1. Write down the equation of the following spirals and describe them as logarithmic or Archimedean

   (a) A spiral that starts with a radius of 10 cm and grows by 2 cm every revolution.
   This is an Archimedean spiral because the radius increases by a constant amount each revolution. Therefore,

$$r = 10 + 2\frac{\theta}{360} = 10 + \frac{\theta}{180}\ .$$

   (b) A spiral that starts with a radius of 2 cm and grows by a factor of 3 every half turn.
   This is a logarithmic spiral because it grows by a constant factor every 180 degrees. Therefore,

$$r = 2(3)^{\frac{\theta}{180}}\ .$$

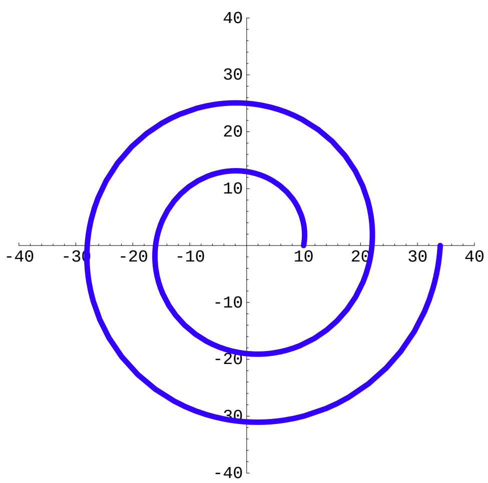
   (c) A spiral that starts at 2 cm and grows by 50% every 90^o.
   Since the spiral grows by 50% every 90^0, the radius grows according to a geometric sequence $2, 3, 4.5, \cdots$ which growth factor 1.5. Therefore,

$$r = 2(1.5)^{\frac{\theta}{90}}\ .$$

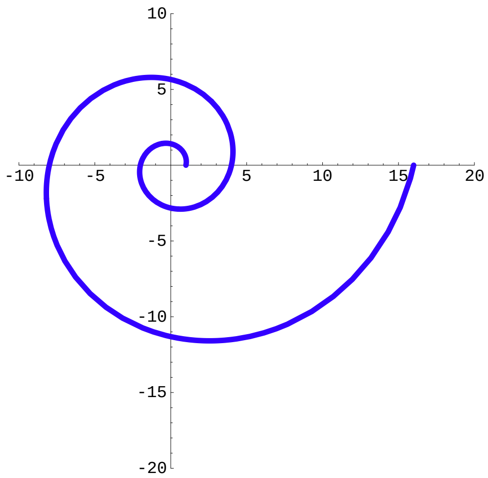
   Note: If the radius grows by x% then the growth factor is $1 + \frac{x}{100}$.

2. The mathematical expressions for the spirals we have discussed are examples of polar functions. Such functions give the coordinates of a point on the plane by specifying a radius, r and and angle θ, rather than the traditional cartesian coordinates x and y. Polar functions are plotted using a polar coordinates system and often reveal interesting graphs. Each member of your workgroup should choose one of the following functions to plot. Make a table of values for the radius r and the angle θ. Start with $\theta = 0^o$ and go up to at least 720^o in increments of 30^o. Plot the points on the polar graph paper provided and connect the points in sequential order. (Note: In some cases you will get a negative value for r. In such cases plot the point in the opposite direction to that indicated by the angle θ.)

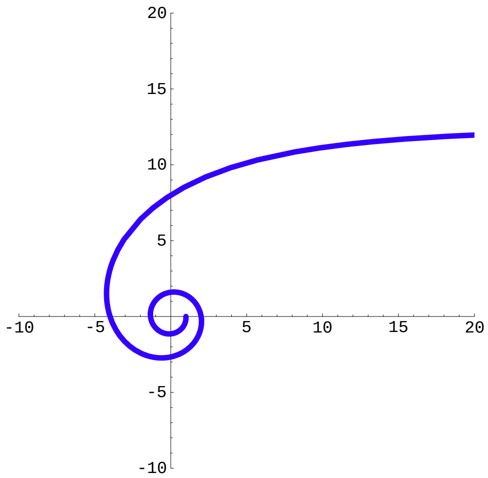
   (a) $r = 10 + \theta/30$. Archimedean Spiral



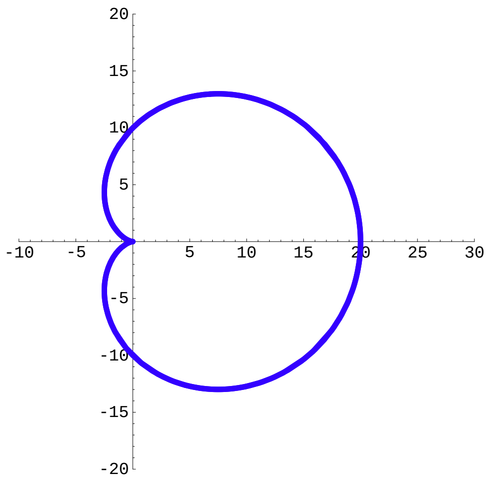
(b) $r = 2^{\theta/180}$. Logarithmic spiral



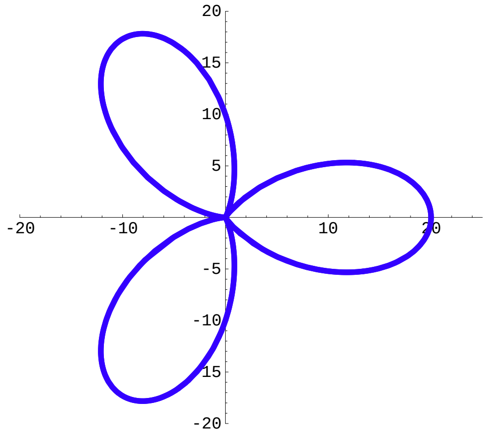
(c) $r = 720/\theta$ and $\theta \neq 0$. Hyperbolic spiral



(d) $r = 10 + 10\cos\theta$. This curve is called a cardioid. What does it remind you of?



(e) $r = 10 + 10\cos 3\theta$. This curve is called a trifolium. What does it remind you of?



Sketch each function plotted by your group members on the back of your graph paper. Describe them with appropriate mathematical language.

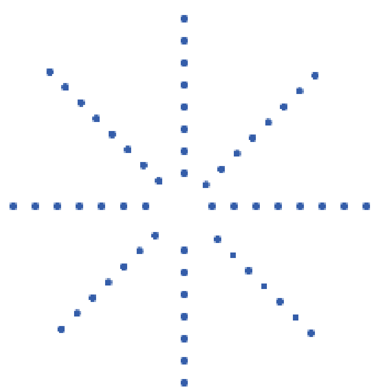
Spiral phyllotaxis is a characteristic arrangement of leaves, petals, scales or seeds that is seen in a variety of plants including daisies, sunflowers, pine cones and cauliflower. The cause of this beautiful natural pattern has only recently been explained fully. The explanation involves an interesting mix of mathematics, biology and physics. At the growth tip of a plant (called the **meristem**) small protusions called **primordia** emerge at regular intervals and move away radially from the center. Eventually these primordia go on to develop into various features of the plant such as its petals or seeds. In this workshop you will investigate how the primordia arrange themselves into a characteristic spiral pattern and why the number of spirals is so often a Fibonacci number.

1. One model of growth for the primordia is to assume that each primordium emerges from the meristem at a fixed angle relative to the previous primordium. The angle separating neighbouring primordia is called the **divergence angle**. In this activity we will try out various divergence angles to see which gives the most efficient packing. We will do this by plotting points (or preferably coloured dots) representing primordia on polar graph paper. Plot the first dot at $\theta = 0$ and $r = 5$. For each subsequent point increase θ by the chosen divergence angle and increase r by one unit. **Do not connect the dots.** The spiral that is formed by connecting the dots together in the order they are plotted (i.e. in the order that the primordia grow) is called the **generating spiral** However, we are not interested in the generating spiral, but rather in the spirals or lines that emerge as our eyes make connections between nearest neighbour dots. These spirals are called **parastichies**.

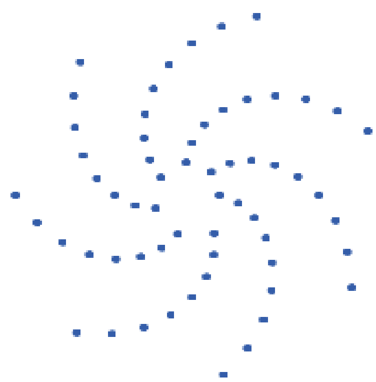
   (a) Each member of the group should choose a different one of the following divergence angles, 45^o, 90^o, 120^o, 135^o and 150^o. Plot enough points so that the parastichies become evident (you will need as many as 36 in some cases) and in pencil connect the dots along each parastichy.

   (b) Now each member of the group should repeat the exercise above but choose an angle that is 5^o larger than the one they chose at first.

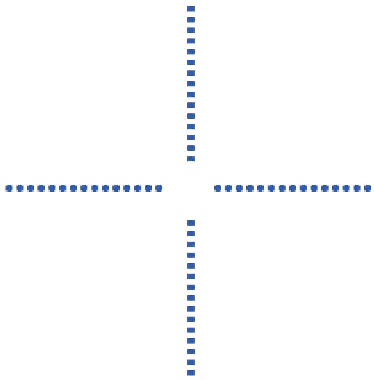
   (c) Finally everyone should repeat the exercise using a divergence angle of 137.5^o (it may be helpful to measure out this angle once on a triangular wedge to use as a template). You will likely need 36 or more primordia to see the parastichies.



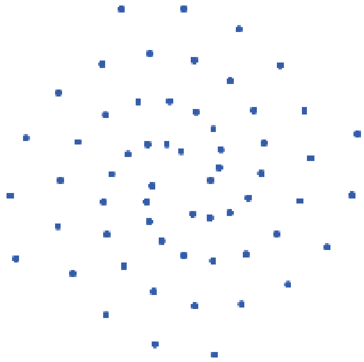
45^{o}



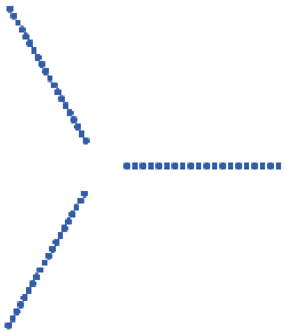
50^{o}



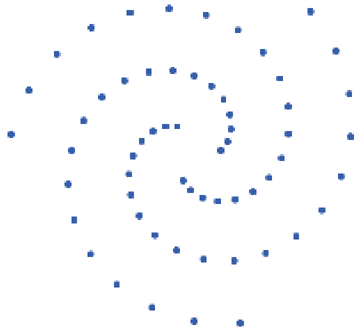
90^{o}



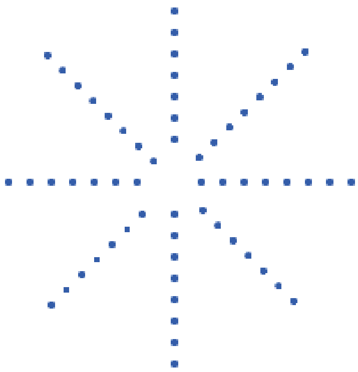
95^{o}



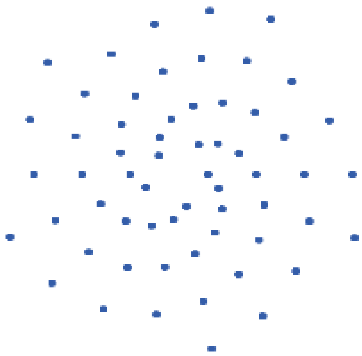
120^{o}



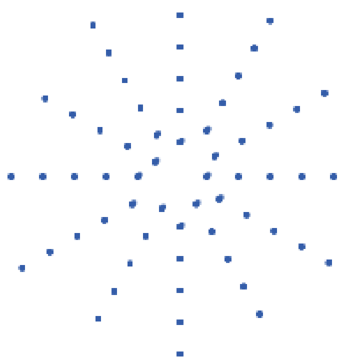
125^{o}



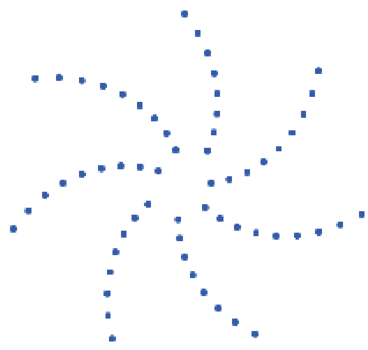
135^{o}



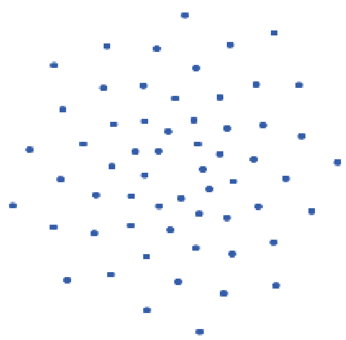
140^{o}



150°



155°



137.5°

2. Draw a rough sketch of each pattern of parastichies created by the members of the group. Record the divergence angle for each diagram carefully, and make sure you accurately represent the number and shape of the parastichies as you will use these results in your homework assignment.