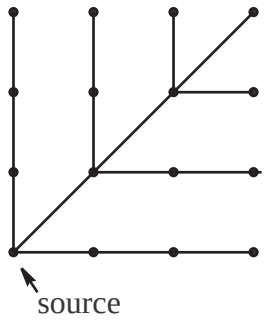
Please show your work on all questions

1.

The diagram on the right shows a branching structure which distributes nutrients from the source at the lower corner to points on a square grid. Find the total length of the branches and the average distance of each point from the source. Show your working.



Assuming 1 unit for a side and, by Pythagoras, $\sqrt{2}$ for a diagonal. The total length is $12 + 3\sqrt{2} \approx 16.24$. For the average there are 15 nodes apart from the source. Taking into account symmetry we add up the distances to the nodes on the left branch and multiply by 2. Then we add up the distance to the nodes on the diagonal.

$$\frac{2(1+2+3+(\sqrt{2}+1)+(\sqrt{2}+2)+(2\sqrt{2}+1))+\sqrt{2}(1+2+3)}{15} = \frac{20+14\sqrt{2}}{15} \approx 2.65$$

2. Suppose the primordia of a flower emerge and grow out radially with a constant divergence of angle of 70°.

   (a) Write the angle 70° as a fraction of a complete revolution in simplest terms.
   70/360=7/36

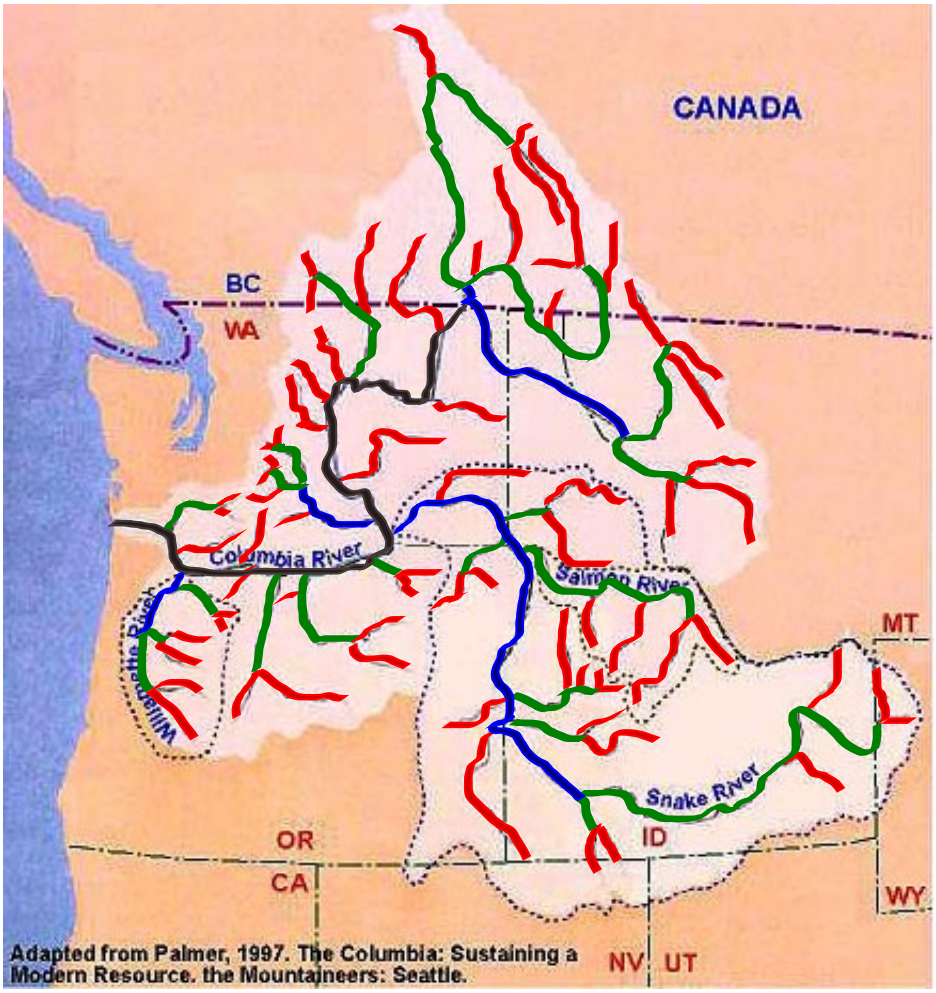
   (b) How many straight line parastichies would emerge in this growth pattern?
   There will be 36 straight line parastichies

   (c) Express the fraction in part (a) as a continued fraction.

$$\frac{7}{36} = \frac{1}{\frac{36}{7}} = \frac{1}{5+\frac{1}{7}}$$

   (d) Find an approximation to the continued fraction and hence write down how many curved parastichies you would expect to see.
   This is approximately $\frac{1}{5}$, so there will be 5 spiral parastichies.

3. The Columbia drainage basin is shown below. Color the first order rivers red, the second order rivers green, the third order rivers blue and the fourth order rivers black. Count the rivers of each order and then find the average bifurcation ratio for this river system. (Note: it is easier to see the river in the color image. Also, there is one place near the northern Washington-Idaho border, where the river appears to be broken. This is the Spokane river which passes through Spokane as it travels from Idaho to the Columbia river. It should not be broken there.



There are 71 first order streams, 21 second order streams, 5 third order streams and 1 fourth order stream. The bifurcation ratios are $71/21 = 3.38$, $21/5 = 4.2$, and $5/1 = 5$. The average bifurcation ratio is $(3.38 + 4.2 + 5)/3 = 4.2$

4. Activity from workshop: Quantitative Analysis of Branching

   (a) Mark off a rectangular array of dots, 10 dots wide by 20 dots high, on a triangular grid (If you didn't get a copy do a web search). For the sake of this activity think of this area as a drainage field for a river. Water at each point will drain down left or right to the next point. For each dot in your grid toss a coin. For heads draw a line in pencil going down to the left to the next point. For tails draw a line to the right. After you have completed this for all points in your grid you will have a collection of lines representing the flow of water down a gradient. (The observant among you may recognize this pattern as similar to one formed by the receding wave at the beach). Some of these lines will have naturally joined into branched networks. Find the longest such network and colour it.

   (b) For this random branching network complete the following analysis. First label the branches as first order, second order or third order. First order branches are those that originate from a point that has no other lines. Second order branches are those that originate when two first order branches join. A third order branch is one which originates when two second order branches and so on. Note when a first order branch joins a second order branch the second order branch does not changes its order. It may be helpful (and even aesthetically pleasing) to shade second order branches thicker than first order branches and so on. To complete the analysis find the ratio of the number of first order to second order branches. Then find the ratio of second order to third order branches and so on. Typically these ratios will range between 3 and 5 for streams. When branching networks are large, this ratios are remarkably constant and quite consistent over a large class of branching phenomena.
   Answers will vary