For this assignment will use the recursion features of Excel to evaluate at least 30 terms of the suggested sequences. Before you start with the questions place the values $1, 2, 3, \ldots 30$ in column A. Remember that when you change a recursive formula in one cell you must make sure to copy that formula down to all the other cells that use it.

Answer all the questions using complete sentences in a text box in Excel. Upload your completed assignment to our class website using the link on the week 2 schedule, by Tuesday, January 11th at 1:00 pm.

`LastName_FirstName_Recursion_Lab.xls`

1. Use Excel formulas to generate the following sequences
   (a) The triangular numbers; (Recall, these are the sum of the first n counting numbers.
   (b) The tetrahedral numbers (The tetrahedral numbers are are formed when you stack balls which are arranged in layers of triangles – ie the tetrahedral numbers are the sum of the triangular numbers.);
   (c) the Fibonacci Sequence.

2. A sequence is said to **converge** to a **limit** if the terms get closer and closer to a fixed value. For example the sequence 1, 1.5, 1.75, 1.825, 1.9375 … appears to converge to the limit 2. If a sequence does not converge to a limit it is called a **divergent** sequence. A divergent sequence can diverge to infinity (*eg* 2,4,8,16,32 ….). It can oscillate between two or more values (*eg* 0,1,0,1,0,1 …), in which case it is called a **periodic** sequence. Or it may seem to exhibit a apparently random pattern in which case it is probably is **chaotic**.

   Generate each of the following sequences and classify them as above. If the sequence converges write down the limiting value. Note: Sometimes you will need to generate a very large number of terms before you are certain about the convergence of a sequence.

   (a) $u_n = \dfrac{1}{u_{n-1}}$ and $u_1 = 2$
   (b) $u_n = 1 + \dfrac{1}{u_{n-1}}$ and $u_1 = 1$
   (c) $u_n = u_{n-1} + 2n - 1$ and $u_1 = 1$
   (d) $u_n = 3u_{n-1}(1 - u_{n-1})$ and $u_1 = 0.5$
   (e) $u_n = 4u_{n-1}(1 - u_{n-1})$ and $u_1 = 0.5$

3. **Order in two dimensions:** One of themes of this programs is to investigate how basic elements following simple rules can build complex structure. The following example shows how a one in a sea of zeros can generate an amazing two dimensional pattern of numbers, which contains within in it many of the sequences we have been exploring.

   In a new spread sheet (say sheet 2) enter zeros in the first 15 columns of row 1 then put the number 1 in cell P1. (You may need to resize the columns so that it all fits on one screen).

The rule we will use to generate the elements in the next row is a simple one. Add the value in the cell immediately above the one you wish to fill to the one in the next column to the right. For example in cell A2 you should write **=A1+B1**, and in B2 write **=B1+C1**. Use the copying features in Excel to fill row 2 with the same formula. Then select the entire row 1 and copy down to row 16 to apply the same formula to all cells. What you should have generated is called Pascal's Triangle – a fascinating sequences of numbers, discovered in China, that we will keep coming back to. In a text box describe and state a recursive definition for the following sequences:

(a) the sequence in column N

(b) the sequence in column M

(c) the sequence formed by summing the values in each row (use the Excel sum function to generate these numbers)

(d) There are series of diagonals in this matrix of numbers that add up to the Fibonacci numbers. Find these diagonals and format the cells in these diagonals a different color for each Fibonacci number.

(e) Make a separate copy of Pascals triangle and then format all the cells with odd numbers in a shade of your choice. The pattern you get is called the Sierpinski Gasket. This is a fractal pattern that we will revisit at the end of the quarter.