$, so this is the growth factor for the other sides. Hence $x = 3 * 4 = 12$, and $y + 5 = 3 * 5 = 15$, so $y = 10$.



5. Let ABCD be an arbitrary rectangle as shown in the figure on the right. Let AF be perpendicular to the diagonal BD and EF perpendicular to AB. Show that the rectangle BCEF is a gnomon to the rectangle ADEF.




IF ECEF is a gnomon then the new rectangle ABCD is similar to old rectangle ADEF. To show this we notice that the triangle ADB is similar to AFD because they are both right angled triangles and angle DAF is equal to angle ABD. Because those triangles are similar, the ratios of their sides are the same. Thus the ratios of the sides of the rectangles are also the same. So they are similar also.