Answer the following questions in your workshop groups. For question four you may find it helpful to draw diagrams to illustrate your answers.

1. Find the next term for these sequences. Then give a recursion formula for them.

   (a) 24,12,6,3, 1.5
   We are halving each time, so the recursion formula is $u_n = \frac{u_{n-1}}{2}$, $u_1 = 24$. The general formula is $u_n = 48(\frac{1}{2})^n$.

   (b) 10,7,4,1,-2
   We subtract 3 each time, so the recursive formula is $u_n = u_{n-1} - 3, u_1 = 10$. The general formula is $u_n = 10 - 3(n-1)$

   (c) 1,2,4,7,11, 16
   To get the second term we add 1, to get the third term we add 2, to get the fourth term we add 3 and so on. So the recursion formula is $u_n = u_{n-1} + n - 1$, $u_1 = 1$. For the general formula notice that this is a bit like the triangle numbers except we add $n-1$ instead of n each time, so $u_n = 1 + n(n-1)/2$

   (d) 2,3,5,9,17,33.
   Each term can be obtained from the previous one by multiplying it by 2 and then subtracting 1, so the recursive formula is $u_n = 2u_{n-1} - 1$.

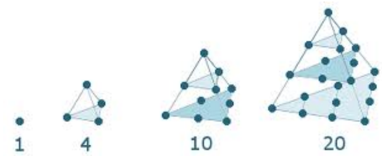
2. Consider the following sequence of patterns.

   (a) Draw the next pattern and write down the number of hexagons in each pattern.
   The next pattern has six hexagons added to the branches. The sequence is 1,7,13,19

   (b) Find a recursive formula for this sequence.
   We add six each time so the recursive formula $u_n = u_{n-1} + 6, u_1 = 1$

   (c) Find a general formula for this sequence.
   This is a linear sequence with a first term of 1 and a constant difference of 6, so $u_n = 1 + 6(n-1)$.

3. Oranges are stacked in the shape of a triangular pyramid (or tetrahedron). A pyramid with one level consists of a single orange, a pyramid with two levels consists of one orange on the top level and three oranges in the shape of a triangle on the next level for a total of four oranges. There is a similar arrangement at higher levels.



   (a) How many oranges do you need to make a tetrahedron with 5 levels?, 6 levels? The sequence of numbers you are generating are called tetrahedral numbers.
   Each level of the tetrahedron is a triangle so the nth tetrahedral number is the sum of the first n triangular numbers. So

   $$u_4 = 1+3+6+10 = 20,\ u_5 = 1+3+6+10+15 = 35,\ u_6 = 1+3+6+10+15+21 = 56$$

   The recursive formula is $u_n = u_{n-1} + \dfrac{n(n+1)}{2}$ since the nth tetrahedral number is $\dfrac{n(n+1)}{2}$. The general formula is a bit of a challenge, but if you arrange six tetrahedrons appropriately you can make a box shape called a parallelipiped made up of a width of n oranges, a height of $n+1$ oranges and a length of $n+2$ oranges. Thus $u_n = \frac{1}{6}n(n+1)(n+2)$.

   (b) If you had 100 oranges, how many levels could you complete and how many oranges would you have left over?
   Using either formula you should find that with seven levels you need 84 oranges, which leaves 16 left over.

4. A tree grows according to the following rule. It starts as a trunk with no branches and grows to a height of one foot in one year. At the start of the next year the trunk produces a branch, then over the course of the next year the new branch and the trunk each grow one foot. The year after a branch is formed it grows in exactly the same way as the original trunk (ie it produces a new one foot branch and grows one foot longer itself.)

   (a) How many branches does the tree have after 2, 3, 4, 5 years (count the end of the trunk as a branch.)? Find a formula for the sequence.
   The number of branches doubles each year, with 2 after 2 years, 4 after 3 years, 8 after 4 years and 16 after 5 years. The recursive formula is $u_n = 2u_{n-1}, u_1 = 1$ and the general formula is $u_n = 2^{n-1}$

   (b) What is the total length of wood in the tree after 5 years of growth?
   The total length of wood is found by summing up each section of branch which gives $1 + 2 + 4 + 8 + 16 = 31$ units of wood.