The Effects of Classroom Community on
Building Conceptual Understanding in Mathematics

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ABSTRACT

This paper reviews the research and literature exploring the question: What are the effects of classroom community on building conceptual understanding in mathematics? Mathematics education in the United States has been under heavy watch over the last few decades. The debate is about whether to teach mathematics following the progressivist curriculum, where conceptual understanding and students' interactions with mathematics is the driving force of the curricula, or to teach basic mathematic facts and algorithms. The history of this debate and the different aspects of mathematics education are presented in this paper. The research explored in this paper consistently pointed to the need for students to have conceptual understanding of mathematics, which is best achieved through a variety of strategies. The research pointed to some effective strategies that promoted conceptual understanding, and also focused on the effects of classroom community. The categories addressed in the paper were: (a) the development of multiple student-invented strategies; (b) student collaboration; (c) mathematical tasks; (d) use of student mistakes/errors for productive learning; and (e) building on student thinking using mathematical reasoning.
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CHAPTER ONE: INTRODUCTION

Introduction

Mathematics is a subject that is unavoidable in our current school system. It is also virtually unavoidable in today’s society. The mathematical foundation built in elementary school is crucial for future mathematical success, both in and out of school. Unfortunately, many children do not succeed, are left behind, and struggle with mathematics throughout the remainder of their schooling, and probably through their lives.

In the past two decades, researchers have attempted to try to resolve this problem, and this research is continuing to increase. They have shown that teaching for conceptual understanding greatly increases students’ mathematical ability, however there is strong resistance from community members, who do not understand or examine the research. Traditional schools taught mathematics as rote memorization, and it was not necessary for students to conceptually understand a problem to be able to solve it. Schools in the United States do not all teach specifically for conceptual understanding today, and students still learn mathematics, but research shows that there are benefits to learning for conceptual understanding, which may give some insight into the success of a wider range of students.

In the following chapters I will present findings that address the question, what are effective strategies for creating a classroom culture that supports students’ learning mathematics for conceptual understanding? Because the primary focus is conceptual understanding, I do not present information about the
implications of traditional mathematics curriculum. The focus of traditional curriculum was memorization of facts and of skill development, whereas reform curriculum includes developing conceptual understanding prior to skill development. Reform curriculum is informed by research that shows the importance of building on students' prior knowledge as well as attending to the role of classroom community in learning. Classroom community and prior knowledge are not the goal, but they are a means to a goal.

In order to fully understand the purpose of this paper, I discuss what learning is and how students learn mathematics. Classroom culture, in an elementary school setting, plays a critical role in how students learn, and how mathematics is taught. The culture within a classroom is the established norms of action and interaction within which a teacher can teach, and students can study (Lampert, 2001). Each classroom has an established community, however, the specific kind of community that best condones learning for conceptual understanding in an elementary classroom is discussed later in this chapter.

In Chapter 2 I discuss some of the history that defined mathematics education in the United States. I begin with the development of mathematics in the United States, discuss mathematics curriculum for elementary school, and look at the "math wars" that have led mathematics education for decades.

In Chapter 3 I present a field of research that informs this paper's question: What are effective strategies for creating a classroom culture that supports students' learning mathematics for conceptual understanding? The
chapter is a review and critique of current literature. The content of the chapter is divided into five sections: development of multiple strategies, student collaboration, mathematical tasks, use of student mistakes/errors for productive learning, and building on student thinking using mathematical reasoning. This body of literature reflects the current research in teaching and learning mathematics that is taking place in our society. The general findings in this chapter show that in order to teach for conceptual understanding in mathematics in elementary school, students should be engaged in tasks that are inquiry based and have many entry points. Students need to be able to make meaning of the mathematical problems in order to understand the concepts, and this is not done by computing standard algorithms or memorizing facts. Productive learning involves collaboration, reasoning, and, above all, understanding.

I will conclude in Chapter 4 with a summary of my findings. These findings have an influence on my future role as an educator and also may impact current educators. The current research is bringing into light new possibilities in mathematics instruction. This paper helps to clarify the new trajectory. There are aspects of the research where I found some holes. Because of this, I also discuss the implications for further research in the field of teaching mathematics.

**Rationale**

I decided to write this paper because I wanted to explore the influence classroom culture had on learning. Mathematics is a major subject that is highly debated in the education community, and has been for over two decades. The way many adults learned math is quite different than what research shows for
learning mathematics for conceptual understanding. This body of research addresses my interest in teaching mathematics so that students have conceptual understanding of what they are learning, and in writing this paper I will explore what current research is finding about this.

**Statement of Purpose**

The purpose of this paper is to find what research has revealed about effective strategies for creating a classroom culture that supports students' learning mathematics for conceptual understanding in an elementary classroom. The culture of an elementary school classroom is one that can have great effect on students, and can bring out their potential to excel in mathematics. Classroom community can also deter students from understanding mathematical concepts, enjoying learning, and can set them up for mathematical failure in future grades. My purpose is to present information that addresses strategies for creating a classroom culture where students can truly learn the concepts of mathematics.

**Learning for Conceptual Understanding**

One of the most important definitions to understand for the purpose of this paper is learning for conceptual understanding. I will define learning, as it is needed for the definition of learning for conceptual understanding. People learn continually throughout the course of everyday life, but learning in schools, and more specific to this paper learning in a mathematics classroom, is intentional.

Learning is a process in which an individual takes new knowledge from the environment and links it to prior knowledge, thereby creating stronger
neuronal and synaptic connections, giving the new knowledge personal relevancy (Zull, 2002; Miller, 2002; Singer & Revenson, 1996). The physical connections that make up this prior knowledge are called neuronal connections. Neurons link together, forming networks which allow new senses or information to enter. Dendrites pick up signals and axons collect the signals and send them away from the cell bodies. There is a myelin sheath covering the axon that grows thicker with use, and promotes efficiency. Synaptic connections also may be strengthened by use, allowing the synapse to fire more frequently. With less use the synapse quiets, and information is not sent as frequently. Synapses are important in relation to learning because, without the connections, networks would not form and there would be no way to connect new information to prior knowledge. Without these connections there would be no learning (Zull, 2002). The brain will change only if the new knowledge presented does not fit into the current schema. When this happens, students become disequilibrated, or cognitively off balance, and they form new schema to make sense of the incoming information.

Learning is a process that occurs throughout normal everyday life. The process explained above contains much of what is needed to understand what learning for conceptual understanding is in mathematics. It is possible to learn aspects of mathematics. Learning mathematics requires that conceptual understanding is made through prior knowledge and personal relevancy.

Conceptual understanding in the field of mathematics is more than learning rote skills. The difference in learning mathematics, and learning
mathematics for conceptual understanding is that when a student learns mathematics for conceptual understanding, he or she understands how and why an answer was achieved. In learning for conceptual understanding, a student's prior knowledge of mathematics, as well as the student's prior knowledge of the world impacts the conclusions the student makes on getting the answer to the problem. Often student-invented strategies, collaboration, and discussion, and argumentation are encouraged so that understanding can be made.

**The Role of Classroom Culture**

An elementary school classroom is a place of childhood. Young students are continually evaluating their abilities and are building schema about what kind of student they are, and want to become (Miller, 2002; Singer & Revenson, 1996). Because learning is dependant on prior knowledge, and because it relies on relevance, students cannot learn what is intended by the teacher unless the classroom teacher creates opportunities for neuronal connections to be formed. Teachers need to give time for puzzlement and create need-to-know situations for the students. In doing this, teachers make learning mathematics relevant.

Each and every classroom has a culture and environment. This is created organically based on the children and the teacher in the class, as well as the preexisting history that the students and teacher bring with them. In an elementary classroom, because students are generally with one teacher throughout the school day, there is time to create and build the existing culture to something that is more contrived and controlled, generally to foster learning.

From this classroom culture, a micro culture can form to create an offshoot
culture that is specific to mathematics. There are different beliefs about the environment’s role in learning. Piaget (Miller, 2002) and Vygotsky (Miller, 2002) differed in their beliefs about the learning environment. Piaget believed that culture was an external factor as far as learning was concerned. There was not a direct connection between the student and the culture that student came from, or the culture that was present within the classroom. Vygotsky, however, believed that the individual and the culture existed simultaneously, and that one cannot exist without the other (Miller, 2002). This is to say that there can be no separation of mind from the individual and the culture in which the individual exists. He believed that learning takes place within the context of the environment.

Within every classroom is a classroom culture that is specific to that particular class. The culture that is formed within a mathematics classroom may be different than that of other aspects of an elementary classroom. This paper will explore the different aspects of classroom culture that occur within elementary mathematics classroom, as well as the impacts, if any, that the mathematical cultures have on the students’ learning for conceptual development.

**Standards-Based Curriculum**

In the late 1980s and early 1990s, the National Council of Teachers of Mathematics (NCTM) began publishing documents addressing recommendations for reform in K-12 mathematics classes. These documents, called Standards, led to the development of curricula that was implemented to align with the
Standards. Standards-based curriculum had great support from The National Science Foundation. In order to fully implement the Standards-based curricula there was a great deal of change that needed to occur within the system of how math was being taught. Schools needed to change their view of what math entailed, as well as change how math was taught and learned. This is a change that is still happening, and difficulties continue to arise with the new curricula. Standards-based curricula are very different in learning goals, assessment, tasks, than the more traditional mathematics curricula.

The underlying goals of the NCTM, which are addressed in all of the Standards-based curricula, are to build students’ understanding of important mathematics through explorations of real-world situations and problems. Small group work and whole class discussions are encouraged for these investigations so that conceptual understanding can be built and reflected upon. Rote memorization of math facts and formulas is not valued in Standards-based curriculum in the same way as the traditional curriculum. Learning skills and facts are valued, but the way in which students arrive at these skills is not through rote memorization.

Summary

I aim, with this paper, to answer the question: What are effective strategies for creating a classroom culture that supports students’ learning mathematics for conceptual understanding in an elementary classroom? Mathematics education is a subject involved in a current debate, and this paper will explore the area conceptual understanding and the effects classroom culture
have on learning mathematics. Often student-invented strategies, collaboration, and discussion, and argumentation, and mathematical reasoning are encouraged so that conceptual understanding can be made. This paper will begin, in Chapter Two, with a brief history of mathematics education and information about the current debate. Chapter Three explores current research that addressed my question, and Chapter Four discusses implications the research has for teachers.
CHAPTER TWO: HISTORY OF MATHEMATICS EDUCATION

Introduction

This chapter will examine several aspects of the history of mathematics education. Mathematics education in the United States has been under heavy watch over the past few decades, wanting to keep up with other countries, and trying to come to an agreement about the most effective way to teach math. I will begin with mathematics education in the United States, discuss elementary curriculum, and touch on the "math wars" that have been occurring for many years. My inquiry is about effective strategies for creating a classroom culture that supports students' learning mathematics for conceptual understanding in an elementary classroom.

Mathematics Education in the United States

The development of mathematics was not a focus of the early United States; however, education was quite important. Harvard College, the first college in the new colonies, was established in 1636, which was more focused on students' abilities to speak Latin, than mathematical abilities. The main purpose of the college was to ensure an educated ministry for the colony (Spring, 2005). Mathematics was only taught to young men in their last year, and the course consisted of arithmetic and geometry for the first three quarters of the year, and astronomy during the last quarter. Algebra was unknown to the new Americans. The curriculum remained as such until the beginning of the eighteenth century (Cajori, 1890). In 1726, one man transmitted 12 hundred pounds of sterling to be applied toward "the instituting and settling a professor of
mathematics and experimental philosophy in Harvard College” (Cajori, 1890, p. 23). This event, and a few similar others, marked the importance of mathematics for our society.

Colleges and Universities taught mathematics during the colonial times as a less sophisticated form of mathematics than existed elsewhere in the world at the same time. Mathematics in elementary schools during colonial times was far less sophisticated than at the college level. Boys were expected to count and perform simple arithmetic. Nothing more was perceived as practical or applicable. In early schools, arithmetic was hardly ever taught to girls. Boys in many early schools were taught reading, writing, and arithmetic, while girls were taught reading, writing, and sewing.

The classroom culture, when math was taught, was very different than what is practiced in schools today. No explanations of processes were given, and no demonstrations of principles were provided. The problems were solved, the answers obtained, and the solutions copied. This was considered complete work (Cajori, 1890).

Mathematics instruction became the norm in American schools after the first mathematics text books were published in America. In 1778 the first edition of Nicholas Pike’s mathematics manual appeared in New England schools, and formed the basis for mathematics in our schools. Pike’s book remained the only American written text book for the schools until 1800. Schools used his book throughout the growing colonies, and only a couple other books were published until 1820, which marked “the end of an old period and the beginning of a new
one" (Cajori, 1890, p. 49). There was a growing reform in mathematics, and after this period mathematics was characterized as rules that were committed to memory without any knowledge or proof of why or how the mathematics worked.

On October 4, 1957 American views of mathematics and science education changed. That was the day that the Soviet Union put a satellite into earth's orbit. This marked a change in political, military, scientific and mathematical implications for Americans (Kilpatrick, J. 1997). America entered a new era for mathematics education.

Gradually the idea began to be accepted that education was necessary for the future of the United States. Each state was deemed responsible for their own curriculum, which marked the beginning of curriculum development in the United States.

Mathematics Curriculum: Elementary

Curriculum for mathematics before Sputnik contained little interactive teaching; it focused primarily on the learning of facts and figures (Howson, Keitel, Kilpatrick, 1981). The rate of change of the American school system expanded rapidly with the growing beliefs about the necessity of education, and states hired agencies to promote and implement change. This marked the birth of institutionalized curriculum development (Howson et al., 1981). The teams assigned to curriculum development designed and implemented the new curriculum, and further developments happened slowly in the following years. Mathematics curriculum remained virtually unchanged in theory and methods until educators called for reform in 1989, bringing about drastic changes.
With the development of the National Council for Teachers of Mathematics (NCTM), in 1920, teachers, students, and curriculum developers could get the support they needed. In 1989 the NCTM released the *Curriculum and Evaluation Standards for School Mathematics*. With this document, the NCTM aimed to improve mathematics based on what students should learn, and what they should know and be able to do, and how this would best be demonstrated in the classroom. It marked the first step by a professional organization to articulate extensive goals for teachers and policymakers (NCTM, 2006).

The standards proposed by the NCTM made mathematics education accountable for the maintenance of a high level of mathematics in students. Their beliefs were grounded in student understanding, and that all students should learn important mathematic material. The NCTM has standards in number and operations, algebra, geometry, measurement, data analysis and probability, process standards, problem solving, reasoning and proof, communications, connections, and representation. Each category has expectations that students should reach that are divided into grade bands. The NCTM rewrote the standard and re-released them in 2000, as the *NCTM PSSM [Principals and Standards for School Mathematics]* 2000.

**Math Wars**

The way in which math is taught in our schools has been the focus of debate for many years. In the 1990s there was a move from teaching basic skills and algorithms to teaching in line with the NCTM Standards. New text books were widely distributed throughout the United States, and the debate continued
(Klein, 2003). Versions of the math wars have been going on since mathematics education became standard in American schools. It is basically an ongoing discussion and occasional shift in power, about whether to teach mathematics following the progressivist curriculum, where conceptual understanding and students’ interactions with mathematics is the driving force of the curricula, and a “back to basics” belief about learning mathematics, where algorithms and rote mathematics is the focus (Klein, 2003). Effective strategies for creating a classroom culture that supports students’ learning mathematics for conceptual understanding in an elementary classroom are addressed in the NCTM standards, which find conceptual understanding highly important. These wars continue today and the issue is prevalent in our society.

Summary

In this chapter I discussed periods of history that affected the current state of mathematics teaching in the United States. Mathematics is now a common standard in the school system that developed in the United States. Curriculum advances, as well as standards for what students should know and be able to do shaped mathematics education as we know it today. The manner in which to best teach mathematics is the center of debate in our current society, and the research I will present in the next chapter will show what is currently being studied centering around my investigation of effective strategies for creating a classroom culture that supports students’ learning mathematics for conceptual understanding in an elementary classroom.
CHAPTER THREE: A REVIEW OF THE LITERATURE

Introduction

The research presented in this chapter begins to answer the question: what are effective strategies for creating a classroom culture that supports students' learning mathematics for conceptual understanding in an elementary classroom? The mathematics research suggests that educators should move away from the rote memorization of facts and computational accuracy, and toward a strong emphasis on conceptual understanding, relations, and a thorough understanding of mathematical ideas, as the studies show.

This chapter will review five major areas that the impact conceptual understanding of mathematics. The chapter will cover: students' development of multiple strategies for solving mathematical problems, student collaboration, mathematical tasks, the use of student mistakes/errors for productive learning, and building on student thinking using mathematical reasoning. All of these categories play a major role in how students learn mathematics.

Development of Multiple Student-Invented Strategies

Most research agrees that in order for children to develop conceptual understanding, students need to identify methods and solutions to problems by making interpersonal connections between their own prior knowledge and relevancies and the mathematical problem. To begin this review of the research discussing effective strategies for creating a classroom culture that supports students' learning mathematics for conceptual understanding in an elementary
classroom, I will first discuss the benefits and shortcomings of developing multiple strategies for solving problems that lead to conceptual understanding.

To begin a review of research literature regarding the invention of strategies for solving problems, Carpenter, Franke, Jacobs, Fennema, and Empson (1998) conducted a three-year longitudinal study that examined the growth of children’s understanding of addition and subtraction involving multidigit numbers. The study investigated how invented strategies effected multidigit addition and subtraction concepts and procedures especially relating to base-ten number concepts. Three invented strategies focused on encompassing both addition and subtraction: sequential, combining units separately, and compensating. The researchers hypothesized that the students who used invented strategies prior to using the standard algorithm would demonstrate greater understanding of the problem than students who started out using the standard algorithm.

The participants in this study came from three different schools. The researchers randomly selected six boys and six girls from each of the ten different first grade classrooms from the initial 120 students. The final sample ended up having 82 students. The study was conducted over a three year period. Researchers interviewed the students five times over the course of the three years where they read mathematical problems to the students and asked them to solve the problems. If the strategy used was unclear to the researcher, the researcher questioned the student on the method used to solve the problem until the strategy was clear. Classroom instruction was not parallel across all of
the classes, however, there were some characteristics that were similar in most of the classrooms. Word problems were used extensively, students were generally given the opportunities to solve problems using a variety of strategies, and alternative strategies were discussed in whole class or small group discussions.

The interview tasks assessed the students’ knowledge of base-ten number concepts, strategies for solving addition and subtraction word problems and computation exercises, abilities to use specific invented strategies, and abilities to extend and use addition and subtraction procedures flexibly.

Carpenter et al (1998) claimed that the results from the study supported their initial belief that the use of invented strategies represented understanding of multidigit addition and subtraction problems. Results showed that students who initially used invented strategies demonstrated knowledge of base-ten number concepts before students who relied primarily on standard algorithms. These students also frequently showed how to use other invented algorithms and standard algorithms when asked. One possible explanation for these differences may be linked in the students’ prior mathematical instruction and knowledge, and not a direct tie to invented strategies. The use of base-ten number concepts may also not fully measure the understanding of the problem. Students in this study could have also had prior knowledge of base ten and chose to use the concrete standard algorithm instead of an invented strategy. The instruction of the students might also dilute the results of the study. Instruction can have a great impact on students’ preferred methods and better
understood methods. Students in the invented strategies group also showed more solid understanding, demonstrated by the ability to flexibly transfer their use to new situations as shown in the extension problems. Students in the algorithm group were significantly less successful at solving the extension problems. Students in the algorithm group also showed significantly more systematic errors than students in the invented strategy group.

This article did show, that in terms of classroom community students' use of multiple strategies for solving multi digit addition and subtraction concepts and procedures especially relating to base-ten understanding, invented or not, is important in gaining base-ten understanding. Carpenter et al (1998) said that this study provided proof that children can invent strategies for adding and subtracting that work with their knowledge of the world. The procedures should be kept by teachers in the form of invented multiple strategies, because if only one or a few strategies are adopted and taught, as opposed to many or all of the strategies, these strategies could become rote, similar to the standard algorithms.

Standard algorithms are often the core of what is taught in mathematics classrooms. In this next study, students developed multiple strategies, individually and collaboratively, before standard algorithms or even mathematical symbols have been presented.

Sharp and Adams (2002) focused on the thinking that took place in children who had the opportunity to construct personal knowledge about the division of fractions. The standard invert and multiply, and common denominator
methods were not taught to them prior to the study, and it was believed that the students did not have knowledge of these techniques prior to the study, and the pretest confirmed their belief. Sharp and Adams (2002) collected data from 23 students in a fifth grade mathematics class. Four of the 23 students were in fourth grade, but had math skills at the fifth grade level. The school, a magnet school for both mathematics and science, was in a mid-level socioeconomic level town. The students of the study were 11 African American, 12 Caucasian, which reflected the population of the school.

For the purpose of the study, both researchers designed and taught a unit on dividing fractions, which first focused on individual thinking, and was followed by whole-class discourse. Students did not enter the unit with no knowledge in the subject area. Sharp and Adams (2002) said that the children already knew several procedures, including the ability to find equivalent fractions, change between improper and mixed-numeral representations, and subtract fractions. They also knew their basic multiplication facts and demonstrated skill at dividing whole numbers. The goal of the study was for the students to accurately describe their thinking of whatever strategy they came up with, both verbally and pictorially in the unit on dividing fractions. They collected data from a pretest and posttest, field notes, video recordings of the classroom discourse, and students’ daily work that related to student thinking to triangulate the findings.

During the teaching experiment, the students used many approaches to solve word problems that required them to divide fractions, and none of the strategies involved the traditional methods. For some students, a key shift in
their procedural development came when Sharp and Adams (2002) removed the context from the problems so that only symbols represented the problem. A possible reason for this shift in procedural development was that students were beginning to generalize their understanding of a procedure for division of fractions, but had not internalized these procedures. Sharp and Adams claimed that keeping the problem dependant on context was helpful because the students could use their prior knowledge, however, they believed that moving students away from the specific contexts was vital if they were to make sound generalizations. They also noted that in some cases context was vital to understanding the thinking of the students in the class.

Sharp and Adams (2002) stated that in order for mathematics to develop in children, the study showed that students had more success if teachers nurtured the preexisting mathematical knowledge within the minds of the students. The students in this class did not have conceptual knowledge or knowledge of the standard algorithm for dividing fractions, but each student, by the end of the study, was able to understand their own methods for solving the problems, and were able to arrive at correct solutions.

This study may be less generalizable to a common elementary school classroom because the school was a mathematics/science magnet school, so the students encountered math in a variety of contexts throughout the day. The students were also a part of a math class, where as in a standard elementary setting, math would be incorporated into the everyday happenings within one class that covered all subjects. The following study also depicts a learning
situation in which students developed multiple strategies before the presentation of standard algorithms.

Cramer, Post, and del Mas' (2002) performed a study that involved 66 fourth- and fifth-grade classrooms in 17 schools in one suburban school district. This study aimed to test a newer curriculum, the Rational Numbers Project (RNP) within the structures of a study in order to evaluate the effectiveness of the curriculum against the other two commercial series curricula used in the district based on the district's goals for teaching fractions (Cramer, Post, del Mas; 2002). The RNP curriculum geared more of its efforts toward conceptual understanding than the other programs being used at the time. The researchers randomly assigned the 66 teachers to either the experimental (RNP) or control groups. They performed the study across the district so the results would be generalizable to the district level.

Students using the experimental curriculum used manipulatives extensively in many contexts and in many ways. They also discussed mathematical ideas before attaching any symbols to the ideas. This facilitated conceptual understanding of fractions. The control group also used manipulatives, though not nearly as extensively, and a student textbook based curriculum. The primary goal of the commercial curriculum targeted the development of student competence at the symbolic level.

Cramer, Post, and del Mas (2002) collected data using two different methods. They gave written tests (a posttest and a retention test) that were parallel, but not identical. They mostly used questions taken from previous RNP
teaching experiments. They also collected data through student interviews, which attempted to identify differences in student’s thinking about fractions. They administered the interviews three to four times throughout the study to 20 randomly selected students (10 from each group) in seven schools. The interview questions covered the same topics as the written test. The study randomly selected two to three students from each class. The students' teachers interviewed them at the end of the study using notes and audiotapes to record their information.

The researchers used both quantitative and qualitative means for analysis. The quantitative analyses involved conducting a factor analysis to assess the viability of the original test subscales, calculating reliabilities, and using multivariate analysis of variance to determine whether treatment differences on total test and revised subscales existed. The qualitative data involved interviews given by project staff and classroom teachers.

The data found significant differences in student achievement between students in the RNP group and the control group. The significant multivariate analysis of variance for total test scores suggested that RNP students significantly outperformed students using the commercial curriculum on both tests (posttest and retention test). RNP students appeared to have significantly stronger conceptual understanding of fractions, were better able to judge the relative sizes of fractions, used this knowledge to estimate sums or differences, and were better able to transfer their understanding of fractions to tasks not directly taught to them. The two groups showed no significant difference on
equivalence items or symbolic addition and subtraction items. Overall, students in the RNP group appeared to have a better conceptual understanding of fractions.

An aspect of this study that may be confounding is that the assessment given to the students was more closely aligned with the RNP curriculum. This may have skewed the data. The test heavily measured conceptual knowledge, with less emphasis on procedural skills. The classroom community that was created in the classrooms that participated in the study reflected the curriculum because the intended community in the RNP curriculum encourages the use of discourse, manipulation, and collaboration. The same results may not have occurred with other curricula that does not have a mathematical community focus.

The Everyday Mathematics curriculum, another reform based curriculum that supports the standards of the NCTM, has been subject of many research studies. It is one of the more innovative curricula available right now, and it is showing promise. In Riordan and Noyce's (2001) study, Everyday Mathematics, along with a middle school mathematics curriculum, called Connected Mathematics, was compared to schools with traditional curriculum. The study aimed to determine the impact these curricula had on test scores when implemented over a period of time. For the purpose of this paper I will focus only on the Everyday Mathematics portion to this study.

The researchers chose schools for this study based on how long the program had been used. These schools were split into two groups, one who had
used the curriculum for two or three years (Group 2) and one for four or more years (Group 1). The study identified 67 schools as using the curriculum in Massachusetts. Group 2 consisted of 48 schools, while Group 1 contained 19. Characteristics of these schools became the benchmarks for choosing the other schools to participate in the study. The comparison groups showed no statistical differences in terms of the schools using Everyday Mathematics. The study also compared the teachers at the different schools. A questionnaire showed only small statistically insignificant differences amongst teachers at $p < .05$. The study only used students from regular education classes and students who lived in the district for three years or longer.

Riordan and Noyce (2001) performed three analyses of the student results and the effect sizes. The first examined mathematics scores between the Everyday Mathematics groups and the comparison groups. Students receiving the Everyday Mathematics curriculum outperformed their counterparts in all areas, and all differences were statistically significant. Students in Group 1 also outperformed the students in Group 2. The second analysis looked at the effect of the Everyday Mathematics curriculum across different student populations (race, free and reduced price lunch, and gender). Riordan and Noyce found that no specific student group scored significantly higher when taught using traditional curriculum than with Everyday Mathematics. In the third analysis, Riordan and Noyce examined students at both ends of the achievement spectrum. They compared these groups statistically, and all students in these groups scored
significantly higher. This showed the range of effectiveness of the standards-based curriculum in this study.

This study showed that fourth graders in the districts using Everyday Mathematics significantly outperformed their very similar counterparts, and the longer they implemented the program, the greater the improvement. Though the study neglected to address classroom community, the Everyday Mathematics curriculum fostered an environment where teachers gave students the opportunity to use a variety of mathematical ideas in a variety of settings, and used a variety of methods to solve mathematical tasks. Though the study failed to track teaching styles and methods which may have made a difference on student achievement, an aspect of classroom community that was imbedded in the curriculum was present that was not present in traditional curriculum.

The research discussed so far shows the benefits of using, presenting, and listening to multiple strategies for mathematical problems. In addressing the use of multiple strategies I have not yet presented any research on how teachers’ use of multiple strategies affects student learning. Bowers (1999) wondered about the different ways in which students developed multiple strategies for solving problems. Bowers’ study addressed one of many ways in which teachers use multiple strategies or approaches for mathematics. In this study, the research team designed a unit of study that supported students’ development of both increasingly sophisticated place value understandings and personally meaningful algorithms for adding and subtracting three-digit numbers.
The unit, called the candy factory, involved the students packing and unpacking boxes, rolls, and pieces of candy. The students used manipulatives as well as a computer program designed for the unit. They were also able to draw out mathematical findings to support their thinking. Twenty-three students participated in the study from a third-grade classroom in a suburban public school. The study took place over a nine-week period, in which 37 lessons were conducted for about 45 minutes each. The researchers collected data by means of a pre- and post-interview, video and audiotapes, field notes, interview data, and the students' written work to examine the multiple strategies students used in solving mathematical problems. The data were analyzed after the research team found critical incidents from the field notes. The critical incidents were based on when the teacher and students appeared to be negotiating new ways of interpreting or solving instructional activities.

The use of the multiple strategies allowed students to work with one problem in a variety of ways, with a goal to make different arrangements by packing and unpacking the boxes of candy. Bowers (1999) assumed that the students would do this, however, when they first attempted to generate new arrangements, and unpacking did not emerge as a means of creating different arrangements until later. The results showed that the progression of the different parts of the unit built on one another, and were necessary in making the latter parts of the unit significant. The use of multiple manipulatives by the students proved to be helpful in the students' conceptual understanding of the mathematics. Findings indicated that the students in this classroom conceptually
benefited in their understanding of place value from the approaching the candy factory unit from a variety of entry points. Two significant trends were revealed in the pre- and postinterviews. The first trend showed an improved proficiency in paper-and-pencil computation for two- and three-digit addition and subtraction tasks. The second trend was that most students developed increasingly sophisticated understandings of place value and numeration.

The developing unit used in the research limited the relevance of the results. The usefulness and the manner in which the students use the computer tool possibly shadowed the actual mathematics the students were doing. It is also a unit that cannot be used in classrooms, so it does not seem to be very generalizable. The results, however, showed that possible potential in using strategies similar to the one presented.

Summary:

The research presented in this section shows the usefulness of a classroom community that encourages conceptual understanding by developing strategies that make sense. Sharing and discussing these strategies can be helpful. The mathematical agenda of the teacher can guide which strategies are shared and discussed in order to benefit students. Research pointed to effective strategies for teachers to elicit multiple strategies from students, and to further conceptual understanding is to have students work collaboratively. In the following section research on collaboration will be discussed.
Student Collaboration

The previous section showed that multiple strategies can prove very useful in getting students to understand a mathematical problem. This section takes these same premises and applies them to work within small groups or pairs. I will begin this section with a study on kindergarten collaboration. These studies will show that effective, efficient collaboration is a skill that is learned and practiced. Students who learn the skills of collaboration at a young age may benefit mathematically in future grades.

According to Fuchs, Fuchs, and Karns (2001), the development of kindergarten students is not frequently researched. Kindergartners come into the education system with different abilities of mathematics, ranging from no mathematical knowledge to above level knowledge. In this study Fuchs, Fuchs, and Karns examined the effects of peer-mediated learning strategies on kindergarten students' mathematical development.

Twenty kindergarten teachers participated in this study from three Title I schools and two non Title I schools. The researchers randomly selected teachers and assigned them to either a peer-assisted learning strategies (PALS) group or the control group. PALS implementation consisted of the students being paired with each other to work on mathematical tasks. Teachers reassigned pairs every two weeks to increase children's exposure to different students, and every third two-week cycle, paired high achievers together.

All of the students in the participating classes, a total of 168, gave the mathematics section of the Stanford Early School Achievement Test (SESAT) for
the purpose of dividing them into subgroups. Fuchs, Fuchs, and Karns (2001) used z scores to identify students scoring in the high achieving, medium achieving, and low achieving groups. They also grouped together special education students.

All twenty teachers followed the district’s core curriculum, and addressed the same content each week of the year. The curriculum focused on a classroom community that relies on a variety of reform based standards, including conceptual understanding, connections to prior knowledge, reasoning, and investigation. Teachers in the PALS group implemented PALS two times a week for 20 minutes for 15 weeks.

All students took pre and posttests using the SESAT. They also took posttests on the Primary 1 level of the Stanford Achievement Test, with content spanning to the end of first grade. Fuchs, Fuchs, and Karns (2001) concluded from the research that PALS improved student learning. A relatively small sample size may account for the absence of a statistically significant mediating effect for student type. The study showed a significantly stronger effect for students of medium initial achievement status and students with disabilities than for students with strong initial mathematical readiness. The researchers concluded that PALS may be an effective strategy for getting kindergartners mathematically ready for older grades.

Though this study did not produce statistically significant results, it addressed possible techniques for preparing students for mathematics in older
grades. Especially for students who enter kindergarten at risk of failing in older
grades, PALS seemed to show potential.

The current reform in teaching mathematics, calls for more partner and
whole group discussion as well as conceptual understanding. Baxter,
Woodward, and Olson (2001) concluded that students should be solving
challenging, open ended problems that can be solved using a variety of methods.
They noted that this contrasted past curriculum that focused on rote
memorization and clear routines. This system valued speed and accuracy, and
often focused on only one way in which to solve a problem.

One critique of the current reform is that low achievers, or at risk students
may be left behind and by the lack of research, have not been adequately
addressed. Baxter, Woodward, and Olson (2001) acknowledged the limited
research and decided to investigate it further. The investigation focused on the
classroom dynamics of reform-based mathematics instruction, and gave special
attention to low achievers and students at risk for mathematical failure. They
used Everyday Mathematics as the curriculum, which aligned closely with the
1989 NCTM standards (Baxter, Woodward, & Olson, 2001). The participants in
this study came from two schools located in the Pacific Northwest, both middle
class with similar socioeconomic status. All five third-grade teachers at the two
schools participated in the study, and all five believed that children learn
mathematics by constructing knowledge, that there was an integrated relation
among skills, understanding, and problem solving, and that mathematics
instruction should be more facilitative and exploratory rather than primarily
teacher directed.

The students in the study included 104 third-graders at the two schools.
They classified seven of the students as learning disabled or on an Individual
Education Plan (IEP). These students received special education services for
mathematics in a mainstreamed setting. Based on teacher interviews, the
research team noted a discrepancy in the number of students who qualified for
special help and the number of students who the teachers believed needed
additional help. The teachers gave the Mathematics portion of the Iowa Test of
Basic Skills (ITBS) to all of the students in the study to determine other students
who may have been at risk. In addition to the seven students who already
qualified, this quantitative analysis of the students' mathematical achievement
yielded nine others that the researchers identified to be part of the target group.
The results from the quasi-experimental comparison study indicated the
effectiveness in the Everyday Mathematics curriculum with statistically significant
increases for average and high-ability students, but the observed only marginal
progress for students in the lowest third. The mean score rose from in the
twenty-fourth percentile in the fall to the twenty-sixth percentile in the spring.
This study presents challenges these students faced in the classroom while
participating in the innovative classroom.

The researchers analyzed the sixteen students used in this study in three
sections: a typical math lesson, similarities across the five classrooms, and
differences across the five classrooms: Baxter, Woodward, and Olson (2001)
used a constant comparative method to focus their data collection and analysis. Students were observed throughout the school year. In the typical math lesson, Baxter, Woodward, and Olson presented their findings which showed little learning in terms of the mathematical agenda for the target students in the class. The similarities for across the five classrooms showed the challenges that the target students faced. They usually listened rather than spoke during class discussions, and seemed disengaged in the conversation. These students often looked out the window, played with objects, or read during classroom discussions. During pair work the target students were more likely to be involved, however, their involvement was often nonmathematical when working with average to high ability students, and while working with other target students, they tended to have a difficult time beginning and understanding the task, and had problems staying involved with the task once started.

The researchers observed some differences among the five classrooms. Generally, these differences consisted of how the formation of groups occurred or how students used manipulatives. The differences that involved the target students more mathematically did not produce significant overall gains in these students.

Baxter, Woodward, and Olson (2001) did not claim to know the answers, and did not claim that reform based mathematics instruction should be discontinued. They claimed that instructional strategies of experienced teachers merits further study so that students with low abilities in mathematics may have more opportunity to learn, and they claimed that it is equally important for
researchers to try to understand the underlying causes of these students' difficulties before guidelines are developed for teachers who are working with reform-based mathematics curricula. Their work indicated that two features of reform-based mathematics – the formation of a community of learners and the increased cognitive load of the curricula – were especially important to consider in relation to low achievers.

Though Baxter, Woodward, and Olson (2001) believed that classroom community is an important part of a functioning inclusive classroom, they found that low achieving students were often marginal members who remained silent or distracted during whole group work – a key aspect of reform based curriculum – and tended to be engaged in non-mathematical tasks during pair work. The cognitive demand and mental load of both the mathematics of the curriculum and the verbal and social aspects of the curriculum created problems for the target individuals, proved to be especially challenging. In creating a mental and cognitively demanding mathematics classroom community for all students, low achieving students may need additional support and the instructional and structural changes that may need to be made need to be identified.

Low achievers in a mathematics classrooms need to be considered, and their achievement in a non standards-based classroom may not be significantly different than in an Everyday Mathematics classroom. Either way, it is important for the community of learners within a mathematics classroom to be conscious and consider all students, from high to low achieving. The quantitative and qualitative measures of this study provided insight into third graders in five
classrooms, but did not set out to find an overall solution for mathematics classrooms. Baxter, Woodward, and Olson (2001) wanted to show a gap in improvement, and possibilities for why this gap existed.

The improvement gap may also be related to motivation. Student motivation is necessary for teaching mathematics. Stipek, Salmon, Givvin, and Kazemi (1998) focused on the discussion of the instructional practices geared toward the motivation of students to achieve in mathematics and how that related to mathematics instruction. The researchers used both qualitative and quantitative measures. The study consisted of 24 teachers, (1 male and 23 female) divided into three groups which represented several school districts in a large, ethnically diverse, urban area. Two of the groups implemented reform-oriented mathematics curriculum on fractions, while the other group used traditional textbooks, followed traditional teaching practices, and expressed no interest in reform-oriented mathematics practices. The lesson addressed in the study dealt with fractions. They collected data on all of the 624 children present (274 boys, 271 girls, and 79 that the research team did not have gender information on) on the day of the student assessment. The students represented diverse ethnic backgrounds.

Stipek et al (1998) used videotapes and field notes to gather information and to code the conversations. They coded both teacher and student conversations. Students completed a questionnaire on two different occasions, once a few weeks after school began, and once after they completed the unit on fractions. The questionnaires contained identical questions on both occasions,
and addressed motivation dimensions. The fractions assessment included both procedurally and conceptually oriented problems, representing traditional and reform-oriented curriculum, respectively. Stipek et al. hypothesized that the classroom practices that enhanced a mastery orientation would enhance performance on the conceptually oriented problems but not on the procedurally oriented problems.

The research team conducted a large variety of correlation computations, but for the purpose of this review, I will focus on the results of the regression analysis, which showed that the more teachers themselves both expressed positive emotions while teaching fractions and provided a social context that encouraged risk-taking, the more students reported that they sought help when they had difficulty, focused on learning and understanding, and experienced positive emotions while learning about fractions. Students experienced more positive emotions and enjoyed learning fractions when their teachers focused on their learning and mastery, and also encouraged effort and autonomy. This included extensive feedback on assignments and a classroom community that encouraged risk-taking and conceptual understanding. In climates that promoted risk-taking results were positively associated with students' mastery orientation, help-seeking, and positive emotions associated with learning fractions. These classroom cultures often emphasized effort and learning, while de-emphasizing performance. The other style of teaching, Differential Student Treatment, did not show any significant correlations to student motivation.
Stipek et al. (1998) also calculated correlation coefficients between the frequencies with which teachers gave different kinds of evaluative feedback, ranging from check marks for completeness, and extensive written comments on the content of the answers. The results showed that when students received papers back with indications of the number of errors or correct answers, they tended to experience fewer positive emotions when working on fractions \( (r = -.42, p < .08) \). Students tended to be less mastery-oriented when teachers claimed they put check marks indicating completeness \( (r = -.49, p < .05) \) and the less they claimed to experience positive emotions \( (r = -.51, p < .05) \). Students reported somewhat higher self-ratings of ability on fractions \( (r = .42, p < .09) \) after teachers gave substantive written comments. They also reported more of a mastery orientation \( (r = .67, p < .05) \), and more positive emotions while working with fractions \( (r = .52, p < .05) \).

Stipek et al (1998) found that their findings were consistent with past research. The correlations on this study supported the value of mastery or learning orientation over performance orientation. More positive emotions, greater enjoyment, and fewer negative emotions tended to be associated with mastery orientation more than students who rated their competencies relatively low.

The affective climate, which Stipek et al (1998) claimed to be the least studied of the three teacher-practice dimensions, showed the most powerful predictor of student’ motivation. A classroom community that included a positive
affect that supported risk taking produced more students who associated learning with mastery orientation and help-seeking.

This study showed that in classrooms where teachers gave the students autonomy, and emphasized effort and learning rather than performance, the students had positive learning experiences and had greater motivation to learn fractions. The students made greater gains on items that required conceptual understanding than students in classrooms in which teachers did not focus on the task and learning in the same way.

This study, though it only focused on teaching and learning fractions, seems generalizable to other fields of math for this age group. The conclusions from this study are consistent with the data, and represent a mathematical classroom community that fosters learning for the sake of conceptual understanding over grades. This allows more students who do not understand the mathematical ideas to claim their lack of understanding and get help when needed.

The motivation gap may begin to close based on teacher influences in classroom culture. In a study conducted by Summers (2006) quantitative data showed the impact of peer learning groups on influencing goals in a mathematics classroom. She hypothesized that after participating in a math peer group in a socioconstructivist classroom, group members would have common academic and social goals, which would predict achievement goals.

Two hundred sixth graders from a middle school in a midsized, diverse city participated in the study, along with two female teachers who had responded
positively to a five-point Likert survey addressing constructivist teaching styles and beliefs. Students filled out two self-report surveys, one near the beginning of the school year, and one near the end. The survey included questions about social goals, achievement motivation, and friendship pertaining to their best friend. The researchers computed mean scores, standard deviations, and reliability estimates for all three sections of the survey.

Summers’ (2006) findings showed that students who valued the importance of group work as a learning activity, not a social activity, tended to have high task orientation toward the end of the school year. Students who reported low social intimacy on the second survey tended to report high performance-approach goals, probably due to a lack of peer influence. Summers stated that students who are more concerned with being popular than having close friends tended to have strategies for “looking smart,” which is a characteristic of students who are performance-oriented in school. Students who tended to not have a best friend who met their validation and caring needs tended to have strategies that helped them to avoid embarrassment in math class. At the peer learning level, the random intercept as well as socially shared achievement goals were significant and positive predictors of change in performance-avoid orientation. This indicated that students who belonged to groups with high levels of shared achievement goals in math tended to change significantly in their performance-avoid orientation goals by the second survey.

An overall decline in motivation during this developmental period may be due to a lack of environmental fit with students’ psychological needs, including
more competitive grading structures. Competition for grades among sixth graders may therefore heighten students’ need for social comparison as a source of information about their own performance or abilities in math.

Overall, Summers’ findings showed that students may have multiple goals pertaining to both individual and group levels. This proved true, however, only for change in performance-avoidance orientation, for which shared achievement goals were a significant predictor at the group level in only one of the teacher’s classes. Not all outcomes of group work are positive in terms of the mathematical agenda, and this is due to the students’ motivation, especially if group work is used too often, because of their particular roles within peer groups outside of the sociomathematical environment.

There are other more reliable methods to retrieve data from the sixth graders than those used in this study. Surveys, in general, may yield inaccuracies. For instance, the students’ peer groups possibly influenced the survey taken by the sixth graders. This may have affected their abilities and willingness to participate in mathematics, therefore impacting the results of the two surveys. The focus of this paper is on elementary school classrooms, and because this study reported consisted of middle school students, the results of classroom community and peer group formation may have been different, had the study been performed in an elementary school, where the students would typically be with each other for the majority of the day, not in just one class.

McClain and Cobb (2001) analyzed a teacher’s proactive role in developing sociomathematical norms in a first grade classroom’s microculture.
They believed that teachers have the influence to develop their students' disposition towards mathematics by guiding the development of sociomathematical norms. Throughout the teaching experiment, McClain and Cobb collaborated with the teacher to attempt to guide the development of the sociomathematical norms. They also traced the development of these norms as they progressed over a four month period. They collected data for the study over the course of one school year. Eighteen students participated in this study (11 girls and 7 boys) in a first grade class from a large suburban area. The female teacher had been teaching for four years. McClain and Cobb recorded data using videotapes from 103 mathematics lessons, copies of student work, three sets of daily field notes, notes from the daily and weekly debriefing and planning sessions, and four clinical interviews conducted with each student done throughout the year.

The mathematics the students engaged in encompassed mental computation and estimation with numbers up to one hundred. For the purposes of this critique, I will focus on the established classroom social norms and the norms that developed throughout the course of the study. The classroom teacher, as well as the students established all of the classroom's social norms, and the community renegotiated the norms when necessary. These norms applied to all subjects, not just math. McClain and Cobb (2001) addressed six classroom norms that arose: (1) Students were expected to explain and justify their answers after being called on because of a raised hand, (2) when student's contributions were judged as invalid by the class, the teacher intervened to clarify
that the student had acted appropriately by attempting to explain his or her thinking, and also emphasized that the situations of this nature did not warrant embarrassment. (3) Students were expected to listen and try to understand others, and to speak loudly and clearly. (4) the teacher often commented on or re-described students' contributions, generally notating the work on the overhead projector or whiteboard. (5) Students were expected to indicate when they did not understand something by posing a clarifying question, and (6) if a student believed a given answer to be invalid, he or she was expected to explain why. It is interesting to note that students rarely claimed invalidity, but challenged solutions by explaining why they did not understand by explaining why the given solution did not make sense.

During the beginning of the study, the teacher regarded all students' responses and answers to questions as equal, and she did not attempt to contrast solutions or indicate which solutions contained more mathematically valuable content. Researchers noted whole class discussions at this time as being disjointed and students tended to repeat each other often. Students also appeared to not listen to others, and only waited for their turns to speak. These discussions, therefore, did not prove useful in gaining conceptual understanding. The researchers addressed this issue with the teacher, and the classroom teacher began making changes in what she deemed as useful in a classroom discussion. Soon the community began showing emerging signs of productive mathematical communication as they began discussing the relevance of their responses. This caused new norms to evolve, the most prevalent being the
sociomathematical norm of offering mathematical difference. Conceptual understanding of the mathematical ideas began to form through the use of negotiation, discussion rules, and the different strategies used by the students.

This study analyzed the teacher’s role in proactively developing sociomathematical norms, however, the norms existing within the classroom did not prove to be mathematically productive, leading McClain and Cobb (2001) to address the teacher and suggest new norms for classroom discussions. The study had not originally aimed to assess these norms, which may have been a confounding variable in the research. However, a transformation occurred in the students’ abilities to have mathematical discussions to gain conceptual understanding. Social norm development, with special attention to mathematical norm development in McClain and Cobb’s study, is crucial in building a productive mathematical community, and is also addressed in the next study.

Nathan and Knuth (2003) presented a study in which the social aspects of classroom community, and the role the teacher played in developing the social norms related to the mathematics community in the class. Nathan and Knuth’s study also examined the influence a teacher has on how a collaborative classroom communicated and functioned. Nathan and Knuth focused on the teacher factors on collaboration and whole class discussions.

Nathan and Knuth (2003) observed one sixth grade mathematics teacher in a public middle school over a two-year period. This included classroom observation as well as summer professional development courses. They collected data using videotapes, written field notes, and audiotapes. Though the
researchers believed that students’ original contributions were the central feature of productive discourse, the role of the teacher who facilitates these discussions had a central roll in the effectiveness and the productivity of the discourse. The research presented consisted of two episodes in which typical discourse occurred in the classroom, one from year one, and one from year two. The two episodes were used to demonstrate the findings of the study.

Typical whole-class discussion that occurred during year one of the study portrayed the teaching style of the teacher, though it did not portray her said beliefs. The teacher believed that students learned best from their peers through class participation. In this first episode, one student presented her answer to the mathematical problem posed to the class at the overhead projector. The teacher frequently interrupted her with statements and questions that the teacher thought would clarify the error the student made to other students. She believed that more conceptual understanding would be gained with her interventions. Nearly all mathematical conversations flowed through the teacher, and generally she spoke to the class as a whole. This teaching method seemed to not fit her teaching philosophy. Nathan and Knuth (2003) as well as the teacher, observed in the year one videotapes that students listened to each other poorly. After the school year ended, the teacher decided to change her approach in teaching mathematics.

The episode used to present the findings in year two of the study showed that the teacher changed her teaching method during whole-class discourse. In the second year the teacher decided to place herself in a place not physically
located at the front, center of the circle of the desks. This marked a drastic
difference in her teaching style. Instead, she remained out of the circle entirely,
at her desk. This, she believed, limited her interruptions of students and kept the
discussions student-led. The teacher limited her intervention to social scaffolding
rather than mathematical scaffolding. Frequent misstatements, and several
competing mathematical hypotheses resulted, and students found no apparent
methods to address their confusions and differences. In essence, no authority
for mathematical value other than student opinion existed. However, the students
listened to each other and had more productive discussions than in the previous
year.

The findings in this study showed the impact that the actions or non-
actions of the teacher had on whole-class collaboration. The class used in this
study consisted of middle school aged students, which represents a different age
group than elementary examined in this study, however, whole-class
mathematical discourse is something that can be done across any ages. This
study presented ways in which teachers can influence the general flow of the
discussions.

The next study went more in depth about how social aspects affect
learning in a classroom. The classroom community greatly impacted the quality
and quantity of the mathematical material learned in this classroom, and the
focus on student collaboration and participation in group discourse made a
difference in students’ learning.
Black's (2004) ethnographic study focused on the factors that influenced the occurrence of effective interactive teaching in the classroom, measured by interaction with the mathematics taught. The nine and ten-year-old students in this study came from a large town in England which contained a concentration of British Asian and working-class children. Black observed 12 girls and 17 boys in the class. The Caucasian teacher instructed a class with a small minority population. The class covered a wide range of abilities. This qualitative study examined student access to beneficial, productive forms of learning based on their cultural background according to class, gender, and ethnicity.

For the purpose of this paper, I will focus on the observations from the mathematics classroom. Classroom research took place over a five-month period where Black (2004) observed a total of 24 hour-long math lessons. She observed the class two mornings each week. Black collected data using videotapes and audio recordings, as well as a teacher interview. She examined teacher-pupil interaction as well as how the interactions contributed to the long term microculture of the classroom.

Black (2004) broke the data from the classroom study into three stages of teacher-pupil interactions: Content Analysis Stage, Practice/Institutional Stage, and Cumulative Stage.

The Content Analysis Stage identified verbal actions that Black (2004) noticed in each interaction. It incorporated the underlying meaning behind the teacher's aims at the time, and the ground rules of classroom discourse. Black used one classroom episode in the analysis of this stage, called Number
Machines, in which students learned about the relationship between calculations. Black showed a situation in which one student, Philip, dominated a classroom discussion about an answer, while the teacher confirmed his ability and allowed him to break classroom norms. The teacher also discounted the method of another student, Erica, without giving her a chance to explain her thinking.

The Practice/Institutional Stage highlighted aspects of the social contact that appeared to affect the meaning behind what the teacher or pupil communicated, such as the teacher’s expectations of a pupil’s abilities. Black (2004) claimed that the interaction stated above indicated that the teacher had high expectations regarding Philip’s ability because she allowed him to break the classroom norms and corrected Erica’s answer. Black argued that this sent out a message of expectation regarding Philip’s ability in relation to Erica’s. The teacher, in this situation, claimed that she used Philip’s answer to move the class along because of time constraints.

Lastly, the Cumulative Stage tracked the student’s history of participation in a series of steps that demonstrated how it contributed to their identity and the social structure of the classroom. This stage quantified the qualitative data analysis. The findings presented by Black (2004) showed that the same student in the Number Machines lesson, Philip, participated in a total of 52 interactions during whole-class discussions, while the class averaged a mere 20. Black coded most of these interactions as productive (30 interactions; class average of 10), but she also coded many as unproductive (22 interactions; class average of 8). Black concludes that this participation contributed greatly to his social
positioning within the class. Philip had a high-ability status, which the class and the teacher reconfirmed for him throughout the school year.

Black (2004) concluded that learning goes beyond the acquisition of knowledge on a subject. Learning is about understanding the behavior, context, and language that goes with a learning opportunity. It is learning how to be perceived as a high ability student. This happens over time, and is a process in which students move toward membership within the community by engaging with members who are older. Philip showed an increase throughout the year in his participation, and in the end was regarded as a high ability student. It is unclear based on this study if, indeed his performance throughout the year showed mathematical gain in ability, but in having the discourse that was rewarded by confirmation of his ability, he was more able than other students to interact with the mathematics.

Standard classroom social interaction in England may differ slightly from the United States. However, it is useful in understanding the effects of teacher instruction and discourse structure and status shaping. Black’s (2004) conclusions are congruent with her findings and warrant further investigation.

Summary:

The NCTM Standards (1989, 2000) place tremendous value on collaborative group work. Learning, as I defined in Chapter 1, involves students taking new knowledge from the environment and linking it to prior knowledge, thereby creating stronger neuronal and synaptic connections, giving the new
knowledge personal relevancy (Zull, 2002; Miller, 2002; Singer & Revenson, 1996). Researchers believed the environment, as it pertained to mathematics, involved the social influences of peers and multiple ideas and strategies for solving problems, through peer collaboration. The research also showed that all group work is not equal. Depending on the norms set up and followed in a class, and depending on the appropriateness of the task, collaboration can either be beneficial to student learning, or it can be ineffective for some students. However, if done appropriately and if understood by the teacher, collaborative group work in mathematics classrooms has shown to be beneficial in students’ conceptual understanding of mathematics.

In the next section I will explore research on mathematical tasks. Mathematical tasks are characterized by the use of a problem or inquiry based question that engages thinking, creates multiple entry points, and has a variety of cognitive demands, helps students to see mathematics as a useful tool. Student collaboration, as discussed previously, is an important factor in the development of mathematical tasks.

**Mathematical Tasks**

Mathematical tasks, characterized by the use of a problem or inquiry based question that engages thinking, creates multiple entry points, and has a variety of cognitive demands, helps students to see mathematics as a useful tool. People often treat mathematics as a subject that requires rote memorization and little relevance to real life situations. Research has shown that structuring an elementary math class with tasks that require students to delve into the
complexities and real-life implications of mathematics promotes an understanding of a high level of mathematics.

Henningsen and Stein (1997) investigated the classroom factors that hindered and/or supported students’ engagement in high-level mathematical thinking and reasoning for “doing mathematics” in terms of high cognitive demand. The researchers looked at ways in which classroom-based factors shaped students’ engagement with high-level mathematical tasks. The participants in this study represented samples of mathematics classrooms that participated in the QUASAR project, as described previously, which focused on the nature of mathematical tasks as vehicles for building student capacity for mathematical thinking and reasoning. The study used for this critique focused on the maintenance of high cognitive demand during mathematics instruction.

The study identified 58 of the 144 mathematical tasks as being set up to encourage the highest cognitive demand of doing mathematics. Henningsen and Stein (1997) used these 58 tasks for their study. They observed students while completing these tasks, and they observed 22 of the students actively engaged in doing mathematics. Students in the remaining 36 tasks did not demonstrate doing mathematics. In these 36 tasks, they observed students focusing on procedures without making connections to the underlying meaning of the task, students engaged in unsystematic exploration, and students who lacked a mathematical focus, and other cognitive phases. Henningsen and Stein identified the factors they associated with the maintenance or decline of the doing-mathematics tasks, and after making these profiles they selected
classroom episodes that reflected (a) the maintenance of high-level cognitive demand and what supported this, and (b) the three identified patterns of decline and the factors that aided to this. Henningssen and Stein created detailed reports describing the nature of the mathematical task in each episode, how the teacher set up the task, how the students implemented it, and how the identified factors influenced the implementation of the task.

Henningsen and Stein (1997) identified five factors as influences in maintaining student engagement in doing mathematics: prior knowledge, scaffolding, appropriate amount of time, modeling of high-level performance, and sustained press for explanation and meaning. They focused on three types of decline of cognitive demand: (1) decline into using procedures without connection to concepts, meaning or understanding, (2) decline into unsystematic exploration, and (3) decline to no mathematical activity.

In their findings, Henningsen and Stein (1997) suggested that time allotment, both too little and too much, provided the greatest influence in the decline of cognitive demand. The removal of challenging aspects of the task after the initial set up of the task, as well as the inappropriateness of the task resulted in a decline in cognitive demand.

In order to maintain cognitive demand, the researchers identified, from their findings, a few factors that influenced the students maintaining a high level of cognitive demand. These included the appropriateness of the task for the students, as well as supportive actions from the teacher, such as scaffolding, and continuous press.
The maintenance of cognitive demand in a mathematical task depends on the students staying engaged with the task. Making tasks applicable to the everyday, real lives of the students in the class can aid in cognitive involvement (Fuches, Fuches, Finelli, and Courey, 2006). The next article addresses the real life implications of mathematics.

The research conducted by Fuches, Fuches, Finelli, and Courey (2006) examined the effects of schema-broadening instruction (SBI) with and without real-life (SBI-RL) complexities. They based their research on the broader the schema of the students, the more likely they will be able to recognize connections between familiar and unfamiliar problems, and would understand when to apply the solution methods they knew. The researchers tested the contribution of explicit instruction in strategies for tackling the complexities involved in real-life problems.

They used the classroom teachers as the unit of analysis in this study. Thirty third-grade teachers volunteered for the study, and they randomly assigned them equally to the three groups (control, SBI, SBI-RL). The groups proved statistically comparable, with insignificant differences, and came from seven schools in an urban district. The students (n=445) in all of the teachers' classes also proved comparable. The researchers administered pretests and posttests to all of the students, and all of the teachers memorized the scripted math lessons.

The study centered on four problem types, all word-problems. All groups addressed one problem type at a time. The control group received a researcher-
designed 3-week instructional unit on general math problem solving strategies. The SBI and SBI-RL groups received the same 3-week general math problem solving unit as well as four researcher-designed 3-week SBI units. They showed videos to the SBI-RL group which portrayed real-life problems in which the students applied their mathematical skills. They also engaged in a discussion about how real-life problems incorporate more information and are more complicated than other problems.

Quantitative results showed that explicit schema-broadening instruction strengthens mathematical problem solving. Effect sizes favoring SBI were large, exceeding 3 standard deviations. Fuches, Fuches, Finelli, and Courey (2006) claimed that when combined with previous findings, these results suggested that explicit instruction broadened schemas to enhance third graders’ mathematical problem solving, and results showed significant improvement over the control group on four of six problem solving scores. The attempt to show the effects of real-life math problem solving by explicitly teaching students strategies for addressing these types of problems produced mixed results, showing positive effects in some cases, and no difference to the SBI students in other cases.

A classroom community that is able to show the steps and procedures of solving a mathematical problem, as well as taking explicit steps in broadening students’ schema, according to this study, improved the mathematical ability of the students.

For mathematical tasks involving real world problems, the solution from a standard algorithm may not always be the most appropriate answer for the
particular situation in the problem. Verschaffel and De Corte (1997) examined four hypotheses based on student ability to solve real-world mathematics problems, and to get answers that represented acknowledgement to the real world. The four hypotheses were: (1) performance on a pretest—students would demonstrate a strong tendency to exclude real-world knowledge while completing a problematic mathematical task where the standard algorithm would not yield the correct answer; (2) predicted a significant increase in the number of realistic reactions on the problematic items from pretest to posttest for the experimental group; (3) the increase in the percentage of realistic reactions in the experimental group from pretest to posttest would not only be significant to the items similar to those learned, but also to items not similar to items used in class; and (4) the positive effect of the experimental program would be lasting.

This study consisted of three classes of children, ages 10-11 in the experimental class, and ages 11-12 in the two control classes. The experimental class enrolled 19 fifth-grade boys, one of the control classes enrolled 18 sixth-grade boys, and the other control class enrolled 17 sixth-grade boys. They held the experiment during the beginning of the school year at a boy’s school in a small town in Belgium, with mostly middle class families. The geographical separation, and the fact they conducted the study at an all boy’s school, may make this study less generalizable than others of the same nature. Verschaffel and De Corte (1997) anticipated no significant difference between the two grades in their tendencies toward unrealistic mathematical modeling at the beginning of the experiment.
Verschaffel and De Corte (1997) incorporated five different teaching/learning units into the study, which they taught only to the experimental class on realistic modeling. Each unit lasted about 2 ½ hours, split into two sessions, and they spread the five units over a 2 ½ week period. Verschaffel taught all of the lessons, though the classroom teacher maintained involvement for the duration of the experiment. The research team videotaped all lessons in this group, and collected all writings of the students during the lessons, which the research team later analyzed. The control classes followed the standard curriculum, which involved several lessons in arithmetic word problem solving. All classes in the study took the same pretest and same posttest which measured students’ disposition to realistic mathematical modeling and interpreting.

In each of the teaching/learning units, students followed a six-phase process: (1) Small group work in mixed ability groups to solve the problem individually and discuss collaboratively, followed by a short guided reflection, (2) whole-class discussion, (3) back to small group with new worksheet with different level of modeling difficulty as the first, (4) whole-class discussion, (5) individual work on a different problem with a typical level of modeling difficulty, to be done in class or as homework, and (6) whole-class discussion, reactions, and difficulties.

Lastly, this study addressed the establishment of new social and sociomathematical norms where, through discussion, feedback, and negotiations, the teacher tried to create new classroom norms that addressed
mathematical relevance and the roles of the teacher and students. They addressed these changed infrequently, and not to the extent intended during the study, however a classroom culture of collaboration and thoughtfulness to the realistic nature of the mathematics evolved within the experimental class.

Verschaffel and De Corte (1997) calculated the results to the four hypotheses. Using the Tukey a posteriori test to measure significance, the first hypothesis did not significance in any of the classes, but did show a tendency to exclude real-world knowledge and considerations in the solutions to the problems. For the second hypothesis, an analysis of variance revealed significance in the Group x Time interaction at $p < .0001$. Tukey a posteriori tests showed a significant increase in the number of realistic reactions from pretest to posttest for the experimental group, whereas it showed no significance for the two control groups. Verschaffel and De Corte claimed that even though they told and showed students in one of the control groups what not to do in real-world mathematics problems the students did not make a significant increase because they did not get a chance to practice the skill. The students did not transform from stereotypical task performers to real world problem solvers (Verschaffel & De Corte, 1997). The third hypotheses also showed significance using the Tukey a posteriori test measuring the two kinds of problems used for the pretest and posttest. The research team confirmed the fourth hypothesis using a test administered to the experimental class one month after the teaching had ended. The test contained five problems even more difficult than the ones administered during the experiment, as well as five parallel to the problems in the posttest.
Retention in the parallel problems measured at 40%, which is only 1% lower than the posttest, and they measured the more difficult problems at 39%. This cannot be interpreted due to not having parallel problems on the posttest, however this score is higher than the 0% - 20% scored by equivalent groups of pupils who took the same test.

This study, as stated above, failed to incorporate classroom culture in the intended fashion. Verschaffel and De Corte (1997) mentioned that not all students remained equally active and productive during the small and large group work, and that the higher achievers tended to be the most active and productive students. This level of action may explain why these students outperformed their classmates. As stated earlier, students need to practice and do what they are being taught in order to understand and retain the lessons. To the same degree that the control class who was told about how to do a real world problem and did not make significant gains in test scores, the students in the class immersed in this kind of problem solving need to practice in order to make a skill part of their schemas. The lack of additional methods to measure realistic reactions hurt the study. Using paper and pencil may leave out a lot of the student’s thought process about a problem, especially if that student possesses poor reading and/or writing skills.

The mathematical task examined in the next study shows the potential students have with intuitive abilities. Mulligan and Mitchelmore (1997) defined an intuitive model of understanding mathematics as an internal mental structure corresponding to a class of calculation strategies. In this model students used
their prior knowledge, or existing schema, to solve mathematical tasks. Mulligan and Mitchelmore sampled 60 girls for this study who started the study in second grade, and continued it through third grade. They randomly selected the students from eight different schools in Australia. The location of the study makes it less generalizable, however, an environment set up in a mathematics classroom is critical no matter what part of the world a student is learning. The researchers chose to use only females for the study so that gender differences would not be cause for any variety in the data, however, that makes this study far less generalizable to boys.

The researchers conducted interviews with the students individually, in which they asked the girls to solve a variety of mathematical problems without paper and pencil. The researchers allowed the use of manipulatives if the students wanted to use them. The questions began with small-number multiplication problems and small-number division problems. They gave the students large-number problems if they succeeded in solving the small-number problems. The team audio-recorded these interviews and then coded the answers for correctness and calculation strategies.

The researchers found no consistency of the students' intuitive models for solving the problems as they related to correctly using the model. Some students used one model correctly on all problems, while others used as many as three different models. They identified three intuitive models for multiplication: direct counting, repeated addition, and multiplicative operations. They also
identified three intuitive models for division: direct counting, repeated subtraction, repeated addition, and multiplicative operations.

Results from Mulligan and Mitchelmore's (1997) study showed that students progressed in the use of intuitive models, as well as an increase in performance. The students showed progress in their ability to interpret word problems, even without specific instruction, students recognized equal-sized group structure in the problems. Conclusions drawn by the researchers suggest that the findings from this study, as well as research from other studies supports their belief that students benefited from being able to solve multiplicative word problems as early as the first year of their schooling, and that multiplication should be taught together, rather than separate, because that is how they are seen by children.

Teachers should create a classroom community that not only allows for the differences in the different models of the students, but also help students to refine their skills and expand their repertoire of calculation strategies.

Differences in student thinking involving steps and strategies in real-life situations for solving a task have been shown in many of the previous studies to be useful in conceptual understanding of the mathematics. What has not been addressed thus far is the impact gender has on the thinking strategies. The next piece of research addressed gender.

Fennema, Carpenter, Jacobs, Franke, and Levi (1998) investigated the differences in problem solving and computational thinking strategies between genders. The longitudinal study tracked 44 boys and 38 girls as they progressed
from first grade to third grade. They interviewed the children individually six times over the course of the three school years, where they solved tasks involving basic number operations and their application to more complex problems. The interviews avoided, for the most part, overt gender bias, and they coded them based on correctness and method used to solve the problem. The initial sample consisted of six girls, and six boys from each of the eleven first-grade classes in the three schools used. A boundary change for one of the schools forced one fourth of the students to drop from the study, and others dropped for other logistical reasons. Eleven percent of the students were non-white, and 11% received free or reduced-price lunch. During the study the teachers participated in a three-year professional development program directed towards teaching mathematics.

The instructional time was not supplemented with a specific curriculum material or set of guidelines, so the teachers and classrooms varied in instruction. Several features characterized the instruction, including most of the instructional time being spent by students solving problems utilizing both single and multidigit numbers, with manipulatives available for use. The teachers also gave the students time to invent their own strategies for solving problems, and to discuss alternative strategies in a whole class or small group discussion.

Fennema, Carpenter, Jacobs, Franke, and Levi (1998) identified two groups of students based on their use of invented (53 students; 35 boys and 18 girls) and standard algorithms (16 students; 14 girls and 2 boys). The students in the latter group possessed no knowledge of invented algorithms by spring of the
second year. Using invented algorithms in the early grades seemed to provide a foundation for solving the extension problems in grade 3 for both boys and girls (Fennema, Carpenter, Jacobs, Franke, & Levi, 1998).

The research did not lead to conclusions that suggested gender differences in accurately solving number fact, addition/subtraction, or nonroutine problems throughout the three years of the study. Differences occurred, however, in the strategies used to solve the problems. Girls tended to use more concrete strategies (modeling, counting), which led to the use of more standard algorithms. The boys tended to use more abstract strategies, which reflected conceptual understanding, but did not tend to use the more standard algorithms by the end of third grade. Both boys and girls who had used invented algorithms in the earlier grades performed better with extension problems, problems that potentially assessed children’s understanding of number concepts as reflected in their ability to operate flexibly with large numbers, than those who did not use invented algorithms.

Though this study neglected to address classroom community, and most of the research came from interview questions, the researchers recorded conceptual understanding found that students who used invented algorithms in the younger grades had better conceptual understanding of the mathematics. A classroom community that can exist with norms that allow for and encourage invented algorithms in both males and females, may result in more overall conceptual understanding of mathematics by the class. The conclusions for this study were congruent with the findings.
Mathematics is not a separate entity from the rest of the subjects covered in a class. The impact on other subjects while being involved in mathematical tasks has not been discussed so far, and in the next study, Pyke (2003) investigated eighth grade students’ strategic representation skills during mathematical tasks. Pyke defined strategic representation as mediators of the influences of the individual characteristics of reading and spatial abilities that can be variously deployed under the moderating influences of task content. The task involved applying geometric knowledge as well as the use of algebraic equations. This study examined how strategic representation influenced characteristics of reading and spatial abilities in cognitive tasks. Pyke stated that other research claimed that reading ability and spatial ability are critical factors that explain the successes and failures of individuals and groups of students on mathematics tests.

The participants in Pyke's (2003) study consisted of 174 eighth grade students from two moderately sized cities, one urban, and one suburban. Two teachers at each school taught the students. Pyke collected data on students’ reading abilities from school’s records, and from a spatial ability test, a test designed by the researcher that included prompts to measure the strategic representation variables. The answers to this assessment gave a sum score that indicated the students’ abilities to apply both algebra and geometry content during problem solving. He measured strategic representation by analyzing students’ responses to prompts embedded in the assessment instrument, Dual Coding Theory (DCT). Students provided this information after each attempt to
solve a task by answering a series of three questions in which they described the problem they just worked on in words, drawings or diagrams, and numbers or symbols. Using Cronbach's alpha test, these prompts were found to assess referential strategies ($\alpha = .77$) more so than representational strategies ($\alpha = .65$) (Pyke, 2003).

Results showed that reading results produced typical reading abilities overall for grade level. These results combined with the spatial abilities showed that students succeeded more with representation strategies and referential strategies than when they used transformation strategies and arrived at correct solutions. He performed many analysis calculations and analyses of the data, and for the purpose of this paper I will focus on the implications of classroom community. It showed that strategies for representation and the prior knowledge, as well as skills associated with reading and spatial ability of the students to develop in coordination with each other. Pyke (2003) claimed that his research showed that the processes by which teaching occurs, that constrain and support the production of external representations. He claimed that multiple strategies without meaningful experiences may not be beneficial in learning math, and will only lead to surface level skills. The constraints of their memory systems limited the students, and too much redundant information interfered with efficient problem solving.

Pyke (2003) discussed two limitations in his study. The first was that Pyke (2003) used a single indicator model, which raised questions because the study lacked convergent evidence for establishing construct validity for the mediating
variables. The second limitation was that the content of the problem-solving tasks were relatively narrow. The tasks did not take the students beyond very basic levels of math, and there were no problems in the tasks that involved real-world contexts. Pyke claimed that meaning needed to be brought into solving a task, but in this case the teacher would need to only be aware of the mathematical meaning of the numbers, but not the context of the task or problem. This excludes the real-world implications that mathematics has, and therefore cannot fully connect to the students' prior knowledge, and it may put the potential for conceptual understanding at risk.

A challenge in mathematical curriculum for the United States is to not fall too far behind in terms of abilities compared to other countries. Japan, one of the leading countries for mathematics education has been used in studies as a comparison tool for measuring mathematical ability in grade school (Fuson, Carroll, Druek, 2000). The premise for this research was to examine the impact Everyday Mathematics (EM) curriculum against students in Tokyo, Japan.

Four hundred ninety six students who were used in a previous study on the effects of EM curriculum were used again for this longitudinal study. In a study discussed previously, Baxter, Woodward, and Olson (2001) showed, on a broad range of questions, that the students using the EM curriculum exceeded the performance of students who were using traditional instruction, and matched or exceeded performance of one or both of the East Asian samples on many of the questions.
The study reported here was twofold. Study 1 followed the progress of some of the same students as the previous study while they were in second grade. Study 2 followed these students while they were in the third grade. In Study 1, 392 second graders participated, of which, 343 students from the original sample were used for data analysis. The students were from 11 school districts, including urban, suburban, and rural districts, and socioeconomic status ranged from low-income to affluent. In Study 2, tests were administered to 620 third graders, of which 236 were part of the original first grade sample.

In Study 1, a whole-class written test was administered. The questions were all read aloud two or more times, which cleared up any reading problems that may have been present. A subset of the test questions were taken from a previous study, with items that reflected number-sense, and mathematics-achievement. That study compared US students (using traditional curriculum) to Japanese students with questions from both schools used in the study. The results of Study 1 compared students who were taught with the EM curriculum to the students in a previous study. For the number-sense portion of the test, the EM students scored significantly better than the US traditional curriculum students on two items, and lower than both the US traditional curriculum students and Japanese students on one item. The question that the Japanese students scored better on was “How many numbers are there between 6 and 2?” a somewhat ambiguous question as reported by the researchers. Fuson, Carroll, and Drueck (2000) suggested that the EM students could have interpreted the question as, “How many steps are there between 6 and 2?” or “What is the
difference between 6 and 2?" There were no other significant differences. In the mathematics-achievement portion, the Japanese students scored nearly perfect on most items, and the EM students scored between the Japanese and the US traditional curriculum students. The Japanese students scores significantly higher than the EM students on six of the most advanced problems. The EM students scored significantly higher than the US comparison on six items.

Fuson, Carroll, and Druek (2000) felt that the problems in Study 1 were largely symbolic, and did not test contextualized computation, which did not capture the whole of the EM students' abilities. The questions for Study 2 were taken from the National Assessment of Education Progress (NAEP), a third-grade cognitively based mathematics test, the EM curriculum, and some were follow-ups to the test used in Study 1. Chi square tests were used to compare performance on all NAEP and Wood and Cobb (1989) items. The EM students were compared, this time, to NEAP comparisons.

Results from this test showed that the EM third graders scored higher overall than did third graders in the NEAP comparison group for number computation, with 6 of the items being significant in favor of the EM group. EM students also did better than the NEAP students on problems that were more conceptual or that had context. Overall, the EM third graders scored higher on items that assessed knowledge of place value and numeration, reasoning, geometry, data, and solving number stories.

The combination of studies presented here shows that with the right test, students can outperform their counterparts. It shows that students will excel if
tested in the manner in which they were taught. The well-roundedness of students’ mathematical abilities may come in teaching for a variety of skills and abilities. The teaching methods and styles were not assessed in this study, however, they were noted as being weak in the areas of whole-class discourse, student responses, description and discussion of solution methods, and extending student thinking. Had the classroom community catered to these areas, test scores may have differed.

**Summary:**

The research presented above shows the role mathematical tasks play in learning for conceptual development. The natures of the tasks influence the cognitive demand and the strategies that students are likely to use (Henningsen & Stein, 1997; Verschaffel & De Corte, 1997). Fuches, Fuches, Finelli, and Courey (2006) found that explicit schema broadening instruction strengthened problem solving ability in mathematical tasks because of the focus on conceptual development. Tasks that involve collaboration put the thinking and the answers that students formulate at the forefront of conversations, and in these cases students will sometimes give incorrect answers. These answers can be just as useful for students’ learning as correct answers, if not more, and will be addressed in the following section.

**Use of Student Mistakes/Errors for Productive Learning**

There is no doubt that when students share answers to others, wrong answers or mathematical flaws will occur. What a teacher or the other students
in the class do with these mistakes or errors can have an impact on the greater classroom community and on the thinking and learning of the students. If students and teachers use mathematical errors for learning, and if they know these errors only as mathematical mistakes and not reflections of the intelligence of the students in the class, they can be used in beneficial ways so that students can gain a more thorough conceptual understanding. The studies in this section outline the effective role student mistakes play in student learning.

Inquiry-based mathematics classrooms can be an effective way to promote conceptual understanding. Battista (1999) showed, in his study, the cognitive development of students' mathematical abilities while working with three dimensional arrays of cubes. The study took place in a fifth grade classroom in which the teacher created a classroom culture of inquiry, problem solving, and sense making.

The math unit she taught for the study was on volume, where students were to find a way to correctly predict the number of cubes that would fill specific boxes described in pictures, patterns, or words. First the students had to predict how many cubes were graphically represented in the problem, then had to check their answers by making a box out of grid paper and filling it with the appropriate amount of cubes. Student work was done in pairs. Three pairs of students were selected for a more extensive analysis, and were given a pre interview and post interview. All sessions were videotaped and transcribed, and the complete task lasted up to three periods.
By the end of the study, all six students in the pair groups were able to properly perform the task. In the post interview all students correctly used a layering strategy for all of the problems except one student, who used a visualization strategy and counted all of the cubes. Results from the total of both classes together showed that the average number correct out of three on the pre interview was 1.30, and the average correct on the post interview was 2.77. Eight students got all answers correct on the pre interview, compared to 37 students on the post interview.

Battista (1999) claimed that one reason that the conceptual understanding was acquired was because the students were able to struggle with the concepts and recreate the procedures several times before internalization. A development in this fashion requires the teacher to recognize that struggle is a means to make understanding. Unlike traditional instruction, where struggles are generally seen as deficiencies, teachers have to recognize the potential benefits of the struggle for sense making and have to allow for this struggle to occur, and have to also know when it is appropriate to intervene.

Batista (1999) documented only a small sample in their struggles and success with problem-based inquiry instruction. The classroom culture within the two classes documented may be reproducible, but it depends on the sociomathematical norms that exist within a classroom. These qualitative results are representative of this study alone, and help us understand how students' ideas develop. More research would be useful in examining inquiry-based classrooms.
As shown previously, discourse is a tool used in reform-based mathematics classrooms to allow students to develop mathematical understanding. The classroom discussions presented in the next study were similar, however, they presented another aspect of discourse that can lead to mathematical understanding. Wood (1999) showed a classroom in which a culture of productive mathematical argumentation helped develop mathematical concepts.

Wood (1999) collected 50 videotaped lessons of one teacher's second grade mathematics classroom over a period of 18 months. She selected specific lessons for further analysis on the teacher's and students' explicit and implicit meanings for mathematics and for doing mathematics. Typical lessons consisted of orienting the students with the activities, pair work, and a class discussion. The teacher explicitly set up, starting on the first day of class, a community that had a mutual set of expectations for mathematical behavior. This allowed the students to engage in disagreements in meaningful ways, and also promoted careful listening, thinking, and social strategies that made these discussions productive and fruitful.

It was normal in this class to have students challenge other students' answers to problems. Mistakes were used as learning tools, and the way in which the students and the teacher handled mathematical mistakes marked flaws in thinking and procedure, not in ability or intelligence. The class was therefore able to defend their thinking, pose alternate strategies, listen to strategies, and gain a better conceptual understanding of mathematics. The findings from this
classroom environment showed that in creating a classroom community in which students struggle with their own thinking and with the thinking of others, requires that teachers understand the relationship between the social process and the mathematical process. If students are not fluent in this kind of exchange, the study showed that more effort would be placed on the social rather than the mathematics, and conceptual development would not come as easily. In essence, when the classroom routines became the tacit patterns of interaction, the children no longer found it necessary to direct their cognitive attention to making sense of their social setting and could direct their mental activity to making sense of their mathematical experiences (Wood, 1999). Wood’s conclusions were congruent with her study and depicted an aspect of student social development that can be a part of every classroom. Her attention to the classroom community in the mathematics class, and student learning and cognitive demand related directly to the focus of my paper.

The next piece of research I will present portrays the role of mathematical discourse in a discussion-oriented classroom. The role of whole-class discussions has been shown, so far, to show positive results on students’ conceptual understanding if done in a community that has mathematical norms that support students in their thinking and reasoning. In Jansen’s (2006) study, seventh graders motivation to participate in the whole class discussions, and the use of mathematical mistakes during the discussions was studied. Because the students in this study were adolescents, their behavior represented typical behavior for students at this age. During adolescence, students generally do not
want to stick out from the group, which can make whole group discussions difficult. The results from this study were more closely aligned with students' feelings and motivation than the role of the teacher on the community.

The seventh graders in this study were from the single middle school in a rural district. The district had used reform-based mathematics curricula in the elementary and middle schools for about 9 years prior to the study, and the middle school was using The Connected Mathematics Project (CMP) for their curriculum, which is a problem-based textbook series that emphasize reasoning and flexibility in representations. Jansen (2006) spent about 100 hours total between the two mathematics classrooms she observed for the study. Though there was data collected that included videotaped discussions, survey data, and interviews, Jansen focused primarily on the data from the interviews. Interviews were conducted with 15 students from both of the classrooms, and the students selected for the interviews represented diversity in gender, achievement, and class participation. The majority of the questions for the interview were about participation in whole-class discussions. For this section of my paper I will focus on the questions from the interviews that dealt with what their reactions were when they gave an answer that was wrong during a whole group discussion. The data were analyzed through a constant comparative process.

Of the students interviewed, eight held beliefs that constrained their participation in whole group discussions, and seven held beliefs that supported their participation. Often students who preferred not to participate did so because they wanted to avoid saying incorrect answers in front of their peers.
Students who did participate in the discussions stated that they often spoke in order to learn the mathematics or to clarify students' thinking who were being misunderstood.

The risks involved in this social setting were present for some, and not for others who were interviewed. When students presented a wrong answer, the teacher generally used the answer and the thinking of the student as a learning opportunity for the whole class. Some students claimed that they learned best from listening to these discussions, however, participating would put them at risk of getting an incorrect answer, and therefore impair their ability to learn the material. The uncomfortable, negative feelings associated with participating in the discussions showed a social structure that did not use all students' thinking and reasoning for the benefit of learning.

The advantages of curricula that support and encourage participation do not always prevail in a classroom setting. There are many factors that can lead to students' lack of participation in whole-class discussions. Some of this may be accounted for in the teaching styles, sociomathematical norms, and some may just be because of the age of the students. It also only reported on 15 white students in a rural school, which may limit the generalizability of the study because it only takes into consideration one microculture. Though this study did not report on an elementary classroom, which makes it less applicable to my inquiry, the issues reported in this study can still be prevalent in elementary mathematics classrooms, especially in upper elementary.
Summary:

The mathematical norms that are present within a classroom may reflect students' willingness to be wrong in front of their peers. Multiple studies presented in this chapter use mistakes as learning devices. Battista (1999) showed the impact of struggling with mathematical concepts in leading to conceptual understanding in a problem-based inquiry setting. Classrooms in which students challenging each other is a norm was presented in Wood's (1999) study that found that students' social lives had an impact on whether or not it was beneficial to challenge or to be challenged in mathematical thinking. The last section in this chapter shows the learning that comes of using mathematical reasoning.

Building on Student Thinking Using Mathematical Reasoning

Students who defend their answers by explaining what they did and why it works build conceptual understanding, and can build the understanding of others in the class as well. Previous sections in this chapter have discussed the importance of classroom norms and student collaboration, and the same applies for this section. The studies in this section will outline how using mathematical reasoning can build on student thinking in a variety of settings.

Student collaboration using mathematical tools can be useful in furthering student thinking and understanding. There does not, however have to be a classroom community that focuses on collaboration and forming a cohesive community in order to use these tools. McClain's (2002) study looks at how computer-based tools can be used in a manner in which to aid in discussion and
argumentation in order to further mathematical reasoning in students. McClain was interested in the influences as well as the confluences of students' and teacher's understandings on the quality of whole class discussions. This study showed how classroom community can be used in conjunction with the mathematical tools, such as computer programs. In essence, classroom community is used to aid in the effectiveness of the tools. McClain used herself and her students as the subjects in this study. The focus of the research was on communication between herself and her students, as well as her students in collaboration. The prime directive of her method was to show student contribution to discussions while attending to the mathematics of the problem. The role of the teacher in this situation was to constantly judge the nature and quality of the students' contributions in smaller groups, and use contributions that adhere to the mathematical agenda. This is in contrast to the teaching method that allows students to share their solutions without concern for the potential mathematical contributions. Social norms within the classroom were set up prior to the study, and included explaining and justifying solutions, attempting to make sense of explanations given by others, and challenging others' thinking.

This study was broken into two parts, the Batteries Episode, and the AIDS Episode. In both episodes students worked in pairs or small groups and analyzed the data they were presented with. The data required the students to analyze information and make a decision about which brand battery was better, in the Batteries Episode, or which treatment was better, in the AIDS Episode. The students were asked to structure and organize the data they were given so
that someone else would be able to make a reasoned decision about which product to choose. The meant that students were asked to investigate patterns in the data and discuss with their partner how to best understand and represent the data. Two computer-based tools were used by the students as analysis tools for the two sets of data. There were multiple features that the students used, including range or variability, data partitioning, identifying the median, or estimating the mean. Both tasks required a computer-based tool, and both required small groups to take a position and argue their solutions. Each group presented their reasoning during a whole group discussion of the episodes. The teacher’s focus, at this point, was not to intervene or to correct students’ errors, or to help with strategies for the task. Her role was to gain an understanding of the varied ways the students were using the tool and approaching the task so that she could use the strategies that aligned with her mathematical agenda and use that for whole class discussions.

Subsequently, a discussion and critique of the students’ analyses was held in the large group. McClain’s (2002) role here was to facilitate the discussion by selecting specific students or groups to share their methods and solutions. She also highlighted the aspects of the solution that were mathematically significant. Classroom norms for discussion and argumentation were also attended to at this time. This addressed the importance of the relation between the negotiation of classroom social and sociomathematical norms, and the students’ mathematical development.
The results of this study remained unclear, though McClain (2002) came to the conclusion that a teacher's understanding of the students' offered explanations and justifications to a mathematical task during a whole group discussion is necessary. Though the mathematical agenda always seemed to be at the forefront of the McClain's lessons and activities, some aspects of her teaching fell short of her goals. McClain noted that she did not acknowledge that for many of the students the activity was about completing a school task, not about analyzing data. Perhaps there was more emphasis on simply using the tools rather than how the students were using the tools for their analysis. The discussion also fell short of McClain's expectations. Constant renegotiation was necessary in order to have the kind of discussion she had planned for. After McClain shifted her focus to the students' activity as supported by the emerging norms did the tasks, tools, and conversations began to support her initial mathematical agenda.

This study points to the necessity of looking at the classroom from many perspectives as to keep the mathematical agenda present so that students will feel motivated to complete a mathematical task for the sake of understanding, not for the sake of completing the task. McClain (2002), however, was clear in what she believed her strengths and weaknesses to be, and they are congruent with what the study shows.

McClain (2002) believed that argumentation could be a useful tool in getting to the core of the mathematics. She claimed that in order to effectively argue a mathematical stance, the math has to make sense to the student.
Reasoning does not have to come in the form of argumentation. Mathematical discourse in a classroom also promoted thinking and reasoning if the classroom norms allowed for students to feel comfortable talking about their mathematical reasoning.

When working with diverse students, teachers may have different approaches to discourse. White (2003) researched the questions, “How do teachers use classroom discourse to teach mathematics?” and “Does the discourse enhance the educational experiences of the diverse student population.” She claimed that productive classroom discourse involves all students, which places responsibility on the teacher to decide the appropriate time and ways in which to encourage students to participate.

In her study, White (2003) examined two third-grade classrooms, and the use of discourse in these classrooms to make educative experiences for the diverse populations. The study took place from January to June on eight occasions. The schools were in a large urban district just outside of Washington D.C. Both teachers were in their second year of teaching and were also both white. Both classes were diverse (88% students of color in one class, and 91% students of color in the other), and were heterogeneous as far as mathematical performance.

Data for this study were collected through transcripts and field notes, in which White (2003) passively observed in the classroom. Transcripts were taken from audiotape via a microphone worn by the teacher, allowing both large group and small group discussion to be recorded. Notes were made during the
instruction of nonverbal actions and selection of students. A second set of data were collected using individual interviews with both classroom teachers on questioning patterns used during classroom discourse.

White (2003) found that four general questioning pattern themes emerged: “(a) valuing students’ ideas, (b) exploring students’ answers, (c) incorporating students’ background knowledge, and (d) encouraging student-to-student communication” (p. 41). Simply asking challenging questions and hearing answers given to students, White claimed, is not enough to allow students to understand conceptually the mathematics of the question for some ethnicities. Questioning has to be used to guide the teacher and to adjust the pedagogy. She claimed, further, that these strategies are not typically seen in diverse classrooms, and should be based on student learning. High press is not addressed in this article; therefore there are no indications if this would be beneficial to diverse groups of students. The themes stated above have potential to lead to discourse that may not be effective, and should be paired with questioning strategies that keep student expectations cognitively demanding. A classroom culture that encourages students to work collaboratively through mistakes and reasoning may still incorporate the findings of this study, and therefore cater to diverse populations within the classroom.

Results from this study found that for the African American and Hispanic students who participated in this study, asking challenging questions and listening to students’ answers and solution strategies alone are not enough to bring change in their mathematical content knowledge. White (2003) suggested
that effective teachers must interpret students’ responses as indications of understanding, and adjust the pedagogy accordingly. Teachers must become skilled listeners who are able to build on their students’ ideas to stimulate further thought, not just provide the curriculum. She claimed that this can only happen when mathematical discussion is a classroom norm.

In the next study, reported by Wood, Williams, and McNeal (2006), children’s mathematical thinking was examined in different classroom cultures to begin to answer what the relationship between children’s verbalized thinking and specific interaction patterns are. The basis for the research was that children’s mathematical thinking for conceptual understanding comes through thinking and reasoning. This construction of knowledge is developed because of a link between social interaction, development of thought, and construction of knowledge.

Five classes were used for this study. Four classes were selected because they were reform oriented mathematics classrooms, two categorized as reform strategy reporting, and two categorized as reform inquiry/argument. The other class was included in the data for comparison, and used a textbook-based curriculum. Two classroom cultures were identified in this class – conventional textbook, and conventional problem solving. The five schools were located in a medium-sized city, and were in the same school district. The researchers chose five lessons from each group for extensive analysis.

Analyses were conducted both qualitatively and quantitatively in which two coding schemes were used. One was for analysis of interaction patterns, and
the other was for children’s mathematical thinking. Transcribed videos of the lessons were used for this process.

Interaction patterns for the four groups – reform strategy reporting, reform inquiry/argument, conventional textbook, and conventional-problem solving – showed that in the four reform classrooms the interaction patterns increased progressively across time, and the types of interaction also changed. Textbook lessons had the fewest interactions (n = 34), where as inquiry/argument had the most (n = 110).

The data for the kinds of children’s mathematical thinking produced similar results as the interaction patterns. Children in the conventional text book group did not have a variety of levels of mathematical thinking. They were limited to recognition and recall skills, which are the least complex of the cognitive operations. There were only 5 incidents of mathematical thinking observed in the lessons analyzed, an average of about 1 in every 100 minutes of class time. The other conventional group had a little more variety in their thinking, and were at a higher level of cognitive demand, however, this group only had 21 incidents of mathematical thinking in the lessons analyzed, about 19 minutes in every 100 minutes of class time. Both reform groups had thinking at all levels. The strategy report group had 92 incidents of mathematical thinking, averaging about 26 minutes per 100 minutes, and the inquiry/argument group had 148 incidents of mathematical thinking, an average of about 41 minutes per 100 minutes.

The interrelationship between types of interaction patterns and the nature of children’s mathematical thinking expressed within these patterns seemed to
have an effect on the outcome of the mathematical thinking in the students. The social interactions that were established in the classroom specifically affected how the students constructed their mathematical knowledge within the classroom. The results of the study showed that higher levels of expressed mathematical thinking happened when there was greater involvement from the students. This is to say, that in a classroom community that stressed involvement and reasoning in a collaborative setting resulted in higher levels of thinking expressed by the students. Wood, Williams, and McNeal (2006) claimed that a classroom community that incorporated a social cognitive process in which attention and understanding was shared by the group as an intentional teaching method, was the means by which mathematical thought emerged and developed.

The role of the teacher in mathematical discourse is important, as stated above. The teacher has to know when and where to intervene with student thinking and to what degree to help a student. It has been shown that students learn better if they come across the solutions themselves, so a teacher who merely shows or tells solutions and methods would not be allowing for the necessary disequilibrium necessary for conceptual understanding and deep learning of the mathematical ideas and content. The next study examined the role of the teacher the mathematics environment present in the classroom. The teacher in the next study did not treat all answers equally, which she believed supported the students attempted to understand the mathematics of the problems conceptually, as the students used reasoning to support their answers.
Ansell and Forman's (2002) aimed to examine the multiple perspectives of educational reform through the discourse of a third grade classroom. They chose the teacher in the study based on her congruence to the standards held by the National Council of Teachers of Mathematics (NCTM). This research focused on the specific standard about mathematical communication, where students make sense of the mathematics by explaining their invented strategies and by listening and reflecting on others' explanations so that conceptual understanding will occur.

The study used third-grade participants that lived in a medium-sized city, and attended a tuition based junior school. The students who attended this school generally came from upper middle class backgrounds with highly educated parents. The class used in the study enrolled 17 students (7 girls, 10 boys), all between the ages of eight and nine. The class chose to sit in same-gender groups of three to four students, all at round tables where they worked on math problems individually, or in pairs. For three and a half months, the study observed two 90-minute blocks of mathematics instruction per week. A typical math lesson consisted of three segments: (1) the teacher introduced one or more problems; (2) students worked individually or in pairs on the problem(s); (3) the teacher led a whole-class discussion on the strategies students used to solve the problem.

The study presented two vignettes of math lessons for analysis, and both revealed similar practices. For students who presented invented strategies, the teacher followed her communication norms, stated above. She devoted a great
deal of time to repeating, revoicing, articulating meaning, and explicating students’ strategies. To do these things, the teacher would interject when a student was speaking to ensure that other students understood what was being explained, checking back with the student doing the explaining to be sure they were relaying the same meaning. These norms diminished when students presented their strategies involving standard algorithms. The teacher avoided teaching multidigit multiplicative algorithms at the point of either vignette, so a majority of the class did not understand them. The teacher only elaborated very minimally on the procedures the students used, which gave the other students no opportunity to try to understand their peers’ strategies, like when students presented invented strategies. The teacher spoke to the class about the disadvantages to the strategy, and urged students to use strategies that made sense. She also told her students not to try the standard algorithm if it did not make sense. When one student came to her individually and asked for further explanation of the standard algorithm she did not comply with his request. She told this student that even though this strategy was fast, it was not the best one if he did not understand it, not giving him a chance to try.

She aimed to support student thinking and wonderment, and helped students to make sense of the mathematics they were dealing with, when she changed voice in how she dealt with standard algorithms, she seemed to go against her classroom’s sociomathematical norms in not supporting and making communal the thinking of all of her students. Ansell and Forman (2002) claimed that the teacher strongly believed that students at this age did not appropriately
understand place value, and needed this before the introduction of the standard, multidigit algorithm.

The data from this study is limited. The study did not analyze lessons that included standard algorithms, which made the full mathematical teaching scope unknown. The research focused on the teacher as part of her social environment (the students). The impact and implications on the students still needs to be examined. The next study examined the role of the teacher the mathematics environment present in the classroom.

Kazemi and Stipek (2001) examined different ways press and questioning affected students’ conceptual mathematical thinking. The primary goal of this study, which used both qualitative and quantitative measures, was to show what a classroom community looks like where students are able to participate and build mathematical conceptual understanding, and where student thinking is valued and developed. The participants in the study included four teachers in grades four and five. They all taught the same lesson on the addition of fractions. These teachers were selected from the Integrated Mathematics Assessment (IMA), a project on reform-minded mathematics instruction. The study ranked teachers in multiple areas, and the four teachers chosen for this study were chosen based on their rankings in press. Two of the teachers were chosen because they ranked highest in press, and the other two were lower in press, but not the lowest. All four teachers also scored high on the positive affect scale. The high-press cases were fifth grade classrooms, and the low-press cases were fourth grade classrooms.
This study was conducted in two phases. The evidence accumulated throughout the study consisted of video tapes, one centered on the teacher, and one on several groups of students which were recorded throughout the lesson. Transcripts were created from each tape. The authors were not present during the classroom instruction. In the first phase, the video tapes focused on the teacher were reviewed, and in the second, those of the students were reviewed.

The goal of this study was to measure the nature of the mathematical conversations, not the understanding of the topic, so grade level did not matter. This study showed that classroom norms allow for the growth in student thinking. The four norms prior to the study were: "(a) students describe their thinking; (b) students find multiple ways to solve problems, and they describe their strategies to their classmates and teacher; (c) Students can make mistakes, which are a normal part of the learning process; and (d) Students collaborate to find solutions to problems."

Kazemi and Stipek (2001) revised these social norms stated above after the study to better fit a mathematical community of learners. They believed that sociomathematical norms, which "concern a set of expectations about what constitutes mathematical thinking" (p. 36), were more appropriate for learning and teaching mathematics. The sociomathematical norms that they put forth were: "(a) an explanation consists of a mathematical argument, not simply a procedural description; (b) mathematical thinking involves understanding relations among multiple strategies; (c) errors provide opportunities to reconceptualize a problem, explore contradictions, and pursue alternative
strategies; and (d) collaborative work involves individual accountability and reaching consensus through mathematical argumentation." They do not claim that this is an extensive list of necessary norms for a high-press classroom, but note that others may research may lead to other norms.

After analysis of the study, Kazemi and Stipek (2001) suggested that the quality of mathematical discourse in the four cases were different, though they all followed the social norms. There was a higher standard of learning achieved in the high-press classrooms, where their sociomathematical norms were met. They do not claim to know how to set up these norms, only that when in place, there are higher standards of learning mathematics. There were clear differences on student conceptualization of mathematical concepts in the study between the high and low-press classrooms.

Though all four classroom teachers showed characteristics of inquiry-based mathematics, using press to further conceptual learning, and social norms were in place in all four classrooms that allowed for students to effectively share and discuss different mathematical strategies, the level of press differed. Kazemi and Stipek (2001) claimed that high-press classrooms allow for students to participate in an intellectual mathematical culture in which all students are accountable for learning mathematics.

O'Connor (2001) conducted a study with the aim of answering how mathematical ideas may emerge out of linguistic and discourse substance, with the primary focus on describing the complex work of the teacher in conducting whole group discussions. When using a position-driven mathematical question
to fuel a discussion, O’Connor believed that this kind of mathematical discourse tool can have an impact on student thinking. This study involved a teacher in her second year of the three year Project Challenge. She had 25 students, of which 60% spoke a language other than English at home, and about 85% qualified for free or reduced lunch. By the third year of the program, a year after the case study was completed, findings showed that the mean score for this class on the California Achievement Test on the mathematics portion, the class scored in the 91st percentile. This portion of the test measured computation and concepts/applications. O’Conner did not mention what percentile these students were in during any years preceding the study. There was no implication of the level of increase or decrease in this percentage.

This fifth grade mathematics class was part of an intervention designed to increase the identification and fostering of mathematical talent in groups of students typically underrepresented in higher-level mathematics classes. The students did not have previous training in decimals and fractions, and believed them to be different things. The question was posed as “Can all fractions be turned into decimals, and can all decimals be turned into fractions?” to elicit students to utilize various computational method of transferring decimals into fractions and vice versa. O’Conner (2001) claimed that because of this kind of question, students were able to extend their thinking and engage with the question at a greater cognitive level.

Three days of classroom discussion surrounding the guiding question were video taped and transcribed. The data for this case study was in the form
of transcripts of talk. O'Connor (2001) claimed that though this is complete and thorough, there is still potential for failure to give evidence of claims. High-press questioning was used during these mathematical discussions in this study, which allowed students to continue to explore the nature of decimals and fractions without immediately or explicitly being told the solution or the means to find the solution. Questions were asked throughout the unit to challenge and disequilibrate the students. Students demonstrated a much greater level of knowledge on the subject after these discussions.

Position-driven discussions, O'Connor (2001) believed, not only gave the students the opportunity to take a position, but also to logically present their claims, explain definitions, and analyze the accuracy of their claims. It is in this presentation of reasoning that students learn and want to learn mathematical concepts. Because the increase in test score is not noted in this study, it is unknown how this kind of sociomathematical culture affected the learning of mathematical concepts for the purpose of quantitative results. O'Connor stated that discussion itself was not the reason for the learning, but that the methods and the application of the discussion fostered the learning, however, a direct discussion without necessary procedures may work for other mathematical ideas.

In the previous studies mathematical discourse and the use of mathematical reasoning by the students showed a way in which students can learn for conceptual understanding. The role of the teacher in all of these cases was crucial. Pesek and Kirshner (2000) wondered if a balancing technique commonly used by teachers who have an obligation to teach for understanding
and to teach to standardized tests, is to teach part time for meaning, and part 
time for recall and procedural skill development. Six fifth-grade mathematics 
classes participated in this study, which was part of a middle-class, semirural 
school. Each class was separated into two groups, using random stratification by 
gender and achievement level. In addition to these groupings, six students from 
each treatment group were interviewed. All interviews were audiotaped, and the 
final interviews were videotaped, transcribed, coded, and analyzed.

In this study Pesek and Kirshner (2000) examined instrumental instruction 
– rules and procedures without reason – and relational understanding – what to 
do and why. They compared instrumental instruction prior to relational 
instruction (I-R treatment) to the effects of relational instruction only (R-O 
treatment. All students took a written pretest, and then 5 days of instrumental 
instruction was given to the I-R group, then there was an intermediate test which 
covered the instrumental instruction for the I-R group, a 3-day relational 
treatment given to all students, a posttest following the relational instruction, and 
a retention test 2 weeks later. Five days were spent on lessons focused on 
facilitating the I-R group’s efforts to memorize and apply formulas. The R-O 
group was not present for this instruction. Relational instruction was taught for 
three days, following the instrumental instruction. Both groups were combined 
for these lessons. The instruction was designed to encourage students to 
construct relationships, and it encouraged invented strategies. Students worked 
in cooperative groups and also had whole-class discussions.
Results showed that after an analysis of covariance, the covariates, pretest and the California Achievement Test were both significant, however, the posttest and the retention tests did not show significant results. This data produced no significant data after instruction had started. Qualitative analysis showed patterns in student learning that provided evidence of interference of instrumental instruction on subsequent relational understanding: cognitive, attitudinal, and metacognitive characteristics. There were some clear differences in the understanding of problems. Conclusions drawn from this study revealed that more instruction does not necessarily mean greater or more learning. In essence, balancing teaching part time for meaning, and part time for recall and procedural skill development did not show significant differences, however, spending more time on skill development may not improve learning.

Whole-class discussions can be useful in creating a learning environment that stimulates thinking and reasoning. Crone (2005) examined how mathematical discussion developed in a fifth-grade classroom. She explored both the teacher’s and the students’ roles in these discussions. Student learning was not addressed directly in this study; the focus was on the development of whole-class mathematical discussions.

The research took place over a six-month period in the fifth grade class mentioned above. Predominantly white students from middle-income families made up the class. Crone (2005) took video and audio recordings, along with written field notes for the data in this study. Crone analyzed three episodes that
she claimed captured the development of the mathematical discussion norms within the classroom.

Results from these analyses showed that in Episode 1, which occurred near the beginning of the school year, the assigned pair groups lacked active listening, and showed difficulties coming to consensus. Students showed this kind of behavior frequently, but the teacher continued to reinforce the norm of justifying personal answers with their mathematical thinking and reasoning. Throughout this process, the teacher frequently conversed about active reflection and participation in mathematical discussions. As the year progressed, so did the discussions. In the next Episode, only about a month after Episode 1, some students spoke more during the whole-group discussion periods. The teacher continued to frequently have explicit conversations about the discourse behavior and norms. In Episode 2, students addressed the norms, as well as the validity of other students’ solutions. Episode 3, in February, which occurred at the end of the study, showed that rather than just sharing answers and methods, students used discussion to look for new ideas. A change the teacher’s role in facilitating the discussion also took place in Episode 3.

These three episodes showed that over the course of the year, sociomathematical norms formed which allowed for discussion and collaboration to flourish. The next study also examined the effects collaboration can have on individuals in the classroom, with a focus on at risk students and students who were high-achievers.
Summary

The studies in this section illustrated instruction in which teachers and students were building thinking using mathematical reasoning. Studies showed the benefits of using position-driven discussions to develop understanding of a task or problem (McClain, 2002; O'Connor, 2001). Students in these studies had to defend their mathematical answers to problems, allowing them to form a logical answer that can be understood by other students, analyze the accuracy of their claims, and therefore have stronger conceptual understanding. Using high-pressure to further conceptual learning also allows for students to participate in an intellectual math culture in which all students are accountable for their learning of the mathematics (Kazemi & Stipek, 2001). The use of mathematical reasoning for conceptual development can prove to be useful in an elementary mathematics classroom.

Summary

The research presented in this chapter showed the effects classroom community has on the learning environment. All students need access to mathematics, and all students need to make sense of what they are taught in order to succeed in their future schooling, as well as in their roles as citizens in our society. Through presenting these studies I showed some of the research that related to my overarching question, what are effective strategies for creating a classroom culture that supports students' learning mathematics for conceptual understanding?
First, I presented research on the impact on learning during the development of multiple student invented strategies, and found that the research showed the usefulness of a classroom community that encourages conceptual understanding by developing strategies that make sense to the students. In the second section, I provided research about student collaboration. Researchers believed the environment, as it pertained to mathematics, involved the social influences of peers and multiple ideas and strategies for solving problems, through peer collaboration. Collaborative group work in mathematics classrooms has shown to be beneficial in students' conceptual understanding of mathematics. The third section involved research centered around mathematical tasks. The nature of the tasks influenced the cognitive demand and the strategies that students were likely to use. In the fourth section of this chapter, research on the use of student mistakes and errors for productive learning was the focus. Mistakes are an inevitable part of learning mathematics and mistakes occur when students share their thinking and their answers. The research in this section showed that the mathematical norms that are present within a classroom may reflect students' willingness to be wrong in front of their peers. Mistakes, in that section were often used as learning devices. In the final section of the chapter, I reviewed research about building student thinking using mathematical reasoning. Studies showed the benefits of using position-driven discussions to develop understanding of a task or problem, as well as using high press to keep all students accountable for their learning.
In the next chapter I will begin to develop the implications of this research for classroom teachers. I will also discuss what further research could be done to better understand the effects of teaching and classroom community on learning mathematics for conceptual understanding.
CHAPTER FOUR: CONCLUSION

Introduction

The goals of elementary mathematics are rapidly changing. We are in a time of transformation, and are at the point where we can either choose to remain in the traditional ways, combine new and old, or we can continue transforming. The research reviewed in this paper provided information about what the current literature says about effective strategies for creating a classroom culture that supports students' learning mathematics for conceptual understanding. All children need to have access to tools and the skills necessary to succeed in mathematics, and the findings in the research showed strategies to accomplish this.

Summary of Findings

After researching the question: what are effective strategies for creating a classroom culture that supports students' learning mathematics for conceptual understanding in an elementary classroom? I found that most of the research pointed to a mathematics classroom that had discourse, collaboration, students' prior knowledge and conceptual understanding at the forefront of the teaching.

I found that developing a community that encourages students to come up with strategies that make sense to them, and that allows the students to use their prior knowledge and schema was useful in developing conceptual understanding (Carpenter et al., 1998; Cramer, Post, & del Mas, 2002; Sharp & Adams, 2002). Sharing and discussing these strategies can be helpful in many cases, especially if a community develops where students listen to each other and try to make
sense of their peers' strategies. The mathematical agenda of the teacher can guide which strategies are shared and discussed in order to benefit students. Allowing select strategies to be shared that point out common mistakes or strategies that will make sense to a large number of the students, and spending time on only these strategies will make effective use of the mathematical period.

Research also showed that student collaboration can be a useful teaching tool. The NCTM Standards (2000) currently place value on collaborative group work for the purpose of conceptual understanding. The environment that is necessary in mathematics is one that involves the social influences of peers and multiple ideas and strategies for solving problems. Depending on the norms set up and followed in a class, and depending on the appropriateness of the task, collaboration can be beneficial to student learning, if it is done as a means to a particular learning goal, and the students understand their roles within the group. Collaboration can also be ineffective, if the groups do not have a clear understanding of the role the small groups play in their learning, and if the teacher is unclear of what he or she wants from the collaboration. However, if done appropriately and if understood by the teacher, collaborative group work in mathematics classrooms has proven to be beneficial in students' conceptual understanding of mathematics.

The nature of mathematical tasks has an influence on the cognitive demand and the strategies that students are likely to use. Tasks that involve collaboration can put the thinking and the answers that students formulate at the forefront of conversations. It is highly likely that some students will give wrong
answers to the problems when asked to collaborate. These mistakes can be just as useful for students’ learning as the correct answers, and a classroom community that places value on wrong answers, without making students feel inadequate or separate from their community can foster growth. The mathematical norms that are present within a classroom should reflect the students’ willingness to be wrong in front of their peers.

Another aspect of classroom community that the research showed to cultivate learning involved students using mathematical reasoning to discuss solutions. Studies showed that when students shared how they got solutions and the mathematical reasoning for why the solution worked and made sense to them, students were able to further their thinking as well as solidify or modify their thinking and conceptual understanding (McClain, 2002; O’Connor, 2001.)

What I found in the research left me with solid evidence about the impact of classroom culture on teaching and learning mathematics. I can now take that information and begin to apply it to how it will effect my future years of teaching.

**Classroom Implications**

Teachers have challenges. This is not unknown. However, knowing how to go about overcoming these challenges may can often be difficult. With 180 days, a typical teacher is required to help all children learn all subjects that are taught in an elementary classroom. For mathematics, children often have to all learn the same material in the same room at the same time. There is probably no class that exists that has each student in the same spot mathematically, which can make knowing what to teach and to focus on difficult for any teacher, even
the most experienced. Teachers cannot allow students to spend months
developing one concept, and likewise, cannot spend too little time in one area,
leaving behind the students who lack understanding. The most mathematically
able students have to learn the same material at the same pace as the at-risk
students, and the experience for each student needs to be educative. This is
where understanding the research I presented in Chapter Three is useful.
Understanding the learning implications for students in the classroom and how to
work with all of the students so they can all be accountable for understanding the
mathematics is the role of the classroom teacher, and the research presents
ways to do this that allow for conceptual understanding.

The findings showed that in order for students to have conceptual
understanding of the mathematics they are learning, the manner in which they
learn the material makes the greatest impact. If students learn mathematics
through standard algorithms, memorization of facts, and little communication,
then understanding the concepts behind how and why mathematics works will be
more difficult. Mathematics makes sense, and children can make sense of
mathematics.

Based on my findings, there are many implications for teachers, and for
my first year of being a teacher. To teach mathematics for conceptual
understanding, teachers should not necessarily show students how to “do it” but
should teach students how to think about the problem, grapple with it, and make
sense of it (Ball, 1993). This is not an easy task, and can easily lead to
confusion and frustration rather than the intended competence and confidence.
Confusion and frustration are necessary for learning, as I discussed previously, but when students are not able to get past this stage of learning it is very likely that they will surrender. In this instance, mathematics would not be accessible to every student, and would therefore be inequitable.

I have found that the best way to bypass these challenges is to set up a classroom community that allows for the struggle, but that also has outlets for need-to-know information. Small and large group work can often help with the discussion of ideas and solutions, but may not always get students the information they need. As discussed earlier, group work is also not always productive. Mathematics teachers have to know the students in their class and be able to recognize when they need to step in and give answers or methods, when to press, and when to let students continue to struggle. It is necessary for students to feel confidence in their learning, and ultimately it is the teacher's responsibility to see that this happens.

**Implications for Further Research**

The research that I used for this paper generally agreed that in order to be successful in mathematics, students need to understand what they learn conceptually. Students in classes that did not promote conceptual understanding tended to not perform as well as students in classes that promoted conceptual understanding. This was not true in all areas of mathematics, however. On occasion, students in more traditional classes outperformed reform-based classrooms in areas such as algorithm computation. Further research is needed to find the underlying truths about why this is the case.
I also believe that more research should be done concerning assessments. Assessments for both classroom analysis and for research analysis can be skewed based on the content of the test, which may not actually test mathematical ability. It may, however, test knowledge of how to perform a specific problem type. This can skew data results, and can be a confounding variable in all research using tests as a measure of mathematical ability. Students perform better on assessments if the assessments are similar to how and what they learned. It seems unfair to students and to researchers to judge mathematical ability based on tests that one group is more likely to score higher on.

Assessments are one necessary aspect of understanding what students understand and do not understand. Without assessment it would be difficult to map improvement and compare groups of students or kinds of curriculum. It is difficult to form a nonbiased test, and more research could be done on different types of assessments and what they actually measure. It could be argued that many of the results I found may measure problem solving abilities more than mathematical ability, but problem solving is an aspect of mathematics that is necessary in order to have mathematic ability. Problem solving skills are highly important when performing mathematical tasks, and the cognitive demand of the tasks measure the level of mathematics being done. However, rote procedures also have value in our society and only measuring one kind of learning and mathematical knowledge may leave out students who are strong in this area.
This leaves me wondering about the value of memorization. There are many schools in our country that find memorizing facts and algorithms valuable. Often there is no explanation about why or how the facts and algorithms work, leaving students conceptually behind. The belief is that if students know this information they will be more efficient at mathematics. One argument for students inventing strategies is that they will become efficient at what they understand. Both methods value efficiency, so if students are taught conceptually why the standard algorithms and facts work, through many of the classroom community methods discussed in this paper, perhaps they would be able to solve problems in the most efficient manner while also maintaining conceptual understanding. More research could be done on the impact of different methods to teaching standard algorithms and facts.
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