1. (a) When trees grow the radius, $r$, of the branches do not grow in direct proportion with their length, $L$. Longer branches tend to be disproportionately wider than shorter branches. Why do you think this is?
   Since the strength of the branch depends on the cross sectional area, but the mass depends on the volume, a branch which is disproportionately wider will be relatively stronger.

   (b) The square of the radius of a branch is directly proportional to the cube of the length. Express this result as a proportionality, in the form $r \propto L^p$ where you should specify the exponent $p$.
   Since $r^2 \propto L^3$, then $r \propto L^{3/2}$.

   (c) If a branch grows to twice its original length, by what factor has the radius of the branch increased?
   Since the scale factor for length is 2, the scale factor for $r$ is $2^{\frac{3}{2}} = 2.83$.

   (d) Assuming the density of wood is constant, by what factor has its mass increased? Assume a cylindrical branch.
   Mass, $M$, is proportional to volume $V$. Since $V \propto r^2 L$ and $r^2 \propto L^3$, it follows that $M \propto L^3(L) = L^4$. Thus the mass has increased by a factor of $2^4 = 16$.

2. The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, where $r$ is the radius of the base and $h$ is the height. What happens to the volume of a cone if it grows such that

   (a) the radius is doubled but the height remains the same?
   If height remains the same then $V \propto r^2$. So if $r$ is increased by scale factor 2 then $V$ increased by scale factor $2^2 = 4$.

   (b) the height is doubled but the radius remains the same?
   If radius is fixed, then $V \propto h$ so if $h$ is doubled so is $V$.

   (c) both the height and radius are doubled?
   If both $h$ and $r$ are doubled, then $V$ increases by a factor of $2^3 = 8$.

   (d) Suppose a cone grows in such a way that the volume has increased by a factor of 10, but the radius has only doubled. By what factor has the height increased?
   Suppose $h$ grows by scale factor $a$. Then since $V \propto r^2 h$, and $r$ grows by scale factor 2 and $V$ by scale factor 10, we get the equation $10 = 2^2 a$. Thus $a = 10/4 = 2.5$.

3. In the book *Gulliver's Travels*, by Jonathan Swift, the main character, Gulliver, encounters a number of different races of people, including the tiny Lilliputians and the giant Brobdingnagians. In Part II, He describes the people of Brobdingnag as "a comely Race of People" and "very well proportioned", which we can presume means identical in shape to Gulliver, except much taller. Gulliver estimates the farmer to be "as tall as an ordinary Spire-steeple". Assume this is about 60 feet – about 10 times Gulliver's height of 6 feet.

   (a) If Gulliver is 160 lbs, how heavy do you expect the farmer to be?
   Assuming geometrically similar shape, then weight is proportional to the cube of the height. Since height is scaled by factor 10, mass is scaled by factor $10^3 = 1000$. Hence the farmer is $160 \times 1000 = 160,000$ lbs.

   (b) Suppose that Gulliver's thighs are roughly circular with a diameter of about 6 inches. What do you expect the diameter of the farmer's thighs to be?
   Since we assume similar shape, the diameter of the farmer's thighs will be 10 times larger or 60 inches.

   (c) Human legs are built with a certain safety factor so that together they can support up to 4 times the weight of the human body. How much weight can Gulliver's legs support?
   Four times is weight or 640 lb

   (d) Assuming the strength of the leg (ie the weight it can support) scales in proportion to the cross sectional area of the leg, how much weight would the Brobdingnag farmer's legs be able to support? What would happen to the Brobdingnag farmer.
   Since area scales with the square of the diameter, then the cross sectional area would be $10^2 = 100$ times as large and so the strength would be 100 as large times. Hence the farmer's legs would support 100 times what Gulliver's could support, or 64,000 lb. Hence the farmer's legs would not be able to support his own weight.

   (e) What would the diameter of the farmer's leg have to be in order to support his weight with the same safety factor as for the human body?
   Suppose the diameter of his legs increased by a factor of $a$ instead of 10. Then the area would increase by factor $a^2$. So his legs would support $a^2 \times 640$ lbs. We want the his legs to support 4 times his weight of 160,000 lb, or 640,000 lbs. Thus $a^2 \times 640 = 640000$. Which is true if $a^2 = 1000$, so $a = 31.6$. His legs would need to be 31.6 times wider than Guliver's, or about 190 inches.