

DESIGNING CONCEPTUAL WORKSHOPS FOR SMALL GROUPS IN THE CLASSROOM

[BASEBALL VERSION]

June 25, 1996

Don Finkel
The Evergreen State College

You'll be working in groups of four. The workshop alternates between individual and small group activities. The groups will be formed based on your preference for thinking about baseball, the family, or money during Step 2 of the workshop.

- 1:30
1. Introduction (20 minutes):
 - a. Read STEPS TO DEVELOPING A WORKSHEET
 - b. Introductory remarks
 - c. Questions on the point of view, the worksheets in the Worksheet Sampler, or STEPS.
 2. Group activity (30 minutes): Conceptual Analysis of Baseball.
- 2:00
1:50

Alphonso, an exchange student from Moravia, is living with you for six months. He has become fascinated with trying to understand the game of baseball. The three main sports in Moravia are soccer, tennis, and tag (which is played in a series of intricate versions). You and Alphonso have watched baseball games together on TV, but he consistently misunderstands the game, because he relates everything to the three sports he knows. So you undertake a more systematic approach to teaching Alphonso about the game of baseball.

As you begin the instructional sessions, you decide to think through how to do it. Before getting caught up in the rules, strategies, or subtleties of the game, it is a good idea to get clear on a few central concepts around which Alphonso can build his understanding of the game. These concepts will form the core of your teaching; they are the objects you set your sights on.

Describe a network of concepts that are central to an understanding of baseball. This network should consist of a small number of interconnected concepts (4-8) and should provide an underlying framework for understanding. However, don't spend time deciding which concept is THE most important one. On the other hand, you should be able to tell how the concepts are related. As you think of the concepts, you might think of critical moments in the game that are described in terms of them.

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Designing Conceptual Workshops - 2

Each person should write down the set of concepts your group formulates as central to an understanding of baseball.

2:30
~~2:20~~

3. Individual activity (20 minutes): Conceptual Analysis in Your Own Subject.

Choose one course you teach regularly. Choose one topic, issue, or segment of that course in which you have a particular interest and which seems to you to have a conceptual structure underlying it. Your job now is to do the same kind of conceptual analysis for this topic, issue, or segment that you did in Step 2 for baseball: Describe on paper a network of concepts that are central to an understanding of your selected area of your subject. If you have trouble getting started, spend a few minutes following the instructions for "Recalling Puzzling Scenes" in STEPS TO DEVELOPING A WORKSHEET, and then use these scenes to get you to your central concepts.

2:50
~~2:40~~

4. Group activity (20 minutes): Describing Student Conceptions in Baseball.

The following are comments made by Alphonso, the exchange student from Moravia, while watching his first baseball game.

- a. I can tell that the man with the bat is trying to score a goal by hitting the ball into the scoring area -- which is between the two yellow posts and into the seats where the audience sits. The other team tries to get the ball to the opposite end of the field by throwing rather than hitting, but I still don't see where the scoring area for them is.
- b. What is the point of all this running that the batting team does? They stay on and leave the field at the most peculiar times. They seem to be trying to get possession of the ball from the other team, but they don't always seem to have a bat to hit it with. They always run towards the ball; they always seem to know where the thrower is going to pass it off, even before he throws it, but they never seem to take it away. And they never run towards the pitcher, even though the ball always ends up being thrown to him.
- c. The throwing team runs around a little, but not much. Some are bunched and others are spread out. Why don't they cover the field more evenly?
- d. Some of the time everybody is in such a hurry to throw the ball or run, and other times they are so casual in both throwing the ball and strolling around the field.

Designing Conceptual Workshops - 3

- e. What is the pitcher doing for so long each time before he throws the ball? Is he calculating strategy, trying to reach a decision, or what? Can he throw the ball anytime he wants?
- f. Why is there no public clock to indicate when periods are over? How do they know when to switch without having a whistle or anything? And how long are the periods anyway?

Though you won't have time to do a thorough job in this brief step, formulate the conceptions that underlie the Alphonso's remarks above. These conceptions represent the his current level of understanding of baseball before you, his teacher, get to do anything.

- 3:10
~~3:00~~
- 5. Individual activity (20 minutes): Describing Student Conceptions in Your Own Subject.

Drawing on your experience with former students in your field, describe in writing the "average" students' conceptions which underlie their understanding of the topic, issue, or segment you selected to think about in Step 3 above; these are the conceptions they bring with them before you ever get to teach them anything about this area. If you get stuck, write down questions you might pose to students in order to uncover these conceptions. (Refer to the STRUCTURAL QUESTIONS handout for guidance.)

3:30
BREAK - 10 MINUTES

- 3:40
3:30
- 6. Paired activity (~~30~~²⁰ minutes): Getting Started Collaborating on Writing a Worksheet.

Experience has shown that the best way to write a worksheet is for two teachers to collaborate: one who is the subject matter expert and another who is the "naive but intelligent" partner. We will use this method today. Decide which of the two roles you would like to take on, and then find a partner to work with, if possible from your own small group. Remember, each pair will work on a worksheet for only one of its members, the subject matter expert. However, you will learn just as much about designing workshops, perhaps more, by taking on the other role.

Designing Conceptual Workshops - 4

a. (20 minutes): Subject matter expert: Drawing on your notes from Steps 3 and 5, explain to your partner the network of concepts in the area you were focusing on, and then the accompanying student conceptions which pertain to this area.

Partner: Ask questions of clarification until you are satisfied you understand the concepts and conceptions in question.

4:00
~~3:50~~

b. (10 minutes): Together, formulate a conceptual goal for a two hour workshop on the topic, issue, or segment. Get clear on and write down what you will be trying to accomplish conceptually during these two hours. In general, the task will be to *move some part of the students' conceptions in the direction of a central concept* such that this movement represents an advance in understanding. Be as specific as you can in formulating this conceptual goal, and don't be overambitious in imagining what you can accomplish in two hours.

10
4:00

c. (40 minutes): Re-read Step IV of STEPS TO DEVELOPING A WORKSHEET and read STRUCTURAL QUESTIONS. Then start composing the worksheet. All you can hope to do in this time period is to invent a Concrete Context and to begin a series of questions that will enable the student to become engaged with and puzzled by the scene or situation described in the Context. This is a lot to accomplish, and if you get this far, continuing later on your own should not be too difficult.

50
4:40

7. Final Discussion and Questions (¹⁰~~20~~ minutes)

5:00 end.

DESIGNING CONCEPTUAL WORKSHOPS FOR SMALL GROUPS IN THE CLASSROOM

[MONEY VERSION]

June 25, 1996

Don Finkel
The Evergreen State College

You'll be working in groups of four. The workshop alternates between individual and small group activities. The groups will be formed based on your preference for thinking about baseball, the family, or money during Step 2 of the workshop.

1. Introduction (20 minutes):
 - a. Read STEPS TO DEVELOPING WORKSHEET
 - b. Introductory remarks
 - c. Questions on the point of view, the worksheets in the Worksheet Sampler, or STEPS.
2. Group activity (30 minutes): Conceptual Analysis of money.

Imagine that you are planning a course for Junior High School students in which the concept of money is to be studied. The students are likely to see money as the same thing as cash, and are unlikely to see the many functions performed by money in a complex society such as ours, or even a relatively primitive society. You want to give them a much wider, more integrated view of what money is and the purposes it serves.

Describe a network of concepts that is central to a wider understanding of money. These need not be (in fact, should not be) the concepts that economists would use. They should be the concepts that help you as an adult understand the concept of money better than the typical Junior High School student. This network should consist of a small number (4-8) of interconnected concepts which help explain the various forms money can take and how it works in an economy. However, do not try to decide which concept is THE most central one. On the other hand, you should be able to tell how the concepts are related and give some examples of key aspects of money that are explained by them. Each person should write down the set of concepts your group formulates as central to an understanding of money.

3. Individual activity (20 minutes): Conceptual Analysis in Your Own Subject.

Choose one course you teach regularly. Choose one topic, issue, or segment of that course in which you have a particular interest and which seems to you to have a conceptual structure underlying it. Your job now is to do the same kind of conceptual analysis for this topic, issue, or segment that you did in Step 2 for money: Describe on paper a network of concepts that are central to an understanding of your selected area of your subject. If you have trouble getting started, spend a few minutes following the instructions for "Recalling Puzzling Scenes" in STEPS TO DEVELOPING A WORKSHEET, and then use these scenes to get you to your central concepts.

4. Group activity (20 minutes): Describing Student Conceptions of money.

The following are paraphrases of statements made by a twelve-year old child about money.

- a. We can't use cans of mushroom soup for money, because not everyone likes mushroom soup, and so you would always have to bargain with it so that would be trading, and it wouldn't be buying. A dollar bill is worth a dollar, and everyone always knows it.
- b. If an Indian tribe traded, and they wanted to have money instead, then they would have to have a government that made money. Then their money could be almost anything they wanted.
- c. If they WANTED to have money, then the Indians in the Pacific Northwest couldn't use pine cones, because there are too many pine cones. The Indians in the desert could use pine cones, because there are not too many pine cones in the desert. But then maybe in some tribes you could use skins for money, if not that many Indians in the tribe hunted.
- d. When the Indians used rabbit skins to get other things, it wasn't money, because rabbit skins could be used to keep them warm, and you would want them more in the winter than the summer. They couldn't make wagon wheels into money either. We can't use our money for anything.

Though you won't have time to do a thorough job in this brief step, formulate the conceptions that underlie the child's remarks above. These conceptions represent his current level of understanding of money before you, his teacher, get to do anything.

5. Individual activity (20 minutes): Describing Student Conceptions in Your Own Subject.

Drawing on your experience with former students in your field, describe in writing the "average" students' conceptions which underlie their understanding of the topic, issue, or segment you selected to think about in Step 3 above; these are the conceptions they bring with them before you ever get to teach them anything about this area. If you get stuck, write down questions you might pose to students in order to uncover these conceptions. (Refer to the STRUCTURAL QUESTIONS handout for guidance.)

BREAK - 10 MINUTES

6. Paired activity (60 minutes): Getting Started Collaborating on Writing a Worksheet.

Experience has shown that the best way to write a worksheet is for two teachers to collaborate: one who is the subject matter expert and another who is the "naive but intelligent" partner. We will use this method today. Decide which of the two roles you would like to take on, and then find a partner to work with, if possible from your own small group. Remember, each pair will work on a worksheet for only one of its members, the subject matter expert. However, you will learn just as much about designing workshops, perhaps more, by taking on the other role.

a. (20 minutes): Subject matter expert: Drawing on your notes from Steps 3 and 5, explain to your partner the network of concepts in the area you were focusing on, and then the accompanying student conceptions which pertain to this area.

Partner: Ask questions of clarification until you are satisfied you understand the concepts and conceptions in question.

b. (10 minutes): Together, formulate a conceptual goal for a two hour workshop on the topic, issue, or segment. Get clear on and write down what you will be trying to accomplish conceptually during these two hours. In general, the task will be to move some part of the students' conceptions in the direction of a central concept such that this movement represents an advance in understanding. Be as specific as you can in formulating this conceptual goal, and don't be overambitious in imagining what you can accomplish in two hours.

c. (40 minutes): Re-read Step IV of STEPS TO DEVELOPING A WORKSHEET and read STRUCTURAL QUESTIONS. Then start composing the worksheet. All you can hope to do in this time period is to invent a Concrete Context and to begin a series of questions that will enable the student to become engaged with and puzzled by the scene or situation described in the Context. This is a lot to accomplish, and if you get this far, continuing later on your own should not be too difficult.

7. Final Discussion and Questions (20 minutes)

DESIGNING CONCEPTUAL WORKSHOPS FOR SMALL GROUPS IN THE CLASSROOM

[FAMILY VERSION]

June 25, 1996

Don Finkel
The Evergreen State College

You'll be working in groups of four. The workshop alternates between individual and small group activities. The groups will be formed based on your preference for thinking about baseball, the family, or money during Step 2 of the workshop.

1. Introduction (20 minutes):
 - a. Read STEPS TO DEVELOPING WORKSHEET
 - b. Introductory remarks
 - c. Questions on the point of view, the worksheets in the Worksheet Sampler, or STEPS.
2. Group activity (30 minutes): Conceptual Analysis of "the family."

You are a member of a planning team for an interdisciplinary course on the Family, to be taught to college freshmen. Among the other team members are a sociologist, a psychologist, an economist, a historian, and an anthropologist. You all agree that the students will have a strong notion of what a family is, based on their own experience of growing up in one, and on images of the family depicted in the mass media (especially television programs). You agree that you want to considerably widen your students' views, but that you want to do it in such a way that they will be able to rethink their own childhood family experiences (and their TV watching) from these wider perspectives.

Describe a network of concepts that is central to a wider understanding of the family. This network should consist of a small number of interconnected concepts (4-8) which help explain the many functions performed by the family in all societies. However, don't spend time deciding which concept is THE most important one. On the other hand, you should be able to tell how the concepts are related, and give some examples of key aspects of family life that are explained by them. Each person should write down the set of concepts your group formulates as central to an understanding of the family.

Designing Conceptual Workshops - 2

3. Individual activity (20 minutes): Conceptual Analysis in Your Own Subject.

Choose one course you teach regularly. Choose one topic, issue, or segment of that course in which you have a particular interest and which seems to you to have a conceptual structure underlying it. Your job now is to do the same kind of conceptual analysis for this topic, issue, or segment that you did in Step 2 for the family: Describe on paper a network of concepts that are central to an understanding of your selected area of your subject. If you have trouble getting started, spend a few minutes following the instructions for "Recalling Puzzling Scenes" in STEPS TO DEVELOPING A WORKSHEET, and then use these scenes to get you to your central concepts.

4. Group activity (20 minutes): Describing Student Conceptions of the family.

These are the views of "the family" as given by a 17-year old high school senior.

- a. Your family's job is to take care of you until you're old enough to make it on your own. Sometimes, though, parents try to hold onto their children too long. They should let them go--when the children are ready. Otherwise children end up by resenting their family.
- b. When I meet the right person, I want to get married, because I think it would be wonderful to spend the rest of my life with the person I love. I want to have children, too. I'm not going to make the same mistakes my parents did. I'm going to make my home a fantastic place for my children to grow up in.
- c. A family should be the place you can always go to when you're in trouble or need help, no matter how old you are. You don't need to spend that much time with your family (once you're not a kid anymore), but it's important to know it's always there if you need it.
- d. Parents shouldn't try to impose their values on their kids. Times change too fast, and what was right for them growing up doesn't always work in today's world. Let the kids work it out for themselves. Families would be so much happier if parents didn't try to define and enforce right and wrong for their kids.

- e. There is so much divorce these days, I think it's really tragic. The government is making a big mistake by letting people get divorced so easily. People should work their problems out instead of splitting up.

Though you won't have time to do a thorough job in this brief step, formulate the conceptions that underlie the teenager's remarks above. These conceptions represent his current level of understanding of the family before you, his teacher, get to do anything.

5. Individual activity (20 minutes): Describing Student Conceptions in Your Own Subject.

Drawing on your experience with former students in your field, describe in writing the "average" students' conceptions which underlie their understanding of the topic, issue, or segment you selected to think about in Step 3 above; these are the conceptions they bring with them before you ever get to teach them anything about this area. If you get stuck, write down questions you might pose to students in order to uncover these conceptions. (Refer to the STRUCTURAL QUESTIONS handout for guidance.)

BREAK - 10 MINUTES

6. Paired activity (60 minutes): Getting Started Collaborating on Writing a Worksheet.

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Designing Conceptual Workshops - 4

a. (20 minutes): Subject matter expert: Drawing on your notes from Steps 3 and 5, explain to your partner the network of concepts in the area you were focusing on, and then the accompanying student conceptions which pertain to this area.

Partner: Ask questions of clarification until you are satisfied you understand the concepts and conceptions in question.

b. (10 minutes): Together, formulate a conceptual goal for a two hour workshop on the topic, issue, or segment. Get clear on and write down what you will be trying to accomplish conceptually during these two hours. In general, the task will be *to move some part of the students' conceptions in the direction of a central concept* such that this movement represents an advance in understanding. Be as specific as you can in formulating this conceptual goal, and don't be overambitious in imagining what you can accomplish in two hours.

c. (40 minutes): Re-read Step IV of STEPS TO DEVELOPING A WORKSHEET and read STRUCTURAL QUESTIONS. Then start composing the worksheet. All you can hope to do in this time period is to invent a Concrete Context and to begin a series of questions that **will enable the student to become engaged with and puzzled by the scene or situation described in the Context.** This is a lot to accomplish, and if you get this far, continuing later on your own should not be too difficult.

7. Final Discussion and Questions (20 minutes)

Curricular Innovations To Promote Software Skills

The Evergreen State College
June 23-28, 1996

Designing Conceptual Workshops
Don Finkel
Tuesday, June 25, 1996, 1:30-5:00

Before coming to the workshop, please do the following as preparation:

1. Read "The Design of Intellectual Experience" to get the point of view behind the workshop.
2.
 - a. Read through the seven worksheets in the "Worksheet Sampler" and select the two you find most interesting.
 - b. Study those two worksheets carefully. Try to see how they are put together, how they were designed, what makes them tick.

STEPS TO DEVELOPING A WORKSHEET

Like any creative process, writing a worksheet can be done intuitively and can be done in a variety of ways. Moreover, the process can be entered from a number of points. Outlined below are the main steps in this process. But note in particular that the early steps can be done in almost any order. What is important is that they all be carried out.

I. Recall Puzzling Scenes.

Puzzling scenes are those times when questions lying within a subject spark a natural curiosity in students. This can be seen as energetic confusion, frustration to get at the real answers, or a feeling of paradoxical bafflement.

1. Recall exactly what went on publicly in those puzzling scenes.
2. Try to recreate the Puzzling Scene from the student's point of view. What was the concept involved; how do students see this concept; what kinds of questions would they ask about it, if they could get through some of their confusion?

II. Analyze and Focus Your Central Concept.

Whether you have chosen a Central Concept for your worksheet by recalling Puzzling Scenes or by thinking about the subject matter, you need to have a very precise understanding of the concept in order to work with it.

1. Break the Central Concept down into interrelated sub-concepts and neighboring concepts. This network of sub-concepts and neighboring concepts should provide an underlying framework for the students' understanding of the Central Concept.
2. Either from your own imagination, or by recalling Puzzling Scenes, think of Concrete Contexts that exemplify the particular power of this Network of concepts. These Concrete Contexts should be "testing grounds" for the deeper problems in understanding this Central Concept.
3. Select a Conceptual Goal. Focus down to one particular part of the Network of sub-concepts within the Central Concept and a few important Concrete Contexts, and think about one goal of one organized experience you would hold for your students with respect to this aspect of the Central Concept. What do you want your students to see, do, or understand differently about this Central Concept?

III. Describe Student Conceptions.

You need to be able to figure out approximately how your students think about the Central Concept, its sub-concepts, and the phenomena it is supposed to describe. For this you need to do a kind of "fieldwork."

1. Write some Structural Questions you can informally put to students that will reveal their thinking in this area. (See additional handout on Structural Questions.)
2. Do some "Fieldwork." That is, put your Structural Questions from 1. to some students and elicit some responses. Either take notes on their answers or use a tape recorder to have a clear record of what your students say.
3. Study the students' responses in order to describe Student Conceptions about the concepts.
4. Review and perhaps revise your Conceptual Goal in the light of what you have discovered in 3.

IV. Formulate a Progression of Structural Questions.

These questions will be the skeleton for your worksheet. They require a Concrete Context, as well as several questions about that context which engage your students with your Central Concept and lead them to rework their thinking about it. These questions are a way of promoting the process of disequilibrium, a process that will help them take apart their Student Conceptions and reconstruct new, more adequate conceptions in their place. These questions should have the following effect:

- a. After the students have engaged with the scene, they will be puzzled, tweaked, or curious about the questions, because they mean something to the students.
- b. But their puzzlement will not be quite resolved--they will feel a little off balance.
- c. A tension will be created by the questions that will motivate them to rethink some of their ideas. This tension derives from the partial inadequacy of their own ideas about how to solve the problem.
- d. The Progression of Questions will have enough intrinsic interest that the group will want to work together toward a resolution.

STEPS

1. Formulate the Concrete Context with the Progression of Structural Questions.
2. Try out the Progression of Questions on a group of students. Act as much as possible as observer and take notes.
3. Revise the questions in the light of 2.

V. Write a Plot Outline for a Full Worksheet.

As with writing essays, outlines are not always necessary, but they are usually helpful to the beginner. The Plot Outline will help you articulate a direction for the worksheet, will insure that you have a beginning, a middle, and an end, and will help you articulate specific steps on the way to your goal. Here are some tips:

- a. The worksheet should have three distinct phases characterized by:

ENGAGEMENT: Where the students' own ideas about the concepts are elicited and given a first testing.

PROGRESSIVE EXPLORATION: Where the students' ideas about the concept are further stretched, tested, and thrown into disequilibrium or conflict.

PULLING IDEAS TOGETHER: Where the students' new ideas about the concept are stabilized and integrated with old concepts.

- b. It is helpful in the course of a worksheet to come at a set of ideas from many angles; one should avoid a monotonic or purely linear approach to the development of ideas. It is helpful to use several different contexts for the same concept.
- c. If possible, the worksheet should allow students of diverse levels of understanding to interact with it, each in his or her own way.
- d. The worksheet should not try to accomplish too much. It is better to allow for the serious engagement and working out of ideas, which takes considerable time, than to try to cover a lot of ground.
- e. Try to include a variety of modes of work, so that new energy can be generated as the workshop proceeds.

VI. Write the Worksheet.

You have already begun your worksheet with your Concrete Context and Progression of Structural Questions. Now you will fill out your Plot Outline in detail. As you write, keep constantly in mind both your Conceptual Goal, and your student audience and their Student Conceptions. At each step of the way you will have to ask yourself: How are my students likely to respond to this question? Is this question taking me toward my goal?

1. Write the worksheet.
2. Critique the worksheet with the aid of peers.
3. Revise the worksheet in the light of 2.

STRUCTURAL QUESTIONS

A Structural Question is one which invites the listener to actively use his or her Conceptual System on a particular phenomenon. Sometimes we use such questions to get a clearer view of the listener's Conceptual System, sometimes to change his or her Conceptual System, and sometimes to confirm and crystallize the System. In any event, Structural Questions should depend as little as possible on memorization or matters of fact, convention, or pure opinion. We have found that the one certain way NOT to elicit an individual's Conceptual System is to simply ask for an explicit verbal explanation of the given concept.

To devise Structural Questions you must first focus on one Central Concept and understand the sub-concepts that make it up, and the network of related concepts that surround it. In most cases, the Structural Question will not be felt as a question by the students unless you focus it down to a specific CONCRETE CONTEXT or contexts. Thus, you should also have a list of examples from the field of phenomena which the concept illuminates, particularly those that present particular difficulties as one applies the concepts to them.

The following is a partial list of strategies for devising structural Questions, along with one example of each type of question.

1. **Provide disparate examples of phenomena** for students to classify, order, or organize by the Conceptual System.

"Which, if any, of the following are governments: NATO, a Public Utility District, the Communist Party of the former USSR?"

2. **Provide a counterexample, a borderline case, or an unusual example** for students to examine by means of the Conceptual System.

"Charlie watches football for five or six hours every Sunday of the football season. Would you say that Charlie is 'addicted'?"

3. **Introduce a new factor** into the field of phenomena and have the students trace out the consequences.

"Suppose a law were passed forbidding the use of students' college transcripts for job applications, grad school, medical school, etc.. What would be the effect on colleges?"

4. **Provide an illuminating shift of context** which requires students to carry the structure of the Conceptual System across from one field to another.

"A small group is in many ways like a family, with the group leader playing the role of father. If this is the case, how is the 'mother-role' carried out?"

STRUCTURAL QUESTIONS

5. **Introduce a decisive conceptual distinction or a decisive conceptual connection** and have the students explore the effects of using the distinction or connection on the field of the phenomena.

"What are the differences and connections between the concepts 'career' and 'profession'?"

6. **Apply a transformation to the conceptual system** and have the students trace out the consequences.

"Imagine that, because of illness, a person loses 75% of the functioning of one of the following organs: Stomach, Liver, Kidneys, Pancreas, Intestines. For each of the organs describe the short- and long-term effects on the individual."

A WORKSHEET SAMPLER

SHAKESPEARE AND THE AGE OF ELIZABETH
Fall 1990

WORKSHOP I: The Immortal Bard??

Part I (1 hr. 30 mins.)

Form groups of four. Find a comfortable place to work and begin by introducing yourself to each other. Discuss the following questions and try to agree on answers to them as a group. Write down your group's answers (and your own, if you disagree) in your notebook, so you can refer to them in seminar and in the whole group discussion at the end of the workshop. Appoint someone in the group to keep an eye on the time, so you move through the workshop in the designated time.

A. (15 mins.) Your name is Kathy and you are a student in SHAKESPEARE AND THE AGE OF ELIZABETH. It is the night before the first seminar, and you have just completed reading the first reading assignment, The Taming of the Shrew. You put your book down in disgust, sigh deeply, and say to your roommate: "This play sucks. I hated it. Why would they have us read such sexist junk, anyway?"

Your roommate responds, "It's written by Shakespeare, isn't it? It must be great literature."

You start musing to yourself. Your first reaction is that your roommate is right. You are just a hopeless case--a dumb, uncultured woman.

Then, a few minutes later, you find yourself thinking, "No. This play was written by a white European male, a person completely distorted by the patriarchal history and culture that shaped him. The play is therefore worse than worthless, it is positively destructive. We shouldn't have to spend time reading stuff like this."

1. These two extremely opposite responses to the play are both natural. They present us the alternatives of "rejecting the play" vs. "rejecting the reader" (the reader's responses). What other possible responses, between these two extremes can you think of? List as many as you can. (10 mins.)
2. Which 2 or 3 responses on your list most recommend themselves, i.e., which would you give as advice to our friend Kathy? (5 mins.)

B. (5 mins.) In "Comic Structure and the Humanizing of Kate . . .," John Bean refers to Kate's final speech about the duties of wives as "a speech that has embarrassed generations of critics." (p. 67) Why should a critic, whose job it is, after all, to be critical, to give critical response to literature, possibly be embarrassed by something written by someone else? What is going on here? What are the assumptions affecting "generations of critics"?

C. (40 mins.)

1. You have read four different interpretations of the character Kate, her development, and by implication, what the play is all about. Three were by actresses who played the role, and one was by a scholar (a literary critic). Agree on brief (one paragraph) summaries of each of these interpretations, and write them down. (20 mins.)

2. What was your initial reaction to reading these "differing opinions"? (5 mins.)

3. (15 mins.)

a. Is there a "right" way to play Kate (and interpret the play)?

b. Suppose, magically, we had a videotape of Shakespeare's own original production of The Taming of the Shrew. Would that show us the right way to play Kate?

c. Suppose we were to agree--just for the sake of argument--that there is no one right way. Does this mean all interpretations are equally valid, that any way an actress portrays Kate is fine? Give reasons.

d. On p. 10 of "Kate: Interpreting the Silence," Fiona Shaw says, "People have criticized my Kate for not putting up more of a fight. I'm dying to put up a fight but look at the text - it ain't there!" What does this phrase, "look at the text - it ain't there!" mean? What could be in the text (which is just words) that would tell an actress whether she should portray Kate non-verbally as putting up a fight or not? (Remember that Shakespeare uses very sparse stage directions.)

D. (15 mins.)

1. After reading John Bean's essay, "Comic Structure and the humanizing of Kate," what would you conclude are the kinds of things you, as a reader, would need to know to make sense of the play? List four or five. (5 mins.)

2. After reading the three actresses discussions in "Kate: Interpreting the Silence," what would you conclude are the kinds of things you, as a reader, would need to know to make sense of the play? List four or five. (5 mins.)

3. Discuss the seeming conflict between the two types of things you listed in response to the above two questions. (5 mins.)

E. (10 mins.)

1. In "Kate: Interpreting the Silence," you may have been surprised to notice how differently the three different directors used the Christopher Sly "framing device." Review these differences. (5 mins.)

2. Is it appropriate for directors to take such liberties with the text? Why or why not? (5 mins.)

(3. for seminar--skip over for now: How would you deal with Christopher Sly, if you were the director?)

F. (5 mins.)

Finally, what does The Taming of the Shrew have to offer us today? Why should we read it?

BREAK - 15 minutes

After the break, reconvene in CAB 110 for Parts II and III.

Part II (15 mins.)

Writing individually, generate a list of questions about Shakespeare's work, about the Age of Elizabeth, and about today's "Age," that you would hope to answer--or to make progress answering--by the end of this program in June. These should be questions you are genuinely interested in. Your list may be taken as a first draft of your own personal intellectual objectives for this program. Write as many of these questions as you can.

Then read them over, and circle the two that seem the most important to you today. Be prepared to tell these to the class when we discuss the workshop together in Part III.

Part III (45 mins.)

The whole class will discuss the results of the workshop, with particular emphasis on Part I - A, Part I - F, and on Part II.

GREAT STORIES
Fall 1993

The Presocratics: Workshop I - The Milesians

I. (15 minutes) In answering the following questions, please put aside anything you have learned about science. Imagine that there is no science yet, and that all you have to go on is what you can observe, and the conclusions you might draw about what you observe.

Look carefully at the burning candle. Individually, write down your answers (your intuitions) to the following questions about the candle. Take time to ponder, but don't take too long writing for any one question; you have only 15 minutes to get through them all.

- a. What do you see?
- b. Where does the smoke come from?
- c. Where does the smoke go to?
- d. Is the flame you saw on the match the same flame as the one you see on the candle?
- e. As the candle burns, where does the missing wax go?
- f. Where did the heat come from? Where does it go?
- g. Where does the light come from and where does it go to?
- h. Where did the color of the candle go to? Where did it come from originally?
- i. Do the wax and wick vanish from the universe and are the heat and smoke and light created at just the same time? If so, how do you account for the coincidence in time?
- j. If not, do the wax and wick get transformed into heat, light, and smoke? If so, then must they not be made of a common underlying substance-- something which is neither wax, nor wick, nor smoke, nor heat, nor light, but which is more real than any of them?
- k. If we accept the latter idea, that wax and wick on the one hand and smoke, heat, and light, on the other are all made out of the same substance, what makes them each appear different to us?

II. (60 mins.) Divide up into groups of four. Work with people you don't know. Place your chairs so you form a little circle. Discuss each of the following questions and try to agree on an answer. Appoint a scribe to be responsible for writing down the group's answer to each question. Everyone should take some notes of what is going on; you can draw on the scribe's records afterwards to complete your notes. In addition, have one person keep an eye on the time so your group moves the questions according to schedule.

A. Thales (20 mins.)

1. Thales asserted "The first principle and basic nature of all things is water." (F1, p.44) What could he mean by this? How could he possibly have believed such a thing?

2. If Thales were describing what happens as the candle burns down to "nothing," what might he say?

3. In the practice of interpretation, there is a slogan that says, "Make it good!" This is closely related to what is called "The Principle of Charitable Interpretation." The idea behind "Make it good!" is to assume that what you are reading is sensible, intelligent, and has something important to say, even if on your first or second reading it does not appear to. That is, make the text good yourself by finding what is good in it. Use this principle with Thales' assertion F1. Given your current understanding of nature and the world, what is the partial truth expressed in Thales' assertion? Paraphrase his assertion so that it is both true and important in today's world.

4. Wheelwright says that what is important about the Milesians is not the conclusions they reached, but their new way of asking questions. What question is Thales trying to answer with his assertion above?

B. Anaximander (20 mins.)

1. Anaximander claimed "The Unlimited is the first-principle of things that are. It is that from which the coming-to-be [of things and qualities] takes place, and it is that into which they return when they perish, by moral necessity, giving satisfaction to one another and making reparation for their injustice, according to the order of time." (F1, p. 54)

According to one commentator, Anaximander believed that "the most important forces at work... were what were later called 'the opposites': pairs of opposed entities of which the most frequently invoked were 'the hot' and 'the cold', 'the wet' and 'the dry'. ... The 'opposites' were above all forces, agents of physical change, each present in varying degrees at different places." (E. Hussey, The Presocratics, p. 20) Moreover, as Wheelwright, explains, "Each actually existing thing ... is a usurper; for during the time that it exists it "commits injustice" by preventing its opposite from existing; accordingly it must eventually pay the penalty by yielding up its overt existence and returning to its submerged place in the great qualitative reservoir [the Unlimited]." (p. 53)

What could Anaximander have meant by these assertions?

2. If Anaximander were describing what happens as the candle burns down to "nothing," what might he say?
3. Use the "make it good" principle with Anaximander's assertion F1. Given your current understanding of nature and the world, what is the partial truth expressed in this assertion?
4. What question is Anaximander trying to answer with his **assertion F1?**

C. Anaximenes (20 mins.)

1. According to Simplicius, a later Greek commentator, "Anaximenes ... agreed with [Anaximander] that the essence of things is one and unlimited; on the other hand he declared that it is not indeterminate but that it has the specific nature of air, which differs in rarity and density according to the kind of things into which it forms itself. Rarefied it becomes fire, condensed it becomes wind, then cloud, and as the condensation increases it becomes successively water, earth, and then stones. Everything else gets made out of these." (T6, p. 62)

In what way is air an improvement over water as a candidate for the basic element out of which all things are made?

2. If Anaximenes were describing what happens as the candle burns down to "nothing," what might he say?
3. Use the "make it good" principle with Anaximenes' ideas in T6. Consider the possibility that Anaximenes might be trying to combine what is best in Thales and Anaximander and is presenting an improved set of ideas that overcomes the weaknesses in each of their approaches to explanation. Give an account of T6 that shows its strengths, and if possible, that puts to use the preceding hypothesis.
4. What question or questions is Anaximenes trying to answer with his assertions above?

III. (15 mins.)

1. Have your scribe read aloud your group's answers to the #4 questions for the three philosophers (the questions they were trying to answer). How would you characterize these questions as a whole? What kind of questions are they? How do they differ from questions you might guess, based on The Kojiki, that the early Japanese people were asking about their world?
2. Are these questions, or these kind of questions, ones that Homer could have asked? Explain why or why not.

3. Are these questions, or these kinds of questions, more closely linked to oral-traditional culture or to culture that has felt the impact of the closely-associated technologies of writing and counting? Explain your response.

In the great temple of Benares beneath the dome which marks the centre of the world, rests a brass plate in which are fixed 3 diamond needles, each a cubit high and as thick as the body of a bee. On one of these needles, at the creation, God placed 64 discs of pure gold, the largest disk resting on the brass plate, and the others getting smaller and smaller up to the top one. This is the Tower of Bramah.

Day and night unceasingly the priests transfer the discs from one diamond needle to another according to the fixed and immutable laws of Bramah, which require that the priest on duty must not move more than one disc at a time and that he must place this disc on a needle so that there is no smaller disc below it.

When the sixty-four discs shall have been thus transferred from the needle on which at the creation God placed them to one of the other needles, tower, temple, and Brahmins alike will crumble into dust, and with a thunderclap the world will vanish.

(From W.W. Rouse Ball's "Mathematical Recreations and Essays".)

The above legend gives rise to a puzzle which appears in virtually every computer science textbook (Dale and Lilly, Tanenbaum, and Aho et al *all* have it). The puzzle is called the "Towers of Hanoi". Today we will explore solutions to this puzzle. The purpose of the workshop is to gain familiarity with recursive algorithms and to compare recursive and non-recursive algorithms.

Begin this workshop by dividing into groups of three. Select a scribe to record your group's work. This workshop will last about three hours, including a break.

PART I: Solving the Puzzle by Hand ["Be a Brahmin"] (60 minutes)

Read these directions for Part I, then carry them out EXACTLY:

The first step in this workshop is to solve the Towers of Hanoi puzzle by hand. Since the 64-disc tower would take you a while, we will use a smaller version of the problem: a five disc tower.

To work on the problem, you need to build a model. A convenient model is to make five different sized pieces of paper. Number the smallest piece "1", the next smallest "2", and so on. These are the "discs". Next, draw a triangle on a sheet of paper, and label the corners "A", "B", and "C". These are the three "needles".

Stack the five pieces of paper at "A" (largest on the bottom, then the next largest, etc.) and you're ready to go: Move all the discs to "B", following the laws of Bramah:

- (1) move only one "disc" at a time,
- & (2) a "disc" with a higher number may never be placed on top of a "disc" with a lower number.

When you've solved the puzzle, write down your solution algorithm (that is, explain the steps you followed). Remember: An *algorithm* is a finite sequence of steps, each of which has a clear meaning and can be performed with a finite effort in a finite time. Your algorithm should not be limited to 5-disc towers...

PART II: Solving the Puzzle by Machine ["Program a Brahmin"] (90 minutes)

Read these directions for Part II CAREFULLY, then carry them out:

It is possible that you solved the puzzle by inventing a non-recursive algorithm. At least two such algorithms exist; if you discovered one, be proud!

The point of this workshop, however, concerns recursion. To see an example of a recursive solution, find your workshop leader and get a listing of HANOI.PAS for each member of your group.

Make one copy of the executable file "HANOI.PR" in someone's directory by typing:

```
HANDOUT/c DSGRP HANOI.PR
```

Run this program a few times, trying towers of 1, 3, and 5 discs. You can direct this program's output to the printer LPT1, by typing:

```
XLP DSGRP.HANOI
```

Make printed output for towers of 3 and 5 discs. [NOTE: When you use the "XLP" command, no prompt is displayed for your input. Just type the tower height and press the RETURN key...]

TAKE A TEN MINUTE BREAK...

When you return from your break, use your program listings and runs to do the following:

- a) Satisfy yourself that "HANOI.PR" works, by following its output directions to move your 3 and 5-disc paper towers from peg "A" to peg "B".
- b) Hand-run the program for a tower of 3 discs. One person steps through the program. Another maintains a stack on the blackboard: Each time PROCEDURE "towers" is called, record the values of all the variables and the statement where execution will resume when the procedure returns. The third person moves the paper "discs" as the WRITELNs print the steps in the solution. If you have time, also hand-simulate a 5-disc tower.
- c) Write down the algorithm used by PROCEDURE "towers". How does this algorithm compare with your solution from Part I? Which solution is easier to understand? To carry out by hand?
- d) Estimate Big-Oh for PROCEDURE "towers". Using your estimate, and assuming that a Brahmin can move one disc every second, how long will the world exist, given the famed tower of 64 discs?

PART III: Discussion (30 minutes)

Reform as a large group. Share the algorithms you discovered in Part I and your answers to the questions in Part II.

```

PROGRAM Hanoi (input,output);
{ Generates the series of moves for a Tower of Hanoi }
{ Coded by John Aikin Cushing; 2/10/88 }

```

```

VAR

```

```

  n : integer;

```

```

PROCEDURE towers (n : INTEGER; frompeg, topeg, auxpeg : CHAR);

```

```

  BEGIN

```

```

    IF n = 1 THEN

```

```

      { If the tower consists of only the top disk (that is, #1), }

```

```

      { move it from "frompeg" to "toppeg", and return. }

```

```

      WRITELN( 'move disc 1 from peg ', frompeg, ' to peg ', topeg )

```

```

    ELSE BEGIN

```

```

      { Otherwise move the top n-1 discs from "frompeg" to "auxpeg". }

```

```

      { Note that this level's "toppeg" will be used as the "auxpeg", }

```

```

      { and visa versa in the recursive call... }

```

```

      towers( n-1, frompeg, auxpeg, topeg );

```

```

      { Once the top n-1 discs are out of the way, move the last }

```

```

      { remaining disc from "frompeg" to "toppeg". }

```

```

      WRITELN( 'move disc ', n, ' from peg ', frompeg, ' to peg ', topeg );

```

```

      { Then move the top n-1 discs from "auxpeg" to "toppeg". }

```

```

      { Note that this level's "frompeg" will be used as the "auxpeg", }

```

```

      { and visa versa in the recursive call... }

```

```

      towers( n-1, auxpeg, topeg, frompeg )

```

```

    END; {else}

```

```

  END; {towers}

```

```

BEGIN {main}

```

```

  WRITE( 'How big a tower, O Mighty One? ');

```

```

  READLN( n );

```

```

  WRITELN; WRITELN;

```

```

  WRITELN( 'To perform the Brahmins' task for a tower of ', n);

```

```

  WRITELN( ' discs, you must:');

```

```

  WRITELN;

```

```

  towers( n, 'A', 'B', 'C' )

```

```

END. {main}

```

Sample Output:

How big a tower, O Mighty One?

To perform the Brahmins' task for a tower of 3
you must

move disc 1 from peg A to peg B

move disc 2 from peg A to peg C

move disc 1 from peg B to peg C

move disc 3 from peg A to peg B

move disc 1 from peg C to peg A

move disc 2 from peg C to peg B

move disc 1 from peg A to peg B

For most interesting problems, we can find more than one algorithm that can provide a basis for solving the problem. Much of our purpose in studying data structures and algorithms is to become more skillful at finding GOOD algorithms. A good algorithm can usually be characterized as one which:

- 1) is easy to understand, implement, and prove correct.
- 2) makes efficient use of the available resources: users' time, storage, and CPUtime.

In this workshop we will explore one of the ways in which algorithms can be

characterized, namely the rate at which the CPUtime to complete a task grows with the size of the task. Begin this workshop by forming groups of four and selecting a scribe to record your group's work. This workshop will last about 3 hours.

PART I: Running-time Growth Functions (90 minutes)

The running-time of a program usually depends on its number of inputs. We can express this mathematically by saying that the running-time for a program is given by some function $T(n)$, where "n" is the number of inputs. For example, the running-time of some program might be given by the function $T(n) = 3n^2 + 2n + 1$ (where "*" means multiplication and "^" means exponentiation).

The running-time of a program depends on several factors besides the algorithm on which the program is based. Among these are the speed of the machine and the efficiency of the object code produced by the compiler. Since most of these factors are usually completely outside the control of the programmer, we can almost never predict the actual running-time of an algorithm in seconds or hours simply by looking at the algorithm for the program.

Because of this inability to make exact predictions of running-times, we are mostly interested in the form of the function $T(n)$ rather than its exact expression; or, to state it another way, in the "order" of the function $T(n)$. Intuitively, we are looking for a simple way to characterize how fast $T(n)$ grows as "n" gets bigger.

Using our example of $T(n) = 3n^2 + 2n + 1$, when "n" gets very large, the contribution to $T(n)$ made by the first term ($3n^2$) will swamp the contributions from the other two terms ($2n$) and (1).

Thus for large enough "n", it's accurate to say that $T(n)$ is "of order n-squared", or is "Big Oh of n-squared". Using Big Oh notation, this would be written " $O(n^2)$ ".

To say that a program is "O of n-squared" means that its running time will grow as the square of the number of inputs: If we double the number of inputs, we will quadruple the running time. Knowing Big Oh for an algorithm thus tells us a lot about how it will behave with various inputs and enables us to compare its speed with that of other algorithms even before we code and run the programs.

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Working in your small group, complete the following exercises concerning Big Oh and the growth rates of functions:

1) Suppose $T(n) = 9n^3 + 13n^2 + 3$. Calculate the values to fill in this table:

n	T(n)	n^2	n^3
0			
1			
2			
3			
4			
5			
10			
100			

- Can you find some constant (call it "k") such that $T(n) \leq k n^2$ for all the non-zero values of "n" in the table? What is the smallest such constant "k" you can find?
- Can you find some other constant (call it "c") such that $T(n) \leq c n^3$ for all non-zero values of "n" in the table? What is the smallest such constant "c" you can find?
- Now calculate $T(n)$, n^2 , and n^3 for $n = 1000$ and for $n = 1,000,000$. Add these to the table.
- For these larger values of "n", is it still true
 - that $T(n) \leq k n^2$, where "k" is the constant you found above?
 - that $T(n) \leq c n^3$, where "c" is the constant you found above?
- Do you think that there is ANY constant "k" you could pick such that $T(n) \leq k n^2$ for ALL values of "n" greater than 0?
- Do you think that there is ANY constant "c" you could pick such that $T(n) \leq c n^3$ for ALL values of "n" greater than 0?
- What does the preceding lead you to conclude about Big Oh for $T(n) = 9n^3 + 13n^2 + 3$?
- Formally, to assert that $T(n)$ of some program is $O(f(n))$ means that there exist constants "c" and "n0" such that $T(n) \leq c * f(n)$, for all $n \geq n0$.
 - What are "c" and "n0" for the example function $T(n) = 9n^3 + 13n^2 + 3$, above?
 - In your own words, what does the formal definition of Big Oh mean?

2) Let $T(n) = n(n + 1) - 3$.

a) What is Big Oh for this function?

b) What values of "c" and "n₀" did you choose for this example? Why?

c) *Formally*, could we say that $T(n)$ for this function is $O(n^3)$ or $O(n^4)$? Why wouldn't we want to do this?

3) Compare the functions below by computing an estimate of each of their values for $n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 50, 100, 500, 1000, 1000000$. Make a table showing the value of each function for each value of "n". [To save time, use calculators and divide up the computational work among your group members.]

n^5	$\text{SQRT}(n)$	$\log(n)$
$n \cdot \log(n)$	n	$n!$
17	2^n	n^2

a) Which function has the largest Big Oh (fastest growth rate)?

b) Which has the smallest Big Oh (slowest growth rate)?

c) What is the fastest growing function you can think of?

4) A function, T , of n is said to be a "polynomial in n " if it can be expressed in the form $T(n) = a \cdot n^m + b \cdot n^{(m-1)} + c \cdot n^{(m-2)} + \dots + z$, where "a", "b", "c", ..., "z" are constants (as many as needed) and "m" is some integer (called the "degree" of T).

For any such function, $T(n) \leq (\text{ABS}(a) + \text{ABS}(b) + \dots + \text{ABS}(z)) \cdot n^m$, for all values of n . Put another way, T is "bounded" by some constant times n^m ; that is, $T(n) \leq (\text{some constant}) \cdot n^m$, for all values of $n \geq 1$. A function which is bounded by n^m for some "m" is said to be "polynomial" (here an adjective, not a noun as in the preceding paragraph).

For example, the function $T(n) = 9 \cdot n^3 + 13 \cdot n^2 + 3$ which you saw earlier is bounded by $(\text{ABS}(9) + \text{ABS}(13) + \text{ABS}(3)) \cdot n^3$, or $25 \cdot n^3$.

Some functions are NOT polynomial: There are no "m" and "c" such that $c \cdot n^m$ bounds them. Such functions are called "exponential".

a) Which of the functions in (3) above are polynomial and which are exponential?

b) Algorithms whose running times can be bounded by a power of n are said to require "polynomial time"; those whose algorithms can't are said to require "exponential time". Do you think there is much practical difference between the class of algorithms requiring polynomial time and that requiring exponential time? Why or why not?

TAKE A TEN MINUTE BREAK...

PART II: Calculating the Running-time of a Program (1 Hour)

To calculate Big Oh for an entire program, we must calculate Big Oh for each step in the program, then combine these estimates for the individual steps.

A few simple rules suffice for non-recursive procedures:

- a) Assignment and I/O statements are usually $O(1)$. The exception is where there is a function-call within the statement. In such cases, Big Oh must be estimated for the function called.
- b) For an IF...THEN...ELSE block, Big Oh is the time required to evaluate the condition (usually $O(1)$) plus the time for the slowest alternative. Similarly for CASE statements.
- c) For a loop, Big Oh is the sum of the time to evaluate the loop termination condition PLUS the time to execute the loop body, all TIMES the number of times the loop is executed. If the time to execute the loop body varies, we must use the worst case.
- d) Given a sequence of two program steps, where the running-time for the first is $O(f(n))$ and the running-time for the second is $O(g(n))$, the running-time of the sequence is $O(\text{MAX}(f(n),g(n)))$. Put simply, take the larger of the two Big Oh's as Big Oh for the sequence. This is called the "rule for sums".
- e) Given a running-time of order $O(f(n))$ *within* a step having order $O(g(n))$, the running-time of the whole will be of order $O(f(n)*g(n))$. This is called the "rule for products".

Using these rules do each of the following:

- 1) Without worrying about what the following procedure does, calculate its Big Oh:

```
PROCEDURE whazzit (VAR list: ARRAY[1..n] OF INTEGER;
                  n : INTEGER);
VAR
  i,j,entry : INTEGER;
BEGIN
  FOR i := 1 TO n-1 DO BEGIN
    j := n;
    WHILE j > i DO BEGIN
      IF list[j-1] > list[j] THEN DO BEGIN
        entry := list[j-1];
        list[j-1] := list[j];
        list[j] := entry
      END; {if}
      j := j-1
    END {while}
  END {for}
END; {whazzit}
```

- 2) Satisfy yourselves (that is, give a rough proof) that $T(n) = n(n-1)/2$ is $O(n^2)$.
- 3) Under what conditions is it correct to say that $\text{MAX}(n^3, 10n^2)$ is $O(n^3)$?
- 4) Generalize (3), that is, find the running-time for the program consisting of two sequential program fragments whose individual running-times are $T_1(n) = a \cdot n^i$ and $T_2(n) = b \cdot n^j$, where "a", "b", "i", and "j" are constants and $i < j$.
- 5) Suppose you have two programs that produce the same results. Program 1 takes $50n^2$ seconds to run when compiled and executed on the Wombat 200 computer. Program 2 takes $2n^3$ seconds when compiled and run in the same way on the same machine. (As usual, "n" is the number of inputs.)
 - a) Are there circumstances in which the algorithm for program 2 might be preferred even though it has a higher growth rate (greater Big Oh)?
 - b) What's an easy way to find the range of values of "n" over which each algorithm is fastest?

**WHEN YOU COMPLETE PART II,
RETURN TO THE CLASSROOM FOR DISCUSSION.**

Key Question. You own a print shop, and, in order to attract larger orders, you have devised a sliding price scale. You charge \$15 a ream for an order of one ream (=500 pages), \$14 for each ream for an order of two reams, \$13 for each ream for an order of 3 reams, and so forth. The paper you use costs \$4 a ream.

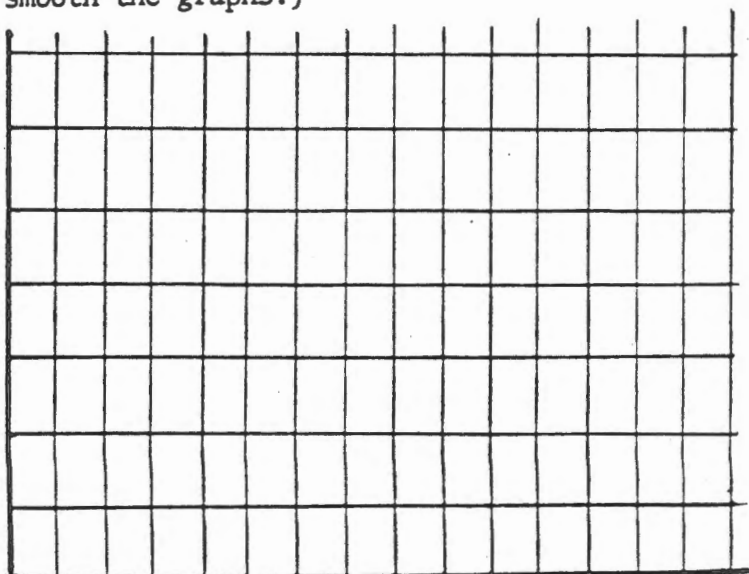
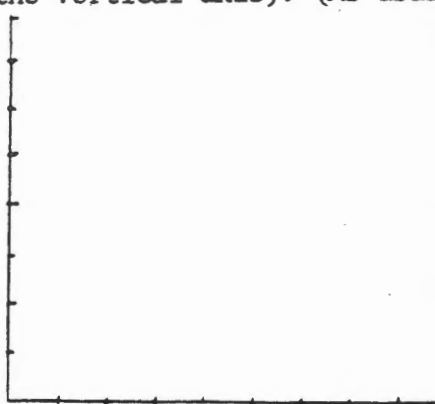
Your partner says that you cannot run a business by charging less for more, and thinks that you will be losing money every time a customer increases his order size. You say that as long as you get more than \$4 a ream, you're still ahead. Who is right? Are both wrong or both right?

1. First deal with your partner's assertion that you are "charging less for more." What is the total bill for an order of 1 ream; for an order of 2 reams; for an order of 3 reams? Is there actually some quantity that is getting smaller as the orders get bigger - other than the price?

2. To explore the Key Question further we need some more systematic tables and graphs. In the table below, q stands for the number of reams in an order, p stands for the price for each ream in the order, and TR stands for the total bill for an order of that size. ("TR" is actually an abbreviation for the economists' term "Total Revenue.") Complete the second and third rows of this table and ignore, for now, the fourth and fifth rows.

q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
p	15	14	13	12											
TR	15	28	39	48				64	63	60	55				
TVC					20	24	28								
<u>P</u>							35	32	27			0			

3. On the axes to the left below plot the graph of p vs q (with p on the vertical axis). On the axes to the right below plot the graph of TR vs q (with TR on the vertical axis). (As usual, smooth the graphs.)



4. Now what do you think of your partner's assertion that you are "charging less for more?" It doesn't look like you want to take a lot of orders for 15 reams, but what about 11 or 13?

5. We have to take into account the cost of the paper, \$4 per ream. What is the cost of 1 ream, 2 reams, 3 reams, etc? We call this new quantity -- the cost of q reams-- TVC (which is an abbreviation for the economists' term Total Variable Cost). Do you agree that the formula for TVC is $TVC = 4q$. Plot the graph of TVC vs q on the same axes as your TR vs q graph. Label the two graphs to keep them straight.

6. Now what would be the largest order you would be willing to take? WHY?

To deal further with the Key Question we need to introduce yet another variable

P = Profit.

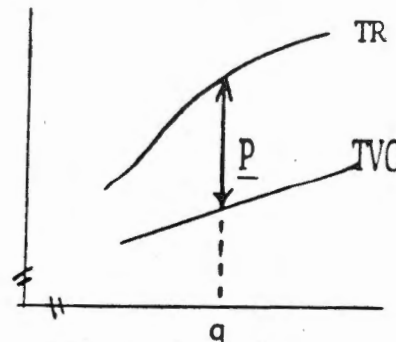
Clearly the profit you make on an order of 3 reams is the \$39 you take in minus the \$12 you pay for the paper. That is

$$P = TR - TVC$$

7. Now go back to the table on the preceding page and fill in the rows labelled TVC and P . (You can ignore all quantities beyond 12.)

8. We could draw on the same set of axes as the TR and TVC graphs, the graph of P vs q . But I claim that it is really "there" in a way. To arrive

a P for a particular value of q , you subtract the corresponding value of TVC from the corresponding value of TR. Do you see that the number you get is the length of the vertical line between the two graphs for that value of q (as in the picture to the right)? Calculate the profit by this graphical method for several values of q and check against the values in your table.



9. Using the TR and TVC graphs, find the quantity q that gives the greatest profit. (This can be hard on your eyes. I find that the best way to do a problem like this is to use a clear plastic ruler and, holding it perpendicular to the horizontal axis, run it back and forth until you find the place where the vertical distance between the graphs is greatest.) Check your answer against your table values.

It seems that you and your partner are probably both right and both wrong. It makes no sense to take an order of 13 reams, because you lose money on it. The best size order is 6 reams, because you make the greatest profit on it. As you take larger and larger orders between 6 and 12, you make less and less money, but you still make money. The odd thing is that your TR hits its peak at 8, but this number is not that significant.

10. Using your old trusty clear plastic ruler on the TR graph, try out the following argument for why Profit is maximized at $q=6$. The slope of the secant line to the TR graph between consecutive values of q (4 & 5, 5 & 6, etc) measures how fast TR is growing. The slope of the TVC graph is always 4. For values less than or equal to $q=5$, the TR graph is steeper than the TVC graph. So the TR graph is "pulling away" and profit is increasing. Starting at $q=6$, the TR graph is less steep than the TVC graph, so the two graphs are coming together, so that profit is decreasing. So, around $q=6$ is the place to be...the profit is greatest there.

In an economics text you would find the following argument for the same phenomenon.

A customer considering an order of 5 or 6 reams is considering spending an additional \$5 for one more ream. It will cost you \$4 for this additional ream. This will increase your profit by \$1 (over what you would have gotten for 4 reams). A customer considering an order of 6 reams or 7 reams is considering spending an additional \$3 for one more ream. It will cost you an additional \$4 for this ream. This will diminish your profits by \$1. You don't want to do that unless you have to.

This notion of additional revenue for one more unit of sales is very important in economics. It is called Marginal Revenue and is defined to be:

Marginal Revenue (MR) is the increase (plus or minus) in Revenue that comes with an increase of 1 unit of sales.

11. Do you see the connection between the economists' argument and the one given about slopes of graphs in question 10?

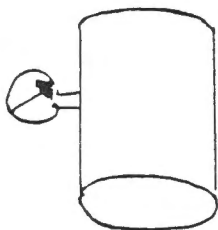
CHEM 101A
February 3, 1976
Meyer

The Ideal Gas Law

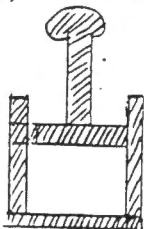
These problems are to be done without the formula. You may use the formula to guide your reasoning if you wish, but give your explanations in your own common sense terms.

We have three devices:

- 1) A given amount of gas (a given number of gas molecules) is put into a steel canister and sealed. The canister has a pressure gauge on its side. This canister keeps the volume constant. We can vary the temperature by immersing it into different vats of water at different temperatures. If I have it at X degrees absolute, and read the pressure as 40 units, then I immerse it into a new vat at $2X$ degrees, what will the new temperature reading be? Explain in your own words the reasoning behind your answer.



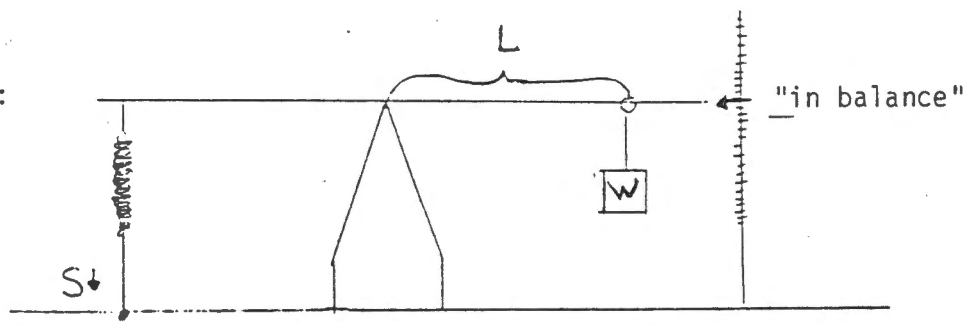
- 2) We have a given amount of gas in a cylinder with an adjustable cap. There is a pressure gauge on the side. The cylinder is kept in a vat of water so that temperature is constant. I can control the volume by moving the cap in and out. If I have an initial pressure of 120 units, and I push the cap in, reducing the volume by $1/2$ (by $1/3$), what will the resulting pressure be? Explain in your own words the reasoning behind your answer.



- 3) We have a given amount of gas in a perfectly flexible balloon. Since the balloon always has 1 atmosphere of pressure against its walls, the pressure is constant in this situation. If the temperature inside the balloon is 300 deg. A, and I heat it up to 600 deg. A, what happens to the volume? Explain.



- 3a) We have a balance beam:



The balance beam has a spring on the left side with a device to vary tension (S), a set of weights that can be put on a hook on the right side which can be moved back and forth. The amount of weight (W) and the length (L) between the fulcrum and the weight are variable, as is the tension (S).

There are three variables and if the system is to be kept in balance, changing one or more means changing some others to compensate for the original change.

In order to keep the beam balanced the three variables must interact in much the same way as the three variables in the ideal gas law: P, V & T.

- i) Suppose you wanted to write some problems for learning about how a balance beam works. Consider problem 1. Give a correspondence (or dictionary) from the set of variables: pressure, volume, and temperature, to our new set: length, weight, and tension.
- ii) Now rewrite problem 1 in terms of the balance beam -- if you have trouble, reconsider your dictionary and see if it needs alteration.
- iii) Repeat i) and ii) for problems 2 and 3.
- iv) As you do the rest of the problems on this sheet, try to keep the analogy of the balance beam in mind. It may help you.

4) I can vary the pressure on the balloon by taking it to different places with different atmospheric pressures (the moon, the top of Mt. Everest, under the ocean, etc.). Suppose I take my sealed balloon to a place with half the pressure, what happens to the volume? Suppose at the same time I increase the temperature four times, what is the resulting volume? Suppose I triple the pressure and double the temperature, what happens to the volume? Explain.

5) I now decide to move my cylinder to different vats of water at different temperature. Suppose I move it to a cooler vat, one that is 200 deg. A instead of its original 400 deg. A, and at the same time I increase the volume from 150 to 450. The original pressure was 10 units, what is the resulting pressure?

6) We have been discussing "a given number" of molecules. Suppose we change the number of molecules: If I put twice as many molecules of the same gas into my balloon, and keep pressure and temperature constant, what happens to the volume? Explain.

If I put three times as many molecules into my canister, keeping temperature constant, what happens to the pressure reading on the gauge?

7) Suppose I put four times as many molecules into the canister and at the same time put it into a vat of water which is 1/3 cooler than its original one. The original pressure reading was 100 units, what is the new one?

LAW for an (ideal) gas: The number of molecules affects the volume, or pressure, regardless of what they weigh. Thus, 423 molecules of a gas made up of light molecules (say helium) and 423 molecules of a gas made up of heavy molecules (say krypton) will occupy the same volume, if temperature and pressure are constant, or will exert the same pressure if volume and temperature are constant.

8) Suppose a balloon full of 2222 molecules of oxygen on the earth at 400 deg. A expands to a volume of 50 units. I fill up a second balloon with 6666 molecules of hydrogen, take it to a planet where the pressure is four times that on earth and the temperature is 400 deg. A. What is the volume of the balloon? Which balloon will weigh more?

The ideal gas law states that $PV=nRT$, where R is some constant depending on units used. If temperature is held constant at 400, explain in your own words what this formula tells you. There are alternative but equivalent ways of writing this. Some of these ways make things easier to understand, depending on what you are using the formula for, e.g.

$$V = \frac{nRT}{P}$$

- 9) We have a given number of molecules at 400° absolute. Using whatever form of the formula you wish, explain how a change in one of the variables P or V affects the other.
- 9a) Remember the balance beam. Write the formula that expresses the relationship of S , W , and L when the beam is in balance. Considering some alternate forms of this formula, which one helps you understand the situation the best? Why? Compare this form with the form(s) of the gas law formula you found most understandable.
- 10) Repeat question 9, keeping P and n fixed and allowing T and V to vary.

SUPPORTING A THESIS

Form into groups of four; choose a scribe to write down your group's answers and report them.

This workshop assumes you have selected a thesis. Its goal is to widen your concept of supporting a thesis. The thesis we are working with is, "American women should be drafted if American men are."

1. Here is a short paper. Its thesis is, "Socrates is mortal."

10 min.

Socrates is a man.
All men are mortal.
Therefore, Socrates is mortal.

Try to agree on a sentence or two which explain why the first two sentences support the third one. Write down your explanation.

10 min.

2. We will hear each group's explanation and work out one we agree on together.

15 min.

3. (Return to your groups of four.) This kind of support, that involved in a logical proof, is not the only kind writers use. Here are some other sentences using the word "support."

She bought a pair of support stockings.
Ronald Reagan increased his support by persuading the teamsters that his measures to fight inflation would not raise unemployment.
The bridge is supported by 960 stainless steel cables.
So then, the bastard stopped making his child support payments.
Sally and I started a women's support group.

Try to find at least one thing which supporting a thesis shares with each of these other kinds of support.

4. Nonetheless, this workshop is about logical support. A list of sentences follows. Select the ones which you think help support the thesis, "American women should be drafted if American men are."

20 min.

1. Women should not be discriminated against because of sex.
2. Russian women help out the Russian army a lot.
3. The draft would be a great opportunity for all women to prove to the United States just how equal we are to men both mentally and physically.
4. Men are born with stronger upper torsos, and therefore have a better physical ability to handle hand-to-hand combat.
5. The draft is bad because it infringes on individual liberty.
6. Our society needs to try and think of women as having the same rights and responsibilities to our country as men do.
7. Women could do a real good job flying aircraft, sailing on ships, being medics, and even working on some of the technology.
8. Women who are expecting babies or have small children need to take care of them and they can't possibly do that in the Army.
9. I'm a member of the Twelve Day Love Church, and my minister says women were made by God to be calm and gentle.

10. Most Americans believe we need some kind of Army.

10 min.

5. If I changed audiences, would that change the way you answered question 2? Try to invent audiences which would mean that a couple of the sentences you said supported the thesis no longer do. Try to invent audiences for whom a couple of the sentences you said did not support the thesis do.

6. Break. End of Part 1.

15 min.

7. Take the sentences which your group thinks would not support the thesis if you were writing for an ordinary American audience. One thing you could do is leave them out. Try to decide as a group if there are any you can get away with omitting. Write a sentence which sets up guidelines for when you can get away with leaving out something which does not support your thesis. (There may be several different kinds of things you can leave out.)

7a. The group as a whole will hear each small group's sentence about when you can leave out something which doesn't support your thesis.

min.

8. There are several other things you might do with the sentences you decide you can't leave out. If the problems they pose for your thesis seem minor, you might modify the thesis in such a way that they are not objections to it any more, but you have not given up the central point you want to make. There is no reason to ask for an argument if you can sidestep one. For example, suppose my thesis were "Moby Dick offers the most profound analysis of pride in American literature," and then I started thinking, "What about The Scarlet Letter?" Unless I'm very concerned about ranking, I might want to handle this problem by changing my thesis a bit, to "Moby Dick offers one of the most profound treatments of pride in American literature" or even "Moby Dick returns again and again to the psychology of pride." Take three of the sentences you could not use and try to modify the thesis so they are not problems any more. Write out the new versions of the thesis.

5 min.

9. Another thing you can do is deal with the problem in passing, if it is not too important a problem. For example, suppose your thesis is, "Everyone should obey the 55 mph speed limit" and one of the objections is, "What about fire trucks and police cars?" You might want to stick a sentence in somewhere which dealt with the issue in passing, say, "Although there will obviously always be emergencies in which people like policemen and firemen should ignore any standard speed limit, whether fifty-five or seventy, the savings in energy, the increased safety, and the greater enjoyment produced by driving at fifty-five make it a much better limit for normal conditions." Take three of the sentences you could not use and write new sentences which deal with the problems they raise like this.

- 15 min.
10. If you have an important objection, you cannot get it out of the reader's mind by ignoring it. Your hard-nosed professor will always complain that you did not deal with it. Therefore, you have to introduce important objections in the paper and try to answer them. For example, "Some people have argued that ... However, they are mistaken because ..." Or, "The number of Vietnamese leaving the country suggests at first glance ... However," This maneuver can take many forms, some considerably more elegant than these. Take two of the sentences you could not use and write a brief treatment like this for each of them.
11. Break. End of Part 2.
- 30 min.
12. Here is a student paper. Read it together carefully and try to identify the places in which the author has tried to deal with sentences like the ones we have been working with in this workshop. Do not worry about other ways you might improve the writing; just concentrate on the ways she dealt with points which did not support her thesis. Take each maneuver and see if you think some other way of dealing with the point might have been more effective. If so, rewrite the paper accordingly.

Draft Women
By Julie Milligan

Although there are certain conditions that could prevent some women from being drafted, the majority of the women should be drafted if men are. I think that some exceptions would have to be made as to just who should be required and who not. Women who are expecting a baby, or have small children, for example, should not be drafted. Another exception would be, of course, if she was not healthy enough or had some kind of a mental or physical disability.

In many other countries around the world, such as Russia, and Israel, they have been proving for some time now just how much a woman can do to help in a war. The United States needs to be shown once and for all, just how much a woman can do to help her country. Women have been labeled as the weaker of the sexes for too long. The draft would be a great opportunity for all women to prove to the United States just how equal we are to men both mentally and physically.

Even though, men seem to have been born with stronger upper torsos than most women, this does not mean they should be denied the right to work in other areas. There are many different positions a woman could take part in and do a real good job in a draft. Women should not be denied the right to fly aircraft, sail on ships, and be medics or even work on some of the technology. Men might have a better physical ability to handle hand-to-hand combat, but there are several other things women are equally as good at, and I believe that women would be an enormous help to the draft.

Women have been fighting for equal rights for a very long time now, so why shouldn't they be drafted if men are? Our society needs to try and think of men and women as having the same rights and responsibilities to our country as men do. In conclusion, I believe the draft is bad but why should men just suffer? Women should not be discriminated against because of sex.