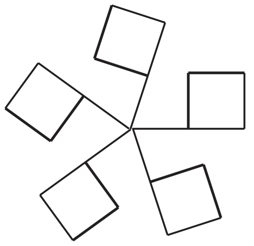
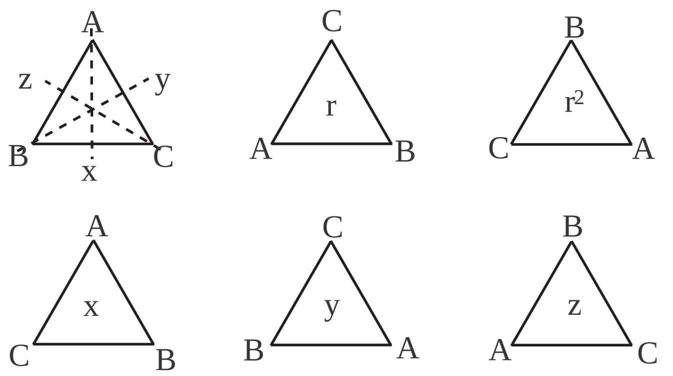
1. Create a group table for the symmetries of the figure below What is its symmetry group called?



The figure has five rotational symmetries and the symmetry group is C_5. Let r be a rotation by 72^o. Then rotation by 144^o is r^2, rotation by 216^o is r^3, and 288^o is r^4. Let I be the identity (or rotation by 0^o). The group table is:

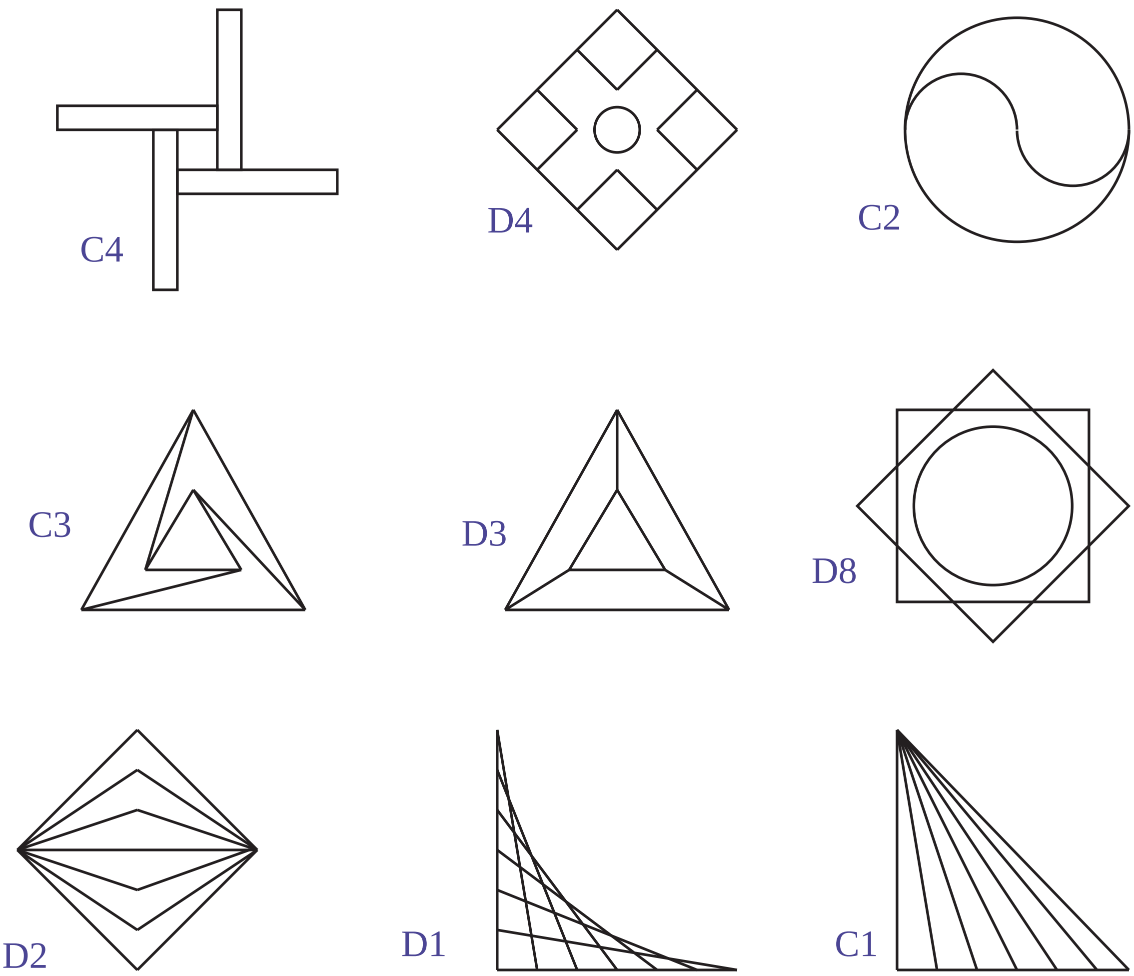
	I	r	r^2	r^3	r^4
I	I	r	r^2	r^3	r^4
r	r	r^2	r^3	r^4	I
r^2	r^2	r^3	r^4	I	r
r^3	r^3	r^4	I	r	r^2
r^4	r^4	I	r	r^2	r^3

2. Create a group table for the symmetries of the equilateral triangle. It will be helpful to cut out such a triangle and label its vertices A,B,C on both sides. What is its symmetry group called? The symmetries of the equilateral triangle are r and r^2, which represent rotations by 120^o and 240^o respectively, and x, y, z, which represent 3 reflection symmetries as shown below in the first diagram on the right. Using these diagrams we can construct the group table on the right. This is the symmetry group D_3.



	I	r	r^2	x	y	z
I	I	r	r^2	x	y	z
r	r	r^2	I	y	z	x
r^2	r^2	I	r	z	x	y
x	x	z	y	I	r^2	r
y	y	x	z	r	I	r^2
z	z	y	x	r^2	r	I

3. What are the symmetry groups of each of the following figures? What is the total number of symmetries in each case? Describe the symmetries which generate each symmetry group?



The symmetry group C_n has no reflection symmetries and n rotation symmetries with rotation angle multiples of $360^o/n$. The symmetry group D_n has n reflection symmetries and n rotation symmetries with rotation angle multiples of $360^o/n$.

4. Count the symmetries of each of the figures below. Now modify each of the figures so as to reduce the symmetry to a smaller symmetry group (this may not always be possible). Try to avoid breaking all the symmetry. Name the new symmetry group you have created and write down how many symmetries it has. The new symmetry group is called a **subgroup** of the original symmetry group. Only certain subgroups are possible. By examining the examples you and the members of your group have created find a rule that will tell you what how many symmetries a subgroup of a group with n symmetries can have.
The first diagram has symmetry group C_4 for a total of 4 symmetries. The second diagram has symmetry group D_5 for a total of 10 symmetries. The last diagram has symmetry group D_6 for a total of 12 symmetries. Examples of symmetry breaking are given on the next page. The symmetry group C_4 has possible subgroups C_2 and C_1, with 2 and 1 symmetries respectively. The symmetry group D_5 has possible subgroups C_5, D_1 and C_1, with 5, 2, and 1 symmetries respectively. These are factors of 10. The symmetry group D_6 has possible subgroups C_6, D_3, C_3, D_2, C_2, D_1 and C_1 with 6, 6, 3, 4, 2, 2 and 1 symmetries respectively. These are all

factors of 12. Based on these examples, it would appear that the number of symmetries of subgroups of a group with n symmetries is some factor of n.