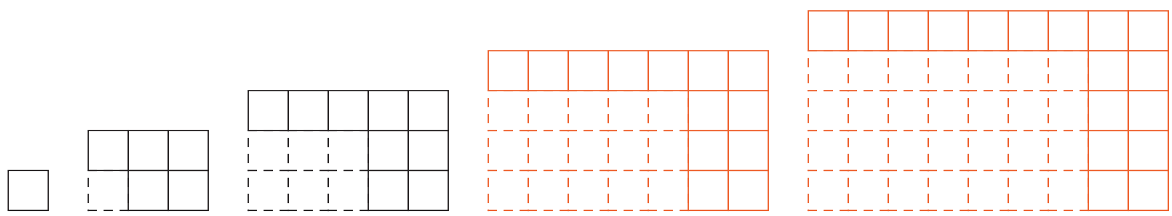
1. Identify each of the following as geometric or arithmetic, write down a general formula for the nth term u_n and the nth partial sum S_n

   (a) $3 + 9 + 15 + \cdots$
   arithmetic with first term $a = 3$ and common difference $d = 6$.
   $u_n = 3 + 6(n-1) = 6n - 3$ and $s_n = \dfrac{(2(3) + 6(n-1))n}{2} = 3n^2$.

   (b) $3 + 9 + 27 + \cdots$
   geometric sequence with first term $a = 3$ and common ratio/growth factor $r = 3$.
   $u_n = 3(3)^{n-1} = 3^n$ and $s_n = \dfrac{3(3^n - 1)}{3-1} = \frac{3}{2}(3^n - 1)$.

2. Consider the following diagrams showing the area added to a seed square at each generation in an accumulated growth process.

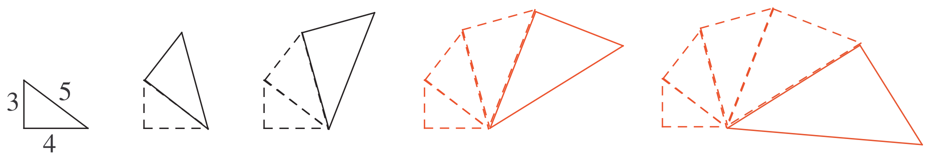


   (a) Draw the next two figures in the series

   (b) Write down a sequence showing the area added each generation.
   1,5,9,13,17

   (c) By finding the sum of the sequence write down a formula for the total area of the figure in the nth generation
   We want to find $1 + 5 + 9 + 13 + 17 + \cdots$. This is the sum of an arithmetic sequence with $a = 1$ and $d = 4$.
   So $s_n = \frac{(2 + 4(n-1))n}{2} = \frac{(2 + 4n - 4))n}{2} = \frac{(4n-2))n}{2} = (2n-1)n$. Which is the area of a rectangle with width n and length $2n - 1$, as required.

3. Consider the following model for accumulated growth of a shell



   (a) Draw the next two figures in the series

(b) Write down a sequence showing the area added each generation.

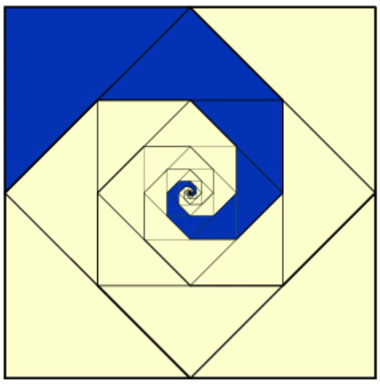
The area of the first triangle is 6 square units. Since the lengths grow with scale factor $\frac{5}{4}$ then the area must grow with growth factor $r = (\frac{5}{4})^2 = \frac{25}{16}$. Hence the area added follows the sequence $6, 6(\frac{25}{16}), 6(\frac{25}{16})^2, \cdots$.

(c) Write down a formula for the total area of the figure in the nth generation.

We want the sum $6 + 6(\frac{25}{16}) + 6(\frac{25}{16})^2 + \cdots$ up to the nth term.

$$s_n = \frac{6((\frac{25}{16})^n - 1)}{\frac{25}{16} - 1} = \frac{6((\frac{25}{16})^n - 1)}{\frac{9}{16}} = \tfrac{32}{3}((\tfrac{25}{16})^n - 1)\ .$$

4. The following figure shows an infinite sum of triangles forming a spiral. If the area of the square is 1, write down an infinite geometric series representing the area of the spiral and find the sum of that series.



The area of the dark spiral is made up of triangles with area $\frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \cdots$. So we want to find the infinite sum $s_\infty = \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$ which is a geometric series with $a = \frac{1}{8}$ and $r = \frac{1}{2}$. Thus

$$s_\infty = \frac{\frac{1}{8}}{1 - \frac{1}{2}} = \frac{1}{4}$$

.