COOPERATIVE LEARNING IN SECONDARY MATH CLASSES

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A Project Submitted to the Faculty of

The Evergreen State College

In Partial Fulfillment of the Requirements

For the Degree

Master in Teaching

2013

This Project for the Master in Teaching Degree

by

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has been approved for

The Evergreen State College

by

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June 2013

ACKNOWLEDGEMENTS

The journey that has brought me to this master program, and to the completion of this project, has been long and complex, and without the support of many people in my life, I would not have made it far as I have. I would now like to take the time to thank the people who have helped along the way.

To my friends and family, for supporting me in everything I do, especially in the times when I haven't been able to spend much time with you, and particularly to my mother for helping me pay for my education and for teaching me that the things that I want in life are worth working for.

To my amazing classmates for contributing to a positive learning environment and sharing your skills and knowledge as we've gone through this program together.

To the many teachers who have impacted my life, for showing me what I could do if I put my mind to it, and helping me grow into the resilient human being that I am now.

Finally, to the amazing faculty in the Evergreen MiT program, for providing me with ideas about what it means to be a socially conscience educator and feedback to help me grow as a professional and as a scholar. In particular to Dr. Terry Ford, for the hours you have spent helping me with this project, and with this program, and to Dr. Sunshine Campbell, for your insights into teaching math, along with the readings you have suggested and assigned, which have transformed the way I think about my own classroom practices, and have brought this project into a new light.

ABSTRACT

This paper examines the effects of cooperative learning on the academic achievement, classroom behaviors and attitudes of students in mathematics. An examination of the history of math education reveals why cooperative learning in the math classroom is relevant. The effects of cooperative learning on three different aspects of schooling are discussed. Research shows: 1) a positive correlation between cooperative learning and increased test scores; 2) with the proper interventions, cooperative learning can help students learn how to be members of a cooperative learning community; 3) a cooperative learning environment can be beneficial to students who have previously had bad experiences with the subject; and 4) there are a variety of ways that cooperative learning can be used in the classroom. Overall, cooperative learning leads to success in the general mathematics classroom. Questions for additional research, as well as concerns regarding the future of cooperative learning are presented.

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CHAPTER 1: INTRODUCTION

Introduction

Math (and science) "education in schools around the world is increasingly required to produce high-order problem solvers," (Harskamp and Ding, 2006, p.1670) and on a regular basis studies show a correlation between success in mathematics (based on test scores) and success in life after school (based on unemployment rates and income.) Meanwhile, many students not majoring in math or the sciences think of mathematics as an abstract field with little application to daily life, while mathematics teachers are dissatisfied with student achievement in mathematics (Sezer, 2010). In other words, neither the teachers not students of mathematics are satisfied.

Rationale

Currently, the teaching of mathematics in secondary and further education is predominantly teacher-centered and transmission-oriented. The teacher lectures for a while and students are expected to replicate the teacher's methods. Teachers seem more concerned that students acquire procedural fluency than conceptual understanding or problem solving strategies, and so in the better case scenarios, students are coming out of the classroom prepared to do well on standardized tests, but not to apply their knowledge of mathematics to problems in the real world. In the worse case scenarios, students are struggling to do either. Their test scores are low and they still have not obtained the skills necessary for higher-level problem solving (Boaler, 2008).

The fact is, low achievement in (American) mathematics education has been discussed in numerous policy reports. For years, administrators have appealed for mathematics education reform but little seems to change. Scores have not gone up, and it appears that the traditional method of teaching mathematics through direct instruction is alienating students from the education system. It seems that in order to increase student success, we must revise the way that mathematics is taught (Ke and Grabowski, 2007).

If mathematics is about problem solving, we don't just want students to memorize formulas. We want them make connections between mathematical ideas. Students therefore must construct math by being active. This includes exploring, justifying, representing, discussing, using, describing, investigating, and predicting, rather than just listening to what an expert has to say on the matter (Sezer, 2010). This paper will answer the question "What is the impact of a cooperative learning environment on students in mathematics classes?"

Cooperative groups give students the opportunity to actively engage in the process of learning mathematics. Cooperative groups create support for student learning as they provide an environment where students can discuss, justify, investigate and challenge their ideas, and allow students to communicate mathematical ideas with one another. They provide a relationship of trust, a community where students can test new theories and provide scaffolding for one another in the learning process. Cooperative groups can also provide peer pressure among students. If one student is not pulling his own weight, the peers relying on him will become frustrated and push him to work harder. Since children and teens in particular are often more responsive to pressure from their own peers than to that from authority figures, collaborative group work can give students an incentive to succeed that direct instruction might not.

A reformed vision of teaching and learning of math also calls for new teacher roles. In the teacher-centered direct instruction model, a math teacher's role is to provide students with formulas and example problems. When the students have practiced the skills used in the lesson, then the teacher might assign a more complex problem to the students. Collaborative learning however is, as the name suggests, collaboration between the members of the community. Later this paper will even explore a case study where a math class was seen as a community project rather than an environment where the teacher was the soul source of knowledge. The teacher therefore must adapt her role in the classroom to be a facilitator who provides students with opportunities to collaborate and learn to problem solve (Cohen, 1994, Lotan, 2003).

Historical Background

In the past, and up to this point in time, lecturing has been the most common form of instruction in many secondary schools and college mathematics classes. This pedagogy assumes that learning happens (or can happen) as a transfer of knowledge from an expert to a passive learner. It also however causes student to learn facts in isolation. This memorization of facts might help students prepare for a test in the immediate future, but often leads to the forgetting of the reasoning behind explanations (Sezer, 2010).

Vygotsky's (1982) theory of learning supported an emphasis on social interaction as a vehicle for learning. According to his work, students are able to solve certain problems cooperatively before they are ready to solve the same problems on their own. Cooperative work is often incorporated into these lessons, but this work is typically a discussion that reiterates the procedure provided by the instructor. Examples are usually identical to those the instructor has already solved and require walking through previously established steps rather than creative thinking on the part of the student (Miller, 2002).

In Britain, each year approximately half of all students entered for the General Certificate of Secondary Education fail to attain a grade of C or higher, which in many cases means they must retake a course if they wish to attend college or university. Meanwhile in the U.S., we regularly hear about how the country is behind in math and science, and far too many students perform below grade level in mathematics (Swan, 2006).

Our country has responded to this by upping the stakes of our standardized tests. Under the No Child Left Behind (NCLB) act, schools can be closed for failing to make adequate yearly progress six years in a row. This adequate yearly progress is based on standardized test scores, so when the schools fail to make progress, the emphasis on testing increases. However, there is little evidence to suggest that an increased emphasis on testing leads to higher test scores among students. Instead, teachers need to focus on adapting their classroom practices to create an equitable learning environment for students. (Johnson and Johnson, 2002)

Studies show that collaborative learning can increase students' performance in math and science classes. In one study, children who had previously worked as collaborative pairs on problem solving in physics were twice as successful on a knowledge test as children who had had the same amount of experience working alone. This is not surprising. Math learning has been shown to be associated with specific types of interactions such as asking questions, discussing problem solving strategies, observing someone else problem solve, and explaining one's thinking. In other words, collaborative learning provides an opportunity for students to learn math through interactions (Esmonde, 2009).

If learning results from student participation in an array of practices of mathematical communities, such as questioning and communicating, continued efforts are needed to expand collective knowledge regarding how to understand teaching that supports the development of students' mathematical proficiencies. Mathematical thinking as form of communication implies that communications with one's self and other leads to deeper understanding of material (Staples, 2007).

While most studies have shown that collaborative learning has a positive impact on student learning in mathematics, it is still not as widely used as whole class learning in which the teacher lectures and provided guided practice. This may be partially due to the fact that some research has demonstrated that students profit from a step-by-step strategy in solving problems independently.

These studies show little to no indication of how collaborative learning compares to individual problem solving however.

Another reason that traditional models of instruction dominate despite compelling success of efforts aimed at collaborative and student centered learning environments, has to do with the teacher's involvement in lesson planning. Collaborative learning among students may require more planning time than direct instruction, as the teacher has to serve as a facilitator rather than just an expert. The teacher may also experience a loss of control in the classroom when she has to focus on multiple small groups of student who can easily get off task when working with peers (Staples, 2007).

There is also evidence that different personality types react differently to group learning. For instance, quiet introverted students might prefer working independently, while extroverts are more likely to enjoy working with classmates but can become distracted by conversation with peers. Similarly, some studies showed that certain groups benefited more than others. This could cause concern as certain groups, such as women and many minorities, are already less likely to go into careers in mathematics or study higher mathematics at the college level, so a strategy that benefits groups who are already succeeding more than those who are not could be counter productive (Galton, Hargreaves and Pell, 2009).

Student learning also depends on how the students interact with one another. One study showed that low and middle achievers benefited more from cooperative learning than high achievers did. This could be because high achieving students have less to gain from the support of peers than lower achieving students. However it could also imply that higher achieving students struggle when forced to work outside of the successful methods that they have already learned to use on their own, or may become frustrated when forced to work with low achieving students who have a harder time grasping the material (Ke and Grabowski, 2007).

Definitions

This paper investigates the effects of collaborative learning on student achievement in a classroom setting, and how educators can use collaborative learning to create opportunities for students in the classroom. Some key terminology needs to be defined in order to do this.

First, in order for this paper to have meaning, the reader must understand what is meant by the phrase *cooperative-learning*. For the purposes of this paper, cooperative learning is a process of interaction among peers in order to reach a common learning goal (Arends, 1997; Cohen, 1994). The term *collaborative learning* falls under the umbrella of cooperative learning, though it specifically implies a joint production of ideas. Students share their thoughts and respond and attend to the ideas of their classmates, to generate shared meaning through their experiences. In contrast, cooperation also does not inherently involve extensive participation, while collaboration does (Boaler and Staples, 2008; Chui, 2004; Galton, Hargreaves and Pell, 2009; Harskamp and Ding, 2006; Kotsopoulos, 2010; and Swan, 2006).

The review of some the studies done by will also refer to *competitive learning* and *strictly cooperative* learning. Both of these fall under the umbrella of cooperative learning. However competitive learning refers to students learning in teams that compete with one another, while strictly cooperative learning does not have a competitive element (Madrid, Canas and Ortega-Medina, 2007).

This paper uses the term *transmission-oriented* to describe teaching in which the teacher delivers the majority of the information to the class rather than learning in which students discover information for themselves. (Swan, 2006). Similarly, the terms *teacher-centered* and *student-centered* will be used to refer to teaching styles found in classrooms. Teacher-centered is used synonymously with transmission-oriented in order to echo the idea that the teacher is the focus of attention within a classroom. A student-centered class however revolves around students learning rather than around teachers teaching (Galton, Hargreaves and Pell, 2009; Madrid, Canas and Ortega-Medina, 2007; Siegel, 1998; and Swan, 2006).

Whole class instruction refers to any method of instruction in which the teacher teaches the class together as a large group. (Clarke, 1997; Galton, Hargreaves and Pell, 2009; Linchevski and Kutscher, 1998; and Shachar and Sharan, 1994). It generally falls under the umbrella of *direct instruction*, a term used to describe a method of teaching in which the teacher give students information on a concept or idea and then provides opportunities for the students to have guided practice working on the strategy they have just learned (Arends, 1996; Cohen, 1999; Lotan, 2003; and Zakaria, Lu Chung and Daud, 2010).

Whole class instruction however can also refer to other whole-class learning activities such as a large group discussion.

This paper also refers to studies that focus on the General Certificate of Secondary Education (*GCSE*). The GCSE is an academic qualification awarded in a specified subject, generally taken by secondary students aged fourteen to sixteen in England, Wales and Northern Ireland (Swan, 2006).

One of the studies reviewed in Chapter two refers to sixth-form colleges. Sixth-form colleges are schools in a variety of countries, including England, where students aged sixteen to nineteen typically study for advanced school level qualifications, including for the GCSE (Swan, 2006).

Finally, *jigsaw* is a specific method of cooperative learning. In a jigsaw structure, groups will have a large task to complete, but will split it up into smaller tasks, each designated to individuals. For example, four students might each become experts on different methods for solving an equation, and then teach the method to the other members of their group (Arends, 1997; Jacques, Wilton and Townsend, 1998; and Moskowitz, Malvin, Schaeffer and Schaps, 1998).

Limitations

There are many factors that impress limitations on our ability to understand the effects of collaborative learning in the mathematics classroom. One limitation is the fact that it is difficult to perform truly random experiments in the education system. Many of the experiments conducted in the studies covered in this paper were done in classrooms. Since most schools draw students primarily from one neighborhood, schools do not represent a random sample of the population in terms of race, ethnicity, or socioeconomic status.

It is also important that participants in a collaborative learning environment have a clear idea of how to reach their common goals as well as how to contribute as individual members of the group. If students do not understand their role in a collaborative learning environment, they will likely have more difficulty. This in turn makes it more difficult to evaluate the effects of collaborative learning because it is not always easy or even possible to distinguish the difference between the effects of the collaborative learning process and the effects of the student understanding his role in collaborative process (Cohen, 1994).

It's important that in collaborative learning, the teacher uses group-worthy problems. Particularly if students are unfamiliar with group learning, the relevance of the group work in what is being taught can affect the outcome at hand. Additionally not all tasks are group-worthy. However, a teacher can determine the group-worthiness of a task based on five design features of a group-worthy task. (1) Group-worthy tasks must be open ended and require complex problem solving. (2) They must provide multiple entry points and allow students multiple opportunities to show intellectual competence. (3) Group-worthy tasks deal with discipline based, intellectually important content. (4) They require both individual accountability and positive interdependence and (5) they include clear criteria for the evaluation of the group's product (Lotan, 2003).

Additionally, it can take time for students to adjust to unfamiliar methods of classroom instruction. Many studies are done in short periods of time relative to the years that students spend in the public, or private, education system. If students have become accustomed to direct instruction as a teaching model, they might react to collaborative learning as a deviation from the norm. It is therefore difficult to tell which effects of collaborative learning make long-term differences, which make a difference only in the short term, and those that make a difference because they are an aberration at the time of the study.

It is true as well that much of the mathematics curriculum in this country, is based on standardized testing and teaching to the tests. We therefore value high scores on tests as an indicator of success in learning mathematics. Many of the studies explored in this paper used test scores as a means of evaluating the effects of collaborative learning. Therefore this paper relies on the success of mathematics being at least somewhat dependant upon test scores, whereas there may be other indicators of successful learning in mathematics, and better indicators of mathematical success in the future (Sezer, 2010).

Finally, because many of the studies on cooperative learning in mathematics classrooms rely on testing, a few studies are included in this paper that were in other types of classes such as social studies or science, where testing was emphasized less and group projects were a larger part of the curriculum. While these studies explore ways that cooperative learning can be used in the classroom, it is unclear whether the same methods used in these classes would apply to a mathematics classroom. Still, these studies are worth including because they provide the reader with a wider range of ideas about what cooperative learning in the classroom might look like.

Statement of Purpose

We live in a society where success in learning mathematics is correlated with success in life after school, and where direct instruction has consistently been the predominant model of teaching used in the mathematics classroom in recent years. Meanwhile mathematics test scores continue to be low and many students view math as disconnected from their lives. The approach to solving this problem seems to be to increase the stakes, rather than to alter the methods by which math is taught, while standardized tests are presented as a reason that students need to learn math.

Studies show that collaborative learning has a positive effect on students' achievements in mathematics, as well as on their classroom behaviors and their attitudes towards mathematics and provides a student-centered learning environment. This in turn can provide support for students and make mathematics more accessible to a variety of learners. Therefore, this paper examines the impacts of cooperative learning on students' achievement, classroom behaviors and attitudes toward mathematics.

Summary

Chapter one introduced the reader to the current state of math education in the United States, stating that we live in a society that has a continually increasing need for individuals who are able to solve problems an reason mathematically, but also a society that is falling behind in mathematics education. Cooperative learning was presented as a potential solution to this problem. The history of mathematics education and cooperative learning were discussed and several key terms were introduced. Chapter two examines several studies on cooperative learning environments in mathematics and other classes, and investigates the effects of cooperative learning on students' academic achievement, as determined by test scores. The chapter also considers the effects of cooperative learning on students' behaviors and attitudes about mathematics, as well as ways that different teachers implemented cooperative learning in their own classes. Then, Chapter three provides a summary of the findings regarding the impact of cooperative learning. The chapter also presents implications for classroom practice as well as suggestions for further research, and concludes with some final thoughts about cooperative learning and the direction of mathematics education in the United States.

CHAPTER 2: CRITICAL REVIEW OF THE LITERATURE

Introduction

Chapter one discussed the history of mathematics education in the United States. Historically, math in the United States has been taught in an individualistic manner, where the teacher lectures and provides guided practice for students. Perhaps, occasionally, the students will check answers with one another, but the majority of class time is spent working alone. The current methods of teaching math are also failing to serve the needs of most students. The students are not learning the problem solving skills they need to become successful in life after school.

Chapter two critically reviews the literature on the effects of cooperative learning in the classroom, with a focus on the secondary math classroom. It first examines the relationship between cooperative learning and academic performance, primarily determined by students' performance on tests. Then, the chapter will show the effects of cooperative learning on student behaviors in the classroom, followed by the relationship between cooperative learning and students' attitudes toward mathematics and school. The chapter concludes with an assessment of ways that different teachers have implemented cooperative learning in their classrooms. Together, these sections review a large portion of the research on the effects of cooperative learning in the classroom, and more specifically in the math classroom.

Academic Achievement

Eight studies in this review compared test scores between students who worked in cooperative groups and those who worked in independent learning situations. In a quantitative study done by Zakaria, Lu Chung & Daud (2010), on the effects of cooperative learning compared to more traditional methods with students from a school in Miri, Sarawak indicated that the cooperative learning approach resulted in higher achievement than the traditional teaching approaches. A quantitative study by Harskamp and Ding (2006) on high school physics students asked whether collaborative learning helped student achievement more than individual learning in both cases where students were given hints, and not given hints and found that students who worked in cooperative groups received higher scores on the post assessment than their peers who had worked independently. A quantitative study of 8th grade students in Israel, by Kramarski and Mevarich (2003) found that of four instructional methods, cooperative learning with metacognitive training produced the best results. A quantitative and qualitative study by Leonard (2001) on the influence of group composition on student interactions in heterogeneous and homogeneous small groups followed 95 sixth grade students of fairly homogeneous SES a year for two (academic) years and found that while no significant differences existed between student achievement in heterogeneous versus homogeneous groups, low and middle achieving students received significantly higher test scores after working in groups than they had previously. A quantitative study by Linchevski and Kutscher (1998) on the effects of learning mathematics in mixed ability settings on students' achievement studied 1730 seventh grade students in twelve Israeli junior high schools and found that the effects of similar ability grouping were not uniform: five schools had a positive grouping effect, meaning that students close to the cutoff point for one ability group gained more by being hypothetically part of a lower ability group than of the next higher one while seven schools had a negative grouping effect. A quantitative study by Madrid, Canas, and Ortega-Medina (2007) examined the effects of competitive team peer-tutoring compared to the effects of a cooperative team peer tutoring procedure in academically at-risk, Hispanic, bilingual children and found that both peer tutoring methods lead to significantly higher post-test scores than the traditional teacher-led procedure, but that the cooperative method led to even higher scores than the competitive method. A quantitative study by Retnowati, Ayers, and Sweller (2010) on the effects of worked example and problem solving approaches in individual versus group settings studied 108 seventh graders in three math classes in on Indonesian junior high school and found that group setting did not appear to have any statistically significant impact on student achievement. Finally, a quantitative study by Sherman and Thomas (1986) on the effects of cooperative versus individual learning strategies with general mathematics classrooms taught by two different teachers in a rural, Midwestern, predominantly Caucasian, middle class high school found that the cooperative group obtained significantly higher achievement on the posttest than the individualistic group.

In a quantitative study, Zakaria, Lu Chung & Daud (2010), asked “What are the effects of the Student Teams- Achievement Divisions (STAD) cooperative learning method versus more traditional methods (direct instruction with students working independently) on student achievement in mathematics?” The researchers studied 82 students from a Form 1 school in Miri, Sarawak. 38 were in a control class that was taught using traditional teacher-centered instruction, while 44 students in an experimental class were taught in small groups where they were given the opportunity to discuss potential solutions. The results indicated that the cooperative learning approach resulted in higher achievement than the traditional teaching approaches.

For this study, an achievement test was used to measure the students' mastery of a topic—fractions. The pre and post-test each contained sixteen objectives questions and ten subjective questions, which each student had 60 minutes to complete. To determine the effects of cooperative learning on students' achievement, an analysis of students' pre and posttest mean scores was carried out.

The results of the pre-test indicated that the mean score for the experimental group was 50.34 with a standard deviation of 10.92 and that of control group was 47.68 with a standard deviation of 11.18. The results also indicated that the difference between the mean achievement scores for experimental and control groups $t(80) = 0.281$ was not significant at the alpha level of 0.05. Therefore, the researchers were able to conclude that the experimental and control groups were at the same level of achievement at the start of the study.

The post-test achievement mean scores of the experimental and the control group indicated that the mean score for the experimental group was 56.18 and that of control group was 50.18. The results also indicated that the difference between the achievement mean scores for experimental and control groups $t(80) = 0.031$ was significant at the alpha level of 0.05.

The results of this study indicated that the cooperative learning approach resulted in higher achievement than the traditional teaching approaches. Both groups of students started out at comparable achievement levels, but by the end of the study, the class that had been working in cooperative groups was performing at a statistically significantly higher level than their peers who had been instructed using more traditional methods.

There are a few things to take into consideration when examining the results. Since the researchers only studied two classes, it is possible that the results shown for the two classes were affected by the individual teacher and not by the method of traditional versus cooperative instruction. Additionally, the classes chosen for the study were preexisting learning communities—the students knew each other before the study took place. Therefore, it is possible that the results for the cooperative group would be different if the study had been done with students who had to get to know one another while they worked in groups. However, the two classes were performing at the same level at the start of the study, so any pre-existing conditions that may have impacted the outcome had likely impacted both groups.

A quantitative study by Harskamp and Ding (2006) on high school physics students asked “Does collaborative learning help students’ achievement more than individual learning in both cases where students were given hints, and not given hints?” They found that students who worked in cooperative groups received higher scores on the post assessment than their peers who had worked independently.

In this study, students in a physics class were expected to spend six 50-minute lessons working on six problems about forces and movement and applying knowledge of Newton’s laws that they had been taught the previous year. In the collaborative conditions, students were asked to discuss their responses with classmates, in the “hint” conditions, student were given five hints to help them solve problems. The groups conditions were as follows: (1) cooperative learning without hints (**CL** group): eleven boys, fifteen girls, (2) cooperative learning with hints (**CL + H** group): eleven boys, fourteen girls, (3) Independent learning with hints (**I +H** group): eight boys, fifteen girls, and (4) independent learning without hints (**I** group): sixteen boys, nine girls. All participants completed both a pre-test and a post-test. The pre-test consisted of two problems about forces and the post-test had two similar problems. Both tests were given in open-question format.

The students were grouped as previously mentioned in the experiment. Within each class, students were divided randomly into four groups. All students were required to spend six lessons of 50 minutes working on six problems about force and movement. Students had to apply knowledge of Newton's laws that they had been taught the year before. The researchers gave students log sheets to record the steps to a procedure that the students had previously learned. In the collaborative conditions (CL+H and CL), students had to discuss their individual answers with their peer learner and write down the results on the log sheet. In the other conditions (I+H and I), students filled in the log sheets on their own. In condition I+H the students working individually were provided with five hints.

The ANOVA test for difference between the mean scores of the students in the four conditions yielded a value ($F = 0.31$; $p = 0.82$). These results showed that there was no significant difference among the students in the four conditions on the pre-test. To examine the differences of the treatment effect, an analysis of covariance (ANCOVA) was conducted. The difference in experimental treatment (four conditions) was the independent variable, the pre-test was the covariate, and the post-test score was the dependent variable. A covariate was used to control for small differences between the students of the four conditions that may confound the effect of the experimental condition. The effect of the experimental condition was significant ($F = 2.7$; $p = .048$).

As the differences in means between the four groups indicated, a contrast analysis showed only a significant difference between conditions CL+H, CL, and I+H, on the one hand, and condition I, on the other (contrast estimate = − 3.9; $p = .005$).

The study found that students working in the cooperative groups outscored their peers who had worked independently, but that the use of hints to guide work in collaborative groups did not appear to have an impact on student achievement. Students who had worked in collaborative conditions also reported remembering formulas better.

One of the most-notable shortcomings of this study, in relation to the rest of the paper is that it focused on the effects of collaborative learning in physics rather than mathematics, so it is possible that the results would not apply to another subject. However, physics is also a highly mathematically based science, so effective methods for one would be likely to apply to another. Additionally, the researcher mentioned that the material covered was material that the students had learned the previous year. It is possible that cooperative groups made it easier for students to recall previously learned information, but might not have had as much of an impact had the material been new information.

Finally, the use of hints in the classes in this study, and their inclusion in this paper might seem out of place. However, they help to highlight the significance of the impact of cooperative learning. Compared to the impact of the cooperative versus individual conditions, the impact of the hint condition was negligible. Many teachers tend to lay out step-by-step procedure for their students rather than letting them struggle through a problem with their group. The miniscule effect of the hints compared to the notable impact of the cooperative condition shows that putting students in cooperative groups had a greater impact than other ways that teachers might try to make work easier for students.

A quantitative study by Kramarski and Mevarich (2003) asked “What are the effects of four instructional methods, cooperative learning with meta-cognitive training (COOP+META), cooperative learning without meta-cognitive training (COOP), individualized learning with meta-cognitive training (IND+ META), and individualized learning without meta-cognitive training (IND) on students' mathematical reasoning and meta-cognitive knowledge?” The researchers studied these effects by observing 384 8th grade students from twelve high schools in Israel.

For the study, the students were grouped into four learning conditions as described above. A 36-item test was used to assess students' ability to interpret graphs. Of the questions on the test, 27 items were based on traditional evaluation procedures, they had a correct answer and a standard set of steps students were expected to take in solving the problems. The remaining nine items were open ended and asked students to explain their reasoning. The subjects were selected randomly from twelve classrooms in four junior high schools in Israel. The schools were similar in size and of average socioeconomic status. Within the schools, classes were typically distributed in terms of students' ability and prior knowledge. Each class had one teacher; all teachers were female and had degrees in mathematics education. Students were also given a graph construction test and finally, a meta-cognitive questionnaire that assessed students' meta-cognitive knowledge regarding graph comprehension.

The meta-cognitive training used three sets of self-addressed meta-cognitive questions: comprehension questions, strategic questions, and connection questions. Generally speaking, both cooperative and individualized learning sessions included three parts: teacher introduction to the whole class (about ten minutes), cooperative or individualized seat-work (about 30 minutes), and teacher review with the whole class (about five minutes).

Students were scored on their performance on a graph-construction test, a task chosen because students were previously unfamiliar with the content.

An ANOVA of classrooms nested within conditions indicated no significant differences between conditions before the beginning of the study, $MSe = 5.22$, $F(3, 372) = .2$, $p > .05$, but at the end of the study, significant differences were found between conditions controlling for pretreatment differences, $MSe = 2.95$, $F(3, 371) = 7.19$, $p < .01$.

Post-hoc analyses of the adjusted mean scores based on a pair-wise comparison t-test technique, indicated significant differences between the meta-cognitive groups (COOP+META and IND+META) and the non-meta-cognitive groups (COOP and IND), but indicated no significant differences were found between the two meta-cognitive groups or between the two non-meta-cognitive groups. Results from a second analysis showed that the students in the COOP+META group significantly outperformed the students in the IND+ META group who significantly outperformed the two groups with no meta-cognitive training. The two groups without meta-cognitive training did not perform significantly differently from one another.

Unlike the previously mentioned studies, this was not a study in which cooperative-learning achieved higher results, on its own, for students than did independent learning. With almost 400 students, this study has a larger population than many of the other studies covered in this paper. Therefore, the potential implication that cooperative-learning alone might not be effective has some weight, as does the potential implication that meta-cognitive training is has a greater effect than the cooperative learning. However, it also implies that meta-cognitive training can improve the effects of cooperative learning, and that students need to be able to internalize why they are learning in order for cooperative group-work to be an effective method.

A quantitative and qualitative study by Leonard (2001) asked, "How do heterogeneous and homogeneous groups differ in mathematical achievement?"(176). The study followed 95 sixth grade students of fairly homogeneous SES a year for two (academic) years and found that while no significant differences existed between student achievement in heterogeneous versus homogeneous groups, low and middle achieving students received significantly higher test scores after working in groups than they had previously.

The students selected each academic year (for two years), were 6^{th} grade students and had an average of 32 students per classroom. Each sample (group from each year) was fairly homogeneous in terms of SES with most students coming from working class backgrounds. During the first year of the study, 10.4% of all students in the school were on free or reduced lunch. In the second year this number was 11.1%. The 6^{th} grade population was roughly 78% white, 19% African American, <1% Asian the first year, and 70% white, 27% African American, <1% Asian the second year. The gender profile of the first cohort was 57 girls, 38 boys and the second cohort was 40 girls and 55 boys.

For the study, students' desks were placed in clusters of three or four, with students grouped by ability. Ability was determined by a combination of pretest scores, and fall quarter mathematics grades. The first year (1995-1996) students were placed in heterogeneous groups (differing levels of ability) while the second year (1996-1997); students were placed in homogeneous groups (similar abilities). Lessons were organized by theme, and data from students' pre and posttests were used for quantitative analysis. Students who were absent for either the pretest or the posttest were excluded from statistical analysis.

ANCOVA was used to compare students' posttest scores by cohort, race, gender and ability. The data showed significant main effects for cohort, $F(1, 153) = 6.083$, $p<.05$ and ability, $F(2, 153)=31.097$, $p<.05$. The data also show that lower-achieving ($M=323.60$) and middle-achieving ($M=333.39$) students made significantly higher scores in the heterogeneous group setting than in the homogeneous group setting ($M= 310.40$ and $M= 341.81$ respectively). However, there was no significant difference between high-achieving students in the heterogeneous setting and their counterparts in the homogeneous setting.

The study showed that low and middle achieving students received significantly higher scores on the posttest when working in heterogeneous groups than in homogenous groups. There was no significant difference between high achieving students' scores based on group type.

Since this study was done with just one teacher, it can be concluded that the teacher probably did not have a significant impact on students' scores, though it is possible that the teacher used slightly different methods each year. Further, as the homogeneous grouping was used during the second year, it cannot be argued that the teacher had more experience when teaching the heterogeneous group.

Linchevski, L. and Kutscher, B. (1998) studied students from twelve Israeli junior high schools, asked "what are the effects of learning mathematics in mixed ability settings on students' achievement," and found that mixed-ability grouping had a greater positive impact on students' achievement than did similar ability grouping, and that the effects of similar ability grouping were not uniform among the schools. The researchers studied 1730 seventh grade students from twelve Israeli junior high schools. The schools were all heterogeneous in terms of socioeconomic status. At the beginning of 7^{th} grade, students were assigned to groups based on ability level based and previous tracking, and without the teacher knowing about the procedure. In each class the teaching was conducted within four settings:(a) students working in a whole-class setting;(b) students working in small mixed-ability groups;(c) students working in small homogeneous groups; and (d) students working in large homogeneous groups. In the first and last settings teachers played an active role, whereas in the others they were in a supportive role only. The groups were set up so that each child was a part of two groups at any given time. The heterogeneous groups worked on projects that met all requirements for the general curriculum, while the homogenous groups usually dealt with topics based on students' specific needs. Students remained in the groups for two years.

At the end of the 8th grade, all students took two achievement tests—one that was administered to all students regardless of ability grouping, and one test where students were assigned one of three tests based on ability group levels. T-tests were used to compare, for each level, the achievements of the students in the mixed-ability and same-ability classes. The high-achieving students in the mixed ability classes scored higher than their high-achieving peers in the same-ability groups, though not significantly ($p > .05$) while low and intermediate achieving students in mixed-ability groups did significantly out-perform their similar ability peers in the similar-ability groups. Additionally, many of the lower-achieving students who had worked in similar ability groups had difficulty with the items on the general test that were not included in the individualized topics that had been covered. The effects of similar ability grouping were not uniform. Five schools had a positive grouping effect, meaning that students close to the cutoff point for one ability group gained more by being hypothetically part of a lower ability group than of the next higher one. Seven schools however had a negative grouping effect. This further suggests that mixed-ability groups have a greater positive impact on student learning than do similar ability groups, as intermediate-achieving students in mixed ability groups would be able to work with both higher-achieving and lower-achieving students.

One of the stronger points of this study is that it compares many different potential groupings for students—large and small, heterogeneous and homogeneous. Additionally, since students were given two tests, one administered to all students and one that varied by achievement level, the results allow one to accurately compare the academic achievement of students of similar levels who worked in different settings, and the improvements made by students at all levels of achievement. The study also however had weaknesses—mainly that it compared different types of cooperative groups but did not include a control group of students who worked in a strictly individualistic setting, so while it provides evidence that (mixed-ability) cooperative groups have a positive impact on student achievement, it does not provide evidence of the significance of cooperative learning compared to individualistic learning.

A quantitative study by Madrid, Canas, and Ortega-Medina (2007) asked, "What are the effects of competitive team peer-tutoring compared to the effects of a strictly cooperative team peer tutoring procedure?" The study focused on academically at-risk, Hispanic, bilingual children and found that both peer tutoring methods lead to significantly higher post-test scores than the traditional teacher-led procedure.

The subjects were six boys and ten girls, all third graders who were bilingual in Spanish and English; their ages ranged from 8.0 to 9.5 years. The students attended a local elementary school in a low-income area of a city in the southwest. The teacher identified them because of their history of poor spelling performance and below-average scores on the state standardized tests. The students were taught spelling in three, alternating instructional conditions for fifteen weeks. Each week, one method would be used. The conditions were (a) competitive team peer tutoring, (b) cooperative team peer tutoring, and (c) standard teacher-centered instruction. For the competitive and cooperative conditions, the students were split into two groups of eight, each of which was also split into four pairs. Each day, the students would flip a coin with their partner to determine who would serve as a tutor, and who as a tutee. During the weeks of the competitive condition, the two groups competed to outperform one another. At the end of each week, students were given spelling tests, which they had to take independently.

The data showed that a standard teacher-centered condition resulted in the lowest mean percentage of correct responses, 33%, 40%, 39%, 37%, and 32%, or an average of 36.2%. The weekly posttest means for correct responses for the competitive peer-tutoring condition were 80%, 75%, 80%, 82%, and 84%, or an average of 80.2%. Weekly posttest means for correct responding for the strictly cooperative peer-tutoring condition were higher than both of the other instructional conditions: 90%, 89%, 95%, 94%, and 96%, an average of 92.8%. The mean percentage of correct responses in the strictly cooperative team peer-tutoring condition increased 80.8%, compared with a mean increase of 67.2% in the competitive team peer-tutoring condition. Overall, both peer-tutoring conditions exceeded the teacher-led condition, but the strictly cooperative condition still produced noticeably higher results than the competitive condition.

While this study was not about students learning math or mathematically based content, it is still worth including in this review because it was one of few studies that focused on bilingual or language-learning students. These students are important to consider because they might not speak English as well as their peers, or because their families might not speak English at home and therefore might not be able to help their students if they struggle in school. Subsequently, this study shows that cooperative learning is also important to students who come from other cultures and speak other languages.

The outcome of this study supports cooperative learning, though there are a few reasons that it might not be entirely accurate. First, the sample included only sixteen students; a larger sample would have provided more trust-worthy results. Second, the author made no calculations other than the mean scores on each test. We therefore do not know the probability that the outcome was a fluke. However, the fact that all students were in the same class, and alternated between all three conditions allows us to eliminate the possibility of other influencing factors.

A quantitative study by Retnowati, Ayers and Sweller (2010) asked, “What are the effects of worked example and problem solving approaches in individual versus group settings?” The authors stated that previous studies, some of which had been done by Sweller, had shown that studying worked-examples had been more effective for novice learners while problem solving had been more effective for higher achieving students. This study focused on comparing the effectiveness of worked-examples or problem solving in the contexts of cooperative or individual learning.

The researchers studied 101 seventh graders in three math classes in an Indonesian junior high school. The students had all graduated from primary schools that used the Indonesian National Curriculum, so the researchers concluded that the students had had similar mathematical experiences. The students had previously been taught in a primarily individualistic, teacher-led classroom, but their teacher indicated that they had had some exposure to small-group work. The students were placed into either a group-work on an individual condition, and then within that condition, into a problem solving or a worked-example condition. The subject matter to be learned was multi-step geometry theorems, which were part of the regular school curriculum followed by the school, but which had not previously been taught to these students.

The experiment included five phases: (1) a revision phase, (2) a group skills induction phase, (3) an acquisition phase, (4) a test phase and (5) a questionnaire phase, conducted during three consecutive daily mathematics lessons lasting 70 minutes each. The revision phase and the group skills induction stage were conducted on the first two days. On the third day, the acquisition phase was completed, followed by the test and questionnaire phases. The test phase included two tests, a “similar test” on which students were given problems similar to the practice problems they had worked on in the acquisition phase, and a “transfer” test, intended to measure students’ ability to apply material to problems different from those they had previously practiced, and which were more complex. For both tests, a 2 × 2 (ANOVA) was conducted on both the numeric and reasoning scores. For the Similar test, a significant main effect was found for numeric scores, $F(1, 97) = 7.74$, $MSE = 2.21$, $p < 0.01$, and for reasoning scores, $F(1, 97) = 39.14$, $MSE = 2.57$, $p < 0.001$,. There was no significant effect for learning setting either ($F < 1$). These results showed that students were more procedurally accurate and able to reason in the worked example groups compared with the problem-solving groups in both individual and group settings. For the transfer test, there was no significant main effect for the numeric scores for the learning approach, $F(1, 97) = 1.40$, or setting ($F < 1$) nor an interaction, ($F < 1$). However, for the reasoning scores, a significant main effect for the learning approach was found: $F(1, 97) = 21.56$, $MSE = 1.49$, $p < 0.001$, close to significant interaction (just failing the $p = 0.05$ test) between the learning approach and the setting, $F(1, 97) = 3.43$, $MSE = 1.49$, $p = 0.07$.

The results show that the worked example approach strongly improved reasoning scores on transfer tasks compared with a problem-solving approach for both individual and group settings. This effect was strongest for students who worked collaboratively.

This study showed that students who worked in groups significantly outperformed their peers who worked individually in three out of four conditions, the exception being that students who used worked examples performed similarly to students who engaged in problem solving on the similar test. Additionally, it shows that worked-examples were more effective than problem solving in both the similar and transfer cases, though this effect was greater in the individual cases than in the problem-solving case. The results therefore suggest that group-work is effective for students engaging in problem solving, and is somewhat effective in worked-example cases.

One noteworthy reason to approach this with caution is the previous studies on which it was based. Multiple studies mentioned that had previously been done on the effectiveness of worked-examples compared to problem solving were done by one of the authors of this study. This means that the authors were likely biased toward the previous research and findings. The researcher also states that we could assume the students had similar backgrounds in math as they had been taught a national curriculum. This fails to take into account that even with a national curriculum, teachers adopt their own methods, so it is possible that some students had stronger backgrounds in math than others, which could have skewed the results. However, the researchers did help to eliminate other potential variables by choosing a topic which none of the students had previously studied.

A quantitative study by Sherman and Thomas (1986) compared two high school general math classes in a rural, Midwestern, predominantly Caucasian, middle class high school and asked "what are the effects of cooperative versus individual learning strategies taught by two different teachers?" and found that the cooperative group obtained significantly higher achievement on the posttest than the individualistic group.

One of the classes was taught using the student teams and achievement divisions (STAD) technique of cooperative learning, a model in which students work in heterogeneous groups of four, while the other, the control group, offered students an individualistic goal structure. Most of the students in the study were low academic achievers—many were working to meet the state's minimum mathematics requirements. Students were given the same test, which consisted of thirty items, twice—once as pre-assessment and then again after twenty-five days as a post-assessment. A three-way within-subjects ANOVA (Subjects x Time x Treatment) was the used to analyze the data. The researchers reported that both groups of students obtained significant gains ($p < .05$) from the pretest to the posttest. However, the cooperative group obtained significantly higher achievement ($p < .05$) on the posttest than the individualistic group. The mean score (out of 30) increased from 3.3 to 12.9 for the control group (a failing average score), while the mean score for the STAD group increased from 3.1 to 19.9 (a passing score). The researchers concluded that this was sufficient to support their initial hypothesis—that the students working in cooperative groups would show greater increasing in their scores between the pre and post tests.

While Sherman and Thomas's study did conclude that students working in cooperative groups significantly out-performed their peers who worked in an individual setting, it is important to take the circumstances of the study into account. The study was done with just two classes and each class had a different teacher, so it is possible that the teacher in the cooperative classroom was a better teacher than the teacher for the control class. Additionally, the authors did not describe the activities that students were working on in much detail, so the activities could have influenced the results. However, the two groups did not perform significantly differently on the pre-tests, and the authors reported that the results of the study strongly supported the findings of previous studies that served as a basis for this study, so it seems reasonable to conclude from this study that the STAD model of cooperative teaching proved to be more effective than an individualistic model of instructions.

The eight studies discussed in this section compared test scores between students who worked in cooperative groups and those who worked in independent learning situations. Of these eight studies, five found that students working in cooperative groups had higher test scores than students working in individual or whole-class settings. Zakaria, Lu Chung & Daud (2010), studied students from a school in Miri, Sarawak and found that the cooperative learning approach resulted in higher achievement than the traditional teaching approaches. Similarly, Sherman and Thomas (1986) found that the cooperative group obtained significantly higher achievement on the posttest than the individualistic group, and Harskamp and Ding (2006) found that the same was true of high school physics students who worked in cooperative groups. Meanwhile, Madrid, Canas, and Ortega-Medina (2007) found that both cooperative and competitive team peer tutoring lead to significantly higher post-test scores than the traditional teacher-led procedure, but that the cooperative method led to even higher scores than the competitive method. Leonard (2001) found that no significant differences existed between student achievement in heterogeneous versus homogeneous groups, but that low and middle achieving students received significantly higher test scores after working in groups than they had previously.

The other three studies suggested that cooperative learning alone might not have an effect on student achievement on tests. Kramarski and Mevarich

(2003) found that of four instructional methods, cooperative learning with metacognitive training produced the best results, but that without metacognitive training, cooperative learning had little to no effect on students' test scores. Retnowati, Ayers, and Sweller (2010) found that group setting did not appear to have any statistically significant impact on student achievement among 108 7th graders in an Indonesian junior high. Finally, Linchevski and Kutscher (1998) studied students in twelve Israeli junior high schools and found that five schools had a positive grouping effect, (students gained more by being hypothetically part of a lower ability group than of the next higher one) while seven schools had a negative grouping effect, suggesting that the benefit to a student of working in a group setting depends on the particular group. No study discussed in this paper showed that cooperative learning had a negative effect on student's test scores.

Cooperative Learning and Student Behavior

A cooperative learning environment in the mathematics classroom requires engagement from the students. Since the classroom focuses on students working together to solve problems, the class work does not allow students to simply sit back. As a result, a cooperative math classroom has an impact on student behavior in the classroom. Ten studies in this review examined the relationship between student behaviors and cooperative learning, and the effects of cooperative learning on student interactions in the classroom. A quantitative study done by Bentz and Fuchs (1996) followed twenty general educators and their students in four elementary schools in a southeastern urban school district and found that students were better able to ask for and offer help to their peers when first offered training. A quantitative study by Esmonde (2009) showed that in a cooperative learning environment, students adopted a variety of positions and discussed ways to provide equitable structures for math group work, and while no one strategy for cooperation worked best, the most constructive groups were those that focused on collaboration rather than on just helping, or on working individually. A quantitative study by Galton, Hargreaves and Pell (2009) done with students between the ages of eleven and fourteen examined the effects of cooperative group work versus whole class teaching on student engagement, academic performance and student behavior and showed that group work lead to higher cognitive level interactions than did whole class discussions. A quantitative study by Jacques, Wilton, and Townsend (1998) on the effects of cooperative learning on the social acceptance and self esteem of mildly intellectually disabled students studied 24 children (21 boys, three girls) with mild intellectual disabilities in the nine to eleven year-old age range, each attending one of 21 regular primary schools in Auckland, New Zealand and found that participation by non-disabled children in a cooperative learning program yielded gains in social acceptance of classmates with mild intellectual disabilities. Furthermore, the gains were evident both immediately following the program, and five weeks later, an indication of durability. A quantitative study by Liekin and Zaslavsky (1997) on the effect of an experimental, cooperative small-group-learning setting on students' activeness in class focuses on 98 students in four low-level 9th grade classes and found that when students worked in structured groups, time engaged in active activities increased while time spent engaged in passive activities decreased. A qualitative case study by Pijls, Dekker and Van Hout-Wolters (2006) examined the interactions that took place when an average achieving student and a high achieving student, both age sixteen and studying applied mathematics at a Montessori school in Amsterdam worked together and showed that one student had benefited significantly more from the interactions than the other did. A qualitative study by Ross (1995) followed eighteen Grade 7 math students while studying the effects of giving students feedback on their behavior when the students did cooperative group-work and found that giving students feedback on their behavior had the potential to lead to higher-quality interactions within the group so that the groups could better help one another. A quantitative study by Shachar and Sharan (1994) studied the effects of cooperative learning with the Group method on eighth grade students in ethnically heterogeneous classrooms in a junior high in Israel found that students from the group investigation method expressed themselves more frequently and used more words per turn than their peers taught in classes using a traditional whole class method. A qualitative study by Webb and Cullian (1983) studied what interaction patterns naturally occur when students are instructed to work in small groups, focused on 105 students in four different classrooms and found that the more frequently students received no answers to their questions, the lower their test scores were, that students worked in groups more frequently when working with specific types of activities, and that students spent more time on task when they received answers to their questions than when they did not receive answers. A quantitative study by Webb and Farivar (1994) compared the effects on achievement and verbal interaction of two instructional programs designed to teach students how to work effectively in small groups: cooperative learning with instruction and practice in basic communication skills and academic helping skills (experimental condition) and cooperative learning with instruction and practice in basic communications skills only (comparison condition), studied six 7th-grade general mathematics classes (166 students) at an urban middle school where students had little or no previous experience working in cooperative groups, and found that In the experimental condition, minority students gave and received more elaboration on their answers to questions, however no significant differences were shown for white students.

A quantitative study done by Bentz and Fuchs (1996) asked the question "What are the effects of providing training and practice in helping behaviors to students during mathematics peer-tutoring?" The study followed twenty general educators and their students in four elementary schools in a southeastern urban school district and found that students were better able to ask for and offer help to their peers when first offered training.

The twenty teachers who participated in the study were female; eighteen were Caucasian and two were African-American. The teachers taught in second, third and fourth grades and all the teachers had been participating in a class-wide peer-tutoring project in math operations during the 1992-1993 academic year.

At the beginning of the peer-tutoring implementation, in September, each teacher had identified two students to participate in the study: (a) one student classified as learning disabled (LD) and (b) one average-achieving student (AA). The students classified as LD were identified by school personnel using the criterion of a discrepancy of more than one standard deviation between achievement and cognitive or intellectual functioning when provided with learning experiences appropriate to age and ability level (average IQ scores for LD students = 88.9). For the AA student, each teacher was asked to identify a student whose mathematics performance ranked near the middle of the class. Half of the teachers gave their AA students training in peer tutoring (treatment group) while the other AA student received no training. The teachers in the trained classroom had an averaged of 9.70 years of experience, while the teacher in the untrained classrooms had an average of 15.70 years of experience.

Using materials provided by the researchers, each teacher in the treatment group taught her class peer-mediated instruction procedures. The training for the class-wide peer tutoring included examples of appropriate behavior for the students to engage in while working with a partner such as instruction on how to give corrective feedback and offer specific positive reinforcement for correct answers. Following peer-tutoring training, every student in each class was paired together to work on mathematics operations problems. In the pre-selected dyad, the AA student served as a tutor to the LD student, though all other students in the class participated in peer tutoring. Each AA/LD dyad was videotaped during a structured peer-tutoring activity, which occurred outside the students' class- room. The pairs each received a mathematics operations worksheet similar to the peer-tutoring worksheets used during the class-wide peer tutoring. The researchers coded both the LD and the AA students' helping behaviors and explanations.

The tests indicated that trained tutors (M = 7.90, SD = 10.94) offered significantly more help than untrained tutors (M =0.00, SD = 0.00), t(18) = 2.28, p = .03. Trained (M= 0.00, SD = 00.0) and untrained (M = 0.00, SD = 0.00) tutees offered comparable amounts of help,t(18) = 0.00. Similarly, trained tutees (M = 4.30, SD = 4.35) asked for help significantly more than untrained tutees (M = 0.00, SD = 0.00), t(18) = 3.13, p = .006; trained (M = 00.0, SD = 0.00) and untrained (M = 0.00, SD = 0.00) tutors asked for help comparable amounts of time, t(18) - 0.00, ns. Tutors provided more prompts to tutees (M = 42.70, SD = 28.44) than tutees provided to tutors (M = 1.85, SD = 4.52), t(18) = 3.13, p = .006; moreover, tutors gave more feedback to tutees (M = 48.90, SD = 25.38) than tutees gave to tutors (M= 1.30, SD = 3.57), t(18) = 3.13, p = .006.

The tests indicated that trained tutors offered significantly more help than untrained tutors, while trained and untrained tutees offered comparable amounts of help. Likewise, trained tutees asked for help significantly more than untrained tutees and trained and untrained tutors asked for help comparable amounts of time, and that both tutors and tutees gave more explanations of their work when they had received training, but that all groups of students explained their work more often after working in their peer-mentor pairs.

This study is of interest because few studies have been done on the effects of cooperative learning with disabled students and it also gave detailed descriptions of the methods used. A potential weakness could be that one group of teachers had, on average been teaching for longer than the other, because more experienced teachers might have greater outcomes. However, the experimental group had the lower mean age, and the students in the experimental group, the ones who had received training, engaged in helping behaviors more often. This indicates that the average number of years of teaching experience alone did not result in students engaging in more helping behaviors. This study focuses on the effects of receiving training, rather than on the effects of group-work, so while it does not draw any conclusions about the effectiveness of group-work, it does offer the conclusion that providing students with training on how to work in cooperative situations could be beneficial.

A qualitative study by Esmonde (2009) focused on students from three high school math classes and showed that in a cooperative learning environment, students adopted a variety of positions and discussed ways to provide equitable structures for math group work. While no one strategy for cooperation worked best, the most constructive groups were those that focused on collaboration rather than on just helping, or on working individually.

The research for this study was conducted in three different high school mathematics classes all taught by the same mathematics teacher, Ms. Delack. The classes were on the campus of a large, diverse urban high school. The students were diverse with respect to race, gender, prior achievement, and grade level. In Ms. Delack’s classroom, students regularly worked in groups. In these classes, group work did not mean only one type of activity. In one 55-minute period, there were typically three or four activities, which could include small-group discussion, presentations, whole-class discussions, quizzes, class-work, and homework. The researcher selected two activities to compare.

Group quizzes were paper-and-pencil tests in which students worked in groups and received two grades—one for mathematical correctness and a second for group participation. The researcher also chose to study *presentation preparation* as the students presented in class on a weekly basis. During this time, the researcher took notes on equity within the groups, as well as the roles that students took on.

In both the group quiz and the presentation preparation activities, the researcher observed that the most equitable groups were generally the groups that worked collaboratively rather than individualistically. For the group quiz, these collaborative interactions only occurred when the groups' interactions did not position any of the students as novices or experts. During the presentation preparation, there was more variety in collaborative interactions. Some groups managed to position one or more students as experts while still being very collaborative as they prepared the presentation. The researcher also stated that the helping interactions displayed by students appeared to be oriented toward quickly finding a single correct answer, rather than toward discussing possible answers. The researcher suggested that changing the way that expertise is defined could help foster more equitable cooperative interactions. For example in one group students appeared to see themselves as experts in one domain and as less expert in another. This assignment of competence appeared to show potential for interactions that were not often seen. For example, students who expressed uncertainty about explaining their answers, but who claimed that they understood, were asked to give a practice explanation, thus possibly reducing the stress over not having the right answer.

One of the major advantages of this study, over some of the others, is that the students in Ms. Delack's classes had previously been exposed to regular group-work. As the class had already been an established group-work environment, the observations resemble the longer-term effects of cooperative learning, compared to many studies where it was initially implemented for the study. In other cases, the results could have been due to a change in routine, but that need not be accounted for in this case. The study also however was done with only students taught by one teacher, so the impact of the individual teacher compared to the impact of the cooperative class-structure is not certain.

A quantitative study by Galton, Hargreaves and Pell (2009) done with students between the ages of eleven and fourteen showed that group work led to higher cognitive level interactions than did whole class discussions. The authors were interested in studying the effects of cooperative learning on student engagement and behavior because one "key reason for teachers' reluctance to use grouping to enhance academic performance may be the requirement that to work effectively pupils need to be trained in communication skills." (121). Researchers compared the classroom behavior of pupils when taught new concepts or engaged in problem solving in sessions organized either as cooperative, student-centered group work or as whole class, teacher-centered instruction and made comparisons of attainment among classes of pupils aged eleven to fourteen years in English, Mathematics and Science. Since all of the teachers involved in the study were interested in learning to implement group work in their classes, the researchers decided on three types of grouping to allow all of the teachers to implement cooperative structures. The first kind of grouping was designated collaborative. The second kind of grouping was termed cooperative. Finally, there were "seated groups" in which students worked independently but could compare answers and check each others' work. Teachers used a combination of the "whole class" method of instruction, in which students could use "seated group work" in their classrooms, as well as the cooperative grouping in some circumstances, such as activities that required students to share equipment. During group instruction, students worked in collaborative groups, with some cooperative aspects when projects could be broken down into smaller tasks. The SPRinG project group worked with teacher to increase levels of engagement in the classroom, and in the quality of interactions between students. The observations revealed that over the course of the year improvements were made in both areas, with greater improvements made in the small group settings than in the whole-class settings. The researchers recorded time on task, and frequency of open dialogue and sustained interactions during visits in the spring and again in the summer. This was done for both when the classes were working in groups, and during whole-class instruction.

Time on task was observed during visits to the classrooms to see either whole class teaching or group work. One visit for each subject took place in the spring and one in the summer term, after the class had been trained during the autumn period. For English there was very little difference between students in small groups and those taught as a whole class during the spring visit (80% for both situations) but a slight advantage of around 5% of all observations in favor of groups by the summer. For Mathematics the position changed dramatically over the two terms. During spring term, time on task in the groups was around 70% compared to over 80% for whole-class teaching. By the summer visits, time on task in groups was over 80%. Time on task during whole-class teaching was reduced to 58%. Science showed a similar but less drastic trend to math. The corresponding figures in the spring term were 76% for groups and 81% in class. For summer term, it was 84% compared to 79%.

The researchers also studied students' interactions, including the frequency of open dialogue and the percentage of interactions that were sustained. Because students became more skilled in group-work after training and with practice, an increase in interactions associated with higher level cognitive discourse was predicted. These interactions consisted of a combination of the "asking questions", "making suggestions", "offering explanations", "agreeing" and "disagreeing" categories. In both English and Mathematics in either the spring or summer terms the percent figure for the groups were superior to those of the whole-class. In Science, students in the groups outperformed those in the whole-class situation but the differences were marginal. Overall the researchers concluded that, when given proper training on working in groups, group-work in all three subjects led to more time-on-task and higher quality interactions between students than did whole-class instruction, with the greatest improvements made in Math.

There are a couple of strong points in this study. Since all of the teachers in this study partook in both whole-class and cooperative group structures, it is clear that the quality of the teachers involved did not have an impact on the results of the study. Additionally, this study focused on multiple aspects of student behavior, which gives a broader picture of the potential benefits. The study does also have a few shortcomings. For one, the only numerical data represented are the percentages of time on task, open dialogue and sustained interactions. There is no analysis to determine whether the results are statistically significant or not. The study also does not describe what the teachers did to reinforce norms for group-work in the classroom. Finally, all of the teachers received training to help them implement group-work, and so no conclusions can be drawn about the effects of implementing group-work without training.

A quantitative study by Jacques, Wilton, and Townsend (1998) followed 24 children (21 boys, three girls) with mild intellectual disabilities in the nine to eleven year-old age range, each attending one of twenty-one regular primary schools in Auckland, New Zealand to examine the effects of cooperative learning on the social acceptance and self esteem of mildly intellectually disabled students. Each of the students was given a “social acceptance” rating, determined by a scale in which all students in the classes were asked to name which students they would invite to a birthday party. The mildly disabled students were also given a self-esteem rating, based on the Coopersmith Self-Esteem Inventory, a widely used self-esteem measure.

Each of the children in the study participated in one of two conditions either a six-week cooperative-learning program (experimental) or their usual classroom program (control). The procedures for the cooperative condition were identical across classrooms/schools. Each cooperative class was divided into small (four to six member) heterogeneous groups, one or two of which contained a child with a mild intellectual disability. The group task was to have each child learn an entire social studies unit, and to successfully complete a test on the material at the end of each week. This required each student to take place in jigsaw activities, which required each student to complete a part of a task to bring together as a whole group.

A three-way ANOVA [*treatment* (experimental versus control) x *special class attendance* (former special class pupil versus never attended a special class) x *pre-testing* (pre-tests versus no pretests)] were conducted on the immediate post-tests and again on post-test five weeks later.

The results indicated that the social acceptance scores of students with mild intellectual disabilities who were in the experimental group were significantly higher than those of the students in the control groups. The same was true of the social adjustment scores as rated by students' teachers. Furthermore, the gains were evident both immediately following the program, and five weeks later, an indication of durability. Overall, the study showed that non-disabled classmates were more accepting of their peers with mild intellectual disability when they had worked in cooperative groups.

It must first be noted that this study focused on social-studies activities rather than math. Therefore, the results might not apply directly to a Mathematics classroom. However, as the study focused on how students were perceived by their peers, not on student achievement, it seems unlikely that this would be the case. Since the students with mild intellectual disability were given social acceptance rating by their teachers, as well as by a method based on their classmates' reactions, it also stands that the article presents a well-rounded view of how the students were perceived socially.

A quantitative study by Liekin and Zaslavsky (1997) focused on ninety-eight students in four low-level 9th grade classes and found that when students worked in structured groups, time engaged in active activities increased while time spent engaged in passive activities decreased.

For the study, learning tasks were designed to have students work in pairs to ensure that every student got the opportunity both to study and to teach each type of learning material. The researchers observed students engaging in a variety of active activities (solving a problem independently, copying written material/taking notes, giving an explanation, posing a question/requesting help), passive activities (listening to an explanation, reading the learning material) and off-task activities (playing, daydreaming, engaging in conversations unrelated to activity).The researchers also identified five possible interactions that may occur in the process of learning mathematics: student-student(S-S), student-learning material (S-LM), student-teacher(S-T), student-learning material-student(S-LM-S) and student-learning material-teacher(S-LM-T).

The researchers addressed the findings in terms of (a) students' activeness, and (b) students' mathematical communication in general and their exchanges of help in particular.

The results showed an increase in the time spent on active activities (from 46.3% to 68.3%) when students starting working in cooperative groups. There was a substantial increase in the two most active on-task S-LM-S interactions (from 2% to 23%). There was no change in time spent on S-LM interactions (45%) but there was an increase in time spent on the more active category of solving a problem (from 15% to 29%) and a decrease in the more passive category of copying the problem (from 30% to 16%). There was also an overall decrease in the time spent on passive activities (from 53.6% to 31.6%).

The researchers in this study followed just one class, so it is possible that the outcome was the result of the particular class dynamics rather than just a product of whether lessons were taught as cooperative group-work or independent work. Additionally, it is difficult to know what effect the presence of an observer had on students' engagement in learning activities. It is also difficult for students to engage in certain types of learning activities (S-LM-S) without cooperative group-work lessons. However, this study also showed that there was dramatic change in how class time was spent when cooperative learning was implemented.

A qualitative case study by Pijls, Dekker and Van Hout-Wolters (2006) examined the interactions that took place when an average achieving student, Peter and a high achieving student, Susan, both age sixteen and studying applied mathematics at a Montessori school in Amsterdam, worked together.

The students were given investigation tasks in a computer simulation before being given a textbook assignment to work out of. The teacher played the role of coaching the students, while the researchers helped with mathematics when students asked for it. Students were not given any special training on interaction, but were instructed to work together. The students were audio-taped and written student products were collected. The students were given a pretest and a post-test consisting of open-ended problems.

Each test consisted of eleven open problems. For example, students were given a task with a grid and asked how many shortest paths they could find from point A to point B. On her pre-test, Susan demonstrated that she could solve easy problems, but not more-complex ones. Peter demonstrated a preference for formulas, though he did not always apply the correct formula. Both students started out at a perceptual level on all concepts.

While observing the students working in pairs, the researchers found that the average achieving student, Peter explained and criticized his own work and did not ask the high achieving student, Susan to explain her work. Susan asked Peter to explain his work but did not explain her own work or criticize it.

On the post-test, Susan showed that she had learned a procedure outlined in her textbook. She knew how to calculate relevant probabilities. She also learned how to use a grid as a model for the distance problem, but was unsure when to apply it. Peter showed that he had learned to count routes, how to use the grid, and when to use or not use it. Peter had attained a conceptual level for all concepts, while Susan had attained a conceptual level for only some concepts.

Both students attained mathematical level raising, however Peter, who had explained and criticized his own work showed greater improvement than Susan. One of the students (Peter) clearly profited more than the other. This suggests that students working in groups who explain or are asked to explain their responses benefit more than those who do not. It also shows that when structured properly, group-work can be an effective way for students to practice important mathematical skills such as explaining and criticizing their own work, as well as receiving feedback from their peers.

Weaknesses for this study include a lack of demographic information about the two focal students. The school picked for the study was also a Montessori school, an educational background that may be different from that of many other students. However, the detailed descriptions of Peter and Susan's interactions show a significant contrast between the two students' behaviors and provide an understanding of why Peter had a better understanding of the material than Susan did.

A qualitative study by Ross (1995) studied eighteen Grade 7 math students and found that giving students feedback on their behavior had the potential to lead to higher-quality interactions within the group so that the groups could better help one another.

The students in the class were audio-taped while working in five cooperative learning groups on four occasions over a sixteen-week period. The researchers assessed the effects of a feedback strategy on the frequency and quality of students' attempts to help their group-mates learn. Their teacher, Ms. Bennett had six years of teaching experience, and had been identified by her colleagues as an effective teacher.

During the initial observations, a trained coder found three levels of requests from students—asking for explanation, procedures/facts or evaluations. Then, two rounds of feedback occurred during weeks six and twelve. In the first round of feedback, Ms. Bennett asked her students to define characteristics and give examples of good group work. Students met in the groups they had previously been recorded working and were given transcripts of their conversations. They were asked to review the transcripts and assign their group a score. They were then given feedback via a copy of the coder's judgment of their performance, and asked to devise an improvement plan for their group.

The second of the feedback started with the students reviewing transcripts, supervised by Ms. Bennett. Students received both transcripts from before, and after they had received feedback, and were asked questions to help them compare the transcripts such as "Has the number of requests for help stayed the same, increased or decreased?" Students continued to use the self-assessment once or twice a week for the next four weeks.

A qualitative analysis indicated that a few lower-ability students were more willing to ask for explanations after they had been given feedback. Additionally, there were fewer unnecessary requests, judged by students asking for help and then deciding they didn't need it, after the feedback. There were also changes in the selection of who the students sought help from—a few students would usually ask Ms. Bennett for help before they had been given feedback, but later started asking their peers for help first. Finally, students' requests for help became more persistent and precise.

Additionally, Ms. Bennett was observed interacting with the groups. She did not intervene extensively, though her contributions reinforced the feedback given to students. She would respond to requests by first determining whether the request required her help, and if the required resources existed within the group, she mobilized the group and explained that they did not need her help. When her help was required, she would provide a model of how to explain.

Overall, by the end of the study, students were spending more time helping each other and less receiving help from their teacher. The researchers concluded that the effect of feedback on student might be the result of three factors. (1) Feedback procedures strengthened helpfulness norms that had previously been established in the class. (2) Feedback procedures influenced students' behavior by increasing their skill in asking for and giving help. (3) Recognition of success in achieving the helpfulness goals of the classroom may have improved students' feelings of self-efficacy.

This study shows ways that a teacher can enhance the cooperative structure in her classroom, however a few things must be taken into consideration. To start, the students had previously been introduced to group-norms, though the students had not previously received a high level of feedback on their ability to work as a group. A teacher who is implementing group-work for the first time may have different results. Second, the study was done on just one class, which, as mentioned earlier, is a small sample size and therefore cannot always be considered a representation of the population as a whole. As mentioned earlier, it is also difficult to know how students would react without the presence of an observer. However, the evidence still shows that students were able to apply the feedback they received, and were therefore capable of exhibiting helping behaviors among their groups.

A quantitative study by Shachar and Sharan (1994) investigated the effects of cooperative learning with the Group method on eighth grade students in ethnically heterogeneous classrooms. The researchers found that students from the group investigation method expressed themselves more frequently and used more words per turn than their peers taught in classes using a traditional whole class method.

The students who partook in this study were in a junior high in Israel, located in a middle-class neighborhood with a high concentration of professional people. However, 33% of the students were bussed to the school from a lower class neighborhood in the same city. These students were distributed evenly through the classes in the school. Five out of nine classes in this study were taught with the Group Investigation method (experimental group), and four classes were conducted with the Whole-Class Presentation- Recitation method (control). All teachers had previously used the latter method exclusively. Nine teachers attended ten workshops held during the course of the year preceding the implementation of the experimental portion of the study.

In the cooperative learning classrooms, the students were divided into groups of four. Group composition was largely determined by the students themselves on the basis of their interest in the particular topics to be studied. Group composition changed with each unit. To study the effects of the experimental method compared to those of the traditional method, the researchers conducted group-discussions with groups of six student each, outside the classroom during the last few months of school. For students in the experimental group, these groups were not the groups they had worked with in class. Each group consisted of three students of Western backgrounds, and three of Middle-Eastern backgrounds.

The mean scores indicated that, as expected, students of Western, middle-class backgrounds spoke more often and used more words per turn than did their peers of Middle-Eastern, lower-class background. However, all students from the Group Investigation classes used more words per turn and expressed themselves more frequently during the same length of time than their peers from the Whole-Class method. On the measure of turns of speech, the interaction effect showed that students of Western background took more turns in the discussion than Middle Eastern students, only if they had studied with the Whole-

Class method. This indicated that students working in groups became more comfortable with speaking, and that their responses were more detailed than students who worked using the whole-class method.

Additionally, students in the Group Investigations method had more disagreements within their groups and clarified their answers more frequently than their peers who were taught in the traditional method did. They also needed fewer clarifications, used fewer directives and interrupted their classmates less frequently. In general, while there were differences between the results of Western and Middle Eastern students within each category, the study showed that the Group Investigation Method led to a higher level of student engagement with the content in that students gave more frequent, detailed, clarifying responses when they had been taught using the Group Investigation Method than when taught using the Whole-class method. The same students also interrupted their peers less frequently, indicating an increased respect for what their peers had to say.

One of the biggest biases of this study is that the research that was conducted was on students interacting in a group setting—students who had been taught by both methods were videotaped engaging in conversation. As the students in the experimental group had had practice engaging in group work, it follows that they would be more comfortable in a group setting than their peers from the control group. However, this effect is slightly offset by the fact that the groups that were videotaped were homogeneous in terms of which method they had been taught by, so at least the students from the control group did not have to compete with students from the experimental group for floor-time.

A qualitative research study by Webb and Cullian (1983) studied what interaction patterns naturally occurred when students were instructed to work in small groups. The study focused on 105 students in four different classrooms and found that students worked in groups more frequently when working with specific types of activities, and that students spent more time on task when they received answers to their questions than when they did not receive answers.

This study was set in a special classroom designed to complement the regular mathematics curriculum. Students worked in small groups with instruction from the teacher when needed. At the end of each study, all students took an achievement test that consisted of one, multiple part item corresponding to each activity in the materials for a topic. Two persons served as observers and took notes of the same groups simultaneously. The observers compared notes, discussed discrepancies and made decisions on how to code problematic interactions.

The study found that the frequency of asking questions within a group and receiving no response was higher among uniform ability groups than among mixed ability groups, and was higher among introverted students than among extroverted students. The study also found that the more frequently students received no answers to their questions, the lower their test scores were. Students worked in groups more frequently when working with specific types of activities, and that students spent more time on task when they received answers to their questions.

The qualitative observations made in this study were minimal compared to some of the other studies compared in this paper. The researchers did not go into much detail. However, the researchers claimed that they did replicate the results from a previous study done on the topic.

A quantitative study by Webb and Farivar (1994) examined the effects of two instructional programs designed to teach students how to work effectively in small groups on students' achievement and verbal interaction. The instructional programs were cooperative learning with instruction and practice in basic communication skills and academic helping skills (experimental condition) and cooperative learning with instruction and practice in basic communications skills only (comparison condition).

The researchers studied six 7th-grade general mathematics classes (166 students, 55% Latino, 26% white, 15% African American) at an urban middle school where students had little or no previous experience working in cooperative groups and found that In the experimental condition, minority students gave and received more elaboration on their answers to questions.

Two teachers participated in five full days of training workshops in which they would become familiar with the activities they were to teach the students. The students were given mathematics pre and posttests and instructed in communication and helping skills. The teachers each taught three classes. The classes were comparable in terms of entering students' achievement level and had similar mixes of student gender and ethnic background.

The frequencies of four kinds of indications of a need for help were coded: specific questions ("Where did you get 12 from?"), general questions or indications of confusion ("How do you do number 10?), errors ("I just added all of [the denominators] up!"), and requests for the answer ("What did you get for number 10?"). Additionally, the frequencies of four categories of giving and receiving help were coded for each student: receiving elaboration, giving elaboration, receiving the answer (without any elaboration), and giving the answer. The two teachers did not differ noticeably in how they managed small-group work. Both teachers were careful not to answer questions directly, but encouraged students to consult with each other using general suggestions As the groups worked, neither teacher referred to specific help-giving or help-receiving behavior.

Students in both instructional conditions learned general communication skills to facilitate their work in small groups, but only students in the experimental condition were given specific instructions to give elaboration to students who appeared to need help. Thus, it was expected that students in the experimental condition would give and receive more elaboration than students in the comparison condition.

The instructional conditions produced significantly different patterns of verbal interaction for Latino/African-American students, but not for White students. Minority students gave and received more elaboration in the experimental condition than in the comparison condition, and received answers more frequently in the comparison condition than in the experimental condition. Although Latino/African-American students requested help equally often under both conditions, in the experimental condition they were more likely to receive elaboration than answers when they requested help [paired $t(27) = 3.40$, $p < .002$], whereas in the comparison condition, the opposite was true [paired $t(20) = 4.31$, $p<.001$]. White students did not even show a tendency toward giving and receiving more elaboration in the experimental condition than they did in the comparison condition.

This study did not follow a particularly large group of subjects, focusing on the students taught by just two teachers. However, the sample of 166 students was still more than many of the studies in this review. As only 26% of the students in the study were white, it also stands that the conclusion that White students were not impacted by the addition of the academic helping skills is also less reliable than the conclusion that students of color did benefit, as that would leave a sample of only about 40 or so White students who were studied.

The ten studies in this section examined the relationship between student behaviors and cooperative learning, and the effects of cooperative learning on student interactions in the classroom. Of these studies, four showed that cooperative learning led to a higher level of student engagement in class than did individual or whole class instruction. Esmonde (2009) showed that in a cooperative learning environment, students adopted a variety of positions and discussed ways to provide equitable structures for math group work, and while no one strategy for cooperation worked best, the most constructive groups were those that focused on collaboration rather than on just helping, or on working individually. Galton, Hargreaves and Pell (2009) examined the effects of cooperative group work versus whole class teaching on student engagement and student behavior and showed that group work lead to higher cognitive level interactions than did whole class discussions. Similarly, Shachar and Sharan (1994) found that students taught using the Group Investigation Method expressed themselves more frequently and used more words per turn than their peers taught in classes using a traditional whole class method. Liekin and Zaslavsky (1997) found that when students worked in structured groups, time engaged in active activities increased while time spent engaged in passive activities decreased.

Additionally, three studies found that cooperative learning was most effective when the students were given guidance about their interactions. Bentz and Fuchs (1996) found that students who worked in cooperative groups were better able to ask for and offer help to their peers if they were first offered training. Ross (1995) found that giving students feedback on their behavior had the potential to lead to higher-quality interactions within the group so that the groups could better help one another. Webb and Cullian (1983) studied what interaction patterns naturally occur when students are instructed to work in small groups and found that when students received answers to their questions, the groups spent more time on task.

Finally, three studies showed that cooperative learning was beneficial to students who were not typically high achievers. Jacques, Wilton, and Townsend (1998) studied the effects of cooperative learning on the social acceptance and self esteem of mildly intellectually disabled students found that participation by non-disabled children in a cooperative learning program yielded gains in social acceptance of classmates with mild intellectual disability. Furthermore, the gains were evident both immediately following the program, and five weeks later, an indication of durability. Pijls, Dekker and Van Hout-Wolters (2006) examined the interactions that took place when an average achieving student and a high achieving student, worked together and showed that the middle achieving student had benefited significantly more from the interactions than the higher achiever did, likely as a a result of being asked to elaborate on his answers. Finally, Webb and Farivar (1994) compared the effects on achievement and verbal interaction of two instructional programs designed to teach students how to work effectively in small groups and found that when helping skills were included in the curriculum, minority students gave and received more elaboration on their answers to questions. Overall the cooperative learning environment proved to be beneficial to students as it led to higher levels of engagement, particularly for low and middle achieving students, and particularly when students were given instruction about how to work in cooperative groups.

Cooperative Learning and Students' Attitudes Toward Mathematics

One of the most pressing issues in mathematics education, beyond getting students to pass standardized tests, is the issue of students' attitudes toward mathematics. Six studies in this review examine the effects of cooperative learning on students' attitudes toward mathematics and toward their math classes. A quantitative study by Bilican, Demirtasli and Kilmen (2011) studied 7834 8^{th} grade students in 1999 (Trends in International Mathematics and Science Studies or TIMSS 1999 sample), and 4498 8^{th} grade students in 2007 (TIMSS 2007 sample), from schools in Ankara Turkey and asked whether students' opinions about "teaching activities in maths courses" changed from 1999 to 2007 and showed a correlation between a large increase in the number of students who had experienced cooperative learning and an increase in the number of students who reported agreeing or partially agreeing with the statement "I enjoy learning Maths." A quantitative, quasi-experimental study by Ifamuyiwa and Akinsola (2008) investigated the effects of self and cooperative instructional strategies on senior secondary students' attitudes toward mathematics, focused on Second year students in nineteen public Senior Secondary schools in Ijebu-North Local Government Area, Ogun State, Nigeria and found that overall, students in the study reported more positive attitudes about achievement in mathematics, though the self-instructional students reported a higher increase in attitude than did that cooperative groups. A quantitative study by Ke & Grabowski (2007) examined the effects of combining computer games with cooperative learning in mathematics by studying 125 5th grade students varying by gender, socioeconomic status and race, all with basic computer skills who elected to participate found that cooperative game-playing resulted in greater attitudes about math than either competitive game-playing or no game playing in economically disadvantaged students, but no statistically significant differences in attitudes for economically normal students. A quantitative study by Owens and Barnes (1982) asked about the relationships between cooperative, competitive, and individualized learning preferences of secondary school students and the perceptions of classroom atmospheres in two classes, English and mathematics and studied 279 students in Grades 7 and 11 in two Sydney high schools, and found that that senior high school students express a much greater preference both for more cooperative and more competitive social contact in learning than do first year high school students, with English generally seen as providing more opportunity for cooperative interaction than mathematics, however girls preferred competing in English more than boys did, whereas boys preferred competing in mathematics more than girls did. A quantitative study by Vaughan (2002) asked how cooperative learning affects the achievements and attitudes in math of 5th grade students of color in a culture different from that of the U.S., studied 21 students of color, ten boys, eleven girls, in a self-contained, 5th grade classroom in one of Bermuda's elementary schools. Results strongly suggested that cooperative learning had a positive impact on the attitudes of students of color toward mathematics. A quantitative study by Moskowitz, Malvin, Schaeffer and Schaps (1983) asked about the effects of the Jigsaw strategy on students' attitudes with regard to themselves, peers and school studied fifth and sixth grade classes in eight elementary schools in a middle-class, suburban public school system in northern California and found that jigsaw was implemented in five of six classes where teachers had received training, but that only three of the classes were showing exemplary results.

A quantitative study by Bilican, Demirtasli and Kilmen (2011) studied 7834 8^{th} grade students in 1999 (Trends in International Mathematics and Science Studies or TIMSS 1999 sample), and 4498 8^{th} grade students in 2007 (TIMSS 2007 sample), from schools in Ankara Turkey and asked whether students' opinions about "teaching activities in maths courses" changed from 1999 to 2007. The study showed a correlation between a large increase in the number of students who had experienced cooperative learning and an increase in the number of students who reported agreeing or partially agreeing with the statement "I enjoy learning Maths."

The international mean of the students replying to this statement as "Completely agree" was higher in 2007 than in 1999 ($z (2514) = 3.95$; $p < .05$). As the distribution of the responses to the statement "I enjoy learning math", the percentage of students who fully agreed or partially agreed with the statement "I enjoy learning math" increased remarkably from 1999 to 2007. The authors of the study claimed that this was likely due, at least in part, to an increase in the variety of methods of instruction, including cooperative learning.

With over 12,000 subjects between the two years, this study was by far the study that covered the widest range of students and had the largest sample size. The questions asked of students were clear and this was the only study covered in this review that examined a period of more than five years. Hence, this is probably one of the most reliable studies in the review in terms the data collected and analyzed. However, the authors only hypothesized that an increase in cooperative learning and other non-traditional methods was a contributing factor to the increasing number of students who reported liking math.

A quantitative, quasi-experimental study by Ifamuyiwa and Akinsola (2008) investigated the effects of self and cooperative instructional strategies on senior secondary students' attitudes toward mathematics, focused on second year students in nineteen public senior secondary schools in Ijebu-North Local Government Area, Ogun State, Nigeria and found that overall, students in the study reported more positive attitudes about achievement in mathematics, though the self-instructional students reported a higher increase in attitude than did that cooperative groups. In this study, students' attitudes toward mathematics were measured through use of a students' attitude questionnaire. Trained teachers in experimental and control groups used an operational guide (TIG or Teacher's Instructional Guide) that consisted of activities, behaviors and specific instructions guiding teacher supervision and instruction. Students were split into self-instructional and cooperative group conditions, each of which received their own packages for instruction. Students were given a post-test after ten weeks of instruction.

Overall, students in the study reported more positive attitudes about achievement in mathematics, though the self-instructional students reported a higher increase in attitude than did the cooperative groups. In other words, while the cooperative groups reported less positive attitudes than the self-instructed groups, both groups reported more positive attitudes than the whole-class instructed groups.

One of the major drawbacks of this study was that it was done with students in senior-secondary schools, in other words, students who would soon be leaving the schools. Many of these students would have had preconceived notions about school, that may have been difficult to overcome at that point in their education. Additionally, little information was given about students' previous attitudes toward math, so again, it is difficult to say what the change was. However, this study still presented strong evidence for student-led instruction over teacher-led, which includes cooperative learning in addition to self-led instruction.

A quantitative study by Ke & Grabowski (2007) investigated the effects of combining computer games with cooperative learning in mathematics. The researchers studied 125 fifth grade students varying by gender, socioeconomic status and race, all with basic computer skills, who elected to participate in the study and found that cooperative game-playing resulted in greater math performance than either competitive game-playing or no game playing.

Students participating in the study were randomly assigned by intact classes to one of three groups: cooperative game-playing, interpersonal competitive game-playing or no game-playing. Participants of both game-playing groups took two orientation sessions, during which they read the guidelines and tried each web-based game. They were then required to play one math game during two 40-minute sessions each week for four weeks. Participants participated from their own classrooms, each with an internet- connected laptop. The teachers oversaw the treatments, setting up in-class game-playing sessions and monitoring the participants' activities. The teachers also attended a one hour training session and were given administration job-aids. The researchers observed nearly all game-playing sessions. In order to determine the effects of the game-playing, at the end of the experiment, the researchers had students fill out Tapia's Attitudes Towards Maths Inventory (ATMI), a web-based, inventory consisting of 40 items investigating students' self-confidence, value, enjoyment and motivation towards mathematics. Sample questions included 'I am able to solve mathematics problems without too much difficulty' (self-confidence), 'Mathematics is important in everyday life' (value), 'I am happier in a math class than in any other class' (enjoyment) and 'I plan to take as much mathematics as I can in school' (motivation). According to Ke and Grabowski (2007) "This inventory reliably measured maths attitudes with a Cronbach alpha of 0.97" (253).

A MANCOVA test showed a significant effect, overall, of game-playing on students' math performance and math attitudes ($F = 4.395$, $p = 0.002$). Univariate tests indicated a significant interaction between the treatments and SES on students' attitudes toward math ($F = 3.415$, $p = 0.037$. Economically disadvantaged students who participated in cooperative game-playing showed more positive attitudes about math than those in the other two conditions ($M_{coop} = 81.5$; $M_{comp} = 72.1$; $M_{cont} = 73.3$). For economically normal students there were no significant differences between the two gaming situations in promoting positive math attitudes.

The findings of this study, regarding students' attitudes toward math indicated that economically normal students did not necessarily benefit from either treatment when it comes to attitudes toward math. However, this study also tested two experimental conditions, compared to one, which is more common in studies. This resulted in a smaller number of students in each group in the study. The study also did not state any information about students' attitudes toward mathematics before the experiment took place, and so it is difficult to determine whether or not the game-playing actually resulted in a change in students' attitudes toward math.

A quantitative study by Owens and Barnes (1982) examined the relationships between cooperative, competitive, and individualized learning preferences of secondary school students and the students' perceptions of classroom atmospheres. The researchers followed students in two classes, English and mathematics and studied 279 students in Grades 7 and 11 in two Sydney high schools. They found that that senior high school students express a much greater preference both for more cooperative and more competitive social contact in learning than do first year high school students, with English generally seen as providing more opportunity for cooperative interaction than mathematics, however girls preferred competing in English more than boys did, whereas boys preferred competing in mathematics more than girls did.

Students in the study completed the Learning Preference Scale-Students (LPSS), a survey of 36 items, brief statements about a feature of learning by cooperating with others, by competing with others, or by working alone, to show their preferences for learning. The Grade 11 students, preferred learning that involved greater cooperation learning involving greater individualization did Grade 7 students expressed a greater preference for individualized learning than Grade 11 students. The students overall expressed a stronger preference for more cooperative learning in English than in Mathematics.

Boys preferred more competitive contact in learning mathematics, and girls preferred more competitive contact in learning English. The same trend appeared in overall desire for personal contact while learning these subjects (Combined Involvement).

This study did not focus on how cooperative learning impacted students' attitudes toward mathematics or English, but simply found out what their views of cooperative, competitive and individualized learning were. While this does not tell us whether a cooperative learning environment could improve students' attitudes, it does suggest that as students preferred to work in groups, the implementation of group-work might result in students' attitudes toward math and other subjects seeing a positive increase.

A quantitative study by Vaughan (2002) examined how cooperative learning affected the achievements and attitudes in math of 5th grade students of color in a culture different from that of the U.S., studied 21 students of color, ten boys, eleven girls, in a self-contained, 5th grade classroom in one of Bermuda's elementary schools. Results strongly suggested that cooperative learning had a positive impact on the attitudes of students of color toward mathematics.

Before the study, students spent one hour each day learning about the importance of cooperative learning and the expectations they would be held to for the semester. The CAT Form E Level 14 (a norm-references, standardized test) was used to measure achievement levels while Peterson's attitude toward mathematics scale was used to measure student attitudes. After one week of receiving information on methods of cooperative learning, students completed a pretest. They were then grouped heterogeneously into one group of five and four groups of four. Students worked in groups through the semester; changing groups every two weeks and students were given weekly quizzes to demonstrate whether they had mastered the concept for the week. Data from the CAT were scored and analyzed.

The results of ANOVA statistical procedures indicated that the cooperative learning situation had a positive impact on student achievement. In all cases but one, statistically significant increases were found between the pretest and posttest. Results also strongly suggested that cooperative learning had a positive impact on the attitudes of students of color toward mathematics.

This study was valuable because it was the only study that focused entirely on students of color, a group that overall tends to be lower achieving students than their white peers. However, because the students in the group were an intact-group, the author suggests that the results of the study could have been skewed.

A quantitative study by Moskowitz, Malvin, Schaeffer and Schaps (1983) investigated the effects of the Jigsaw strategy on students' attitudes with regard to themselves, peers and school. The researchers studied fifth and sixth grade classes in eight elementary schools in a middle class, suburban public school system in northern California and found that Jigsaw influenced students 'impressions of their classroom environment but did not produce the hypothesized affective benefits

There were 147 students (33 5th grade males, 38 5th grade females, 37 6th grade males, 39 6th grade females) in the treatment group (experimental schools) and 114 students (38 males and 39 females in grade 5, 23 males and fourteen females in grade 6). Fifteen teachers were assigned to the experimental condition, with 8 of those participating in training on using jigsaw.

The teachers who participated in the Jigsaw in-service training rated the course as interesting, well organized and useful. Observations indicated that jigsaw was implemented in five of the six classes, but only three of the classes were exemplary. Some of the teachers exhibited difficulty implementing the class structure.

The students who participated in Jigsaw reported that their classroom climate was less competitive, while sixth-grade participants perceived their classes as more cooperative. Unfortunately, fifth-grade participants were found to have lower academic self-esteem. Students in the exemplary Jigsaw classes found their classroom atmosphere to be more cooperative, but these classes were in the minority. They had better school attendance than students in the control group. In addition, fifth-grade girls perceived less competition in the exemplary classes. Overall, however, the results were disappointing given the failure to find improvements in students' attitudes toward school.

One major flaw in this study that was pointed out by the authors was that students' attitudes were not measured before the study, so no changes could be seen. Additionally, the teachers had some difficulty implementing the jigsaw activity, which the authors stated could have been due to the fact that the teachers had previously taught in whole-class instruction, and jigsaw required giving up some control of the classroom. However, the teachers who did successfully implement the jigsaw structure saw their classes become less competitive, which could be helpful overall to the students.

The six studies in this section examined the effects of cooperative learning on students' attitudes toward mathematics and toward their math classes. The results of these studies varied. Three studies found that students who worked in cooperative groups had more positive attitudes toward mathematics than students who learned through whole-class instruction. Bilican, Demirtasli and Kilmen (2011) showed a correlation between an increase in the number of students who had experienced cooperative learning and an increase in the number of students who reported agreeing or partially agreeing with the statement "I enjoy learning Maths." Additionally, Vaughan (2002) asked how cooperative learning affects the achievements and attitudes in math of 5th grade students of color in one of Bermuda's elementary schools. Results strongly suggested that cooperative learning had a positive impact on the attitudes of students of color toward mathematics. Meanwhile, Ifamuyiwa and Akinsola (2008) investigated the effects of self and cooperative instructional strategies and found that overall, students who had participated in either method reported more positive attitudes about achievement in mathematics than they had prior to the study. However the self-instructional students reported a higher increase in attitude than did that cooperative groups.

Additionally, one of the studies indicated that cooperative learning had a positive effect on attitudes toward mathematics only for certain groups of students.

Ke & Grabowski (2007) examined the effects of combining computer games with cooperative learning in mathematics and found that cooperative game-playing resulted in more positive attitudes than either competitive game-playing or no game playing for economically disadvantaged students, but showed no significant differences for economically normal students.

Finally, two of the studies had ambiguous results regarding students' attitudes toward mathematics. Owens and Barnes (1982) asked about the relationships between cooperative, competitive, and individualized learning preferences of secondary school students and the perceptions of classroom atmospheres in two classes. They found that that senior high school students expressed a much greater preference both for more cooperative and more competitive social contact in learning than do first year high school students, indicating that older students expressed a preference for cooperative learning, but showing little information about the impact of cooperative learning on students' attitudes. Additionally, Moskowitz, Malvin, Schaeffer and Schaps (1983) asked about the effects of the Jigsaw strategy on students' attitudes with regard to themselves, peers and school and found that jigsaw was implemented in five of six classes where teachers had received training, but that only three of the classes were showing exemplary results, with little information about what was meant by "exemplary" results. However, no studies indicated that cooperative learning had a negative effect on students' attitudes about mathematics.

Cooperative Learning in Action

We have now seen many of the effects of cooperative learning in the classroom. However, cooperative learning can be implemented in the classroom setting in a wide variety of ways. It is therefore worthwhile to consider the role that the teacher plays in the cooperative classroom environment. Six studies in this review focus on the ways that a variety of teachers have implemented cooperative learning in their own classrooms, and the degree of success those teachers had. A qualitative study by Swan (2006) invesigated the effects of learning GCSE mathematics through discussion and reflection, with a sample of 834 students ages sixteen to twenty-one and found that when discussion and reflection were implemented in the classroom, students' existing knowledge and misunderstandings were brought to the surface. A longitudinal, multiple case quantitative study by Boaler and Staples (2008) studied how mathematics can be taught effectively in a variety of different settings and circumstances and studied roughly 700 students in three California schools. After one year students working a mixed-ability, cooperative class, as opposed to the traditional math classes taught at the other two schools were performing as well as their peers from other schools, and after two years those students were significantly outperforming students at the other schools on standardized math tests. A quantitative study Chui (2004) examined how teacher interventions (TI) conducted during cooperative learning tasks affected students' subsequent time on-task (TOT) and studied two female, non-Hispanic, White teachers and their six classes made up of a total 220 9th grade students, and showed that Post-TI TOT generally exceeded pre-TI levels, particularly when a student initiated a TI with a specific question, while greater problem-solving content tended to reduce post-TI TOT. A qualitative case study by Clarke (1997) focused on two teachers and their sixth grade students from a variety of SES backgrounds, the teachers' use of innovative mathematics and how the role of the teacher changed when a unit of instruction based largely on non routine problems was used, and found that one teacher demonstrated little change while the other demonstrated increased comfort with posing non-routine problems to students. A qualitative study by Kotsopoulos (2010) studied the effects of "self-surveillance" in a collaborative learning environment by videotaping students in an 8th grade classroom in a large urban setting, located in a region with a high concentration of families with high socio-economic status and found that over time, self-surveillance generated normalized ways of collaborating. Finally, a qualitative study by Siegel (1998) explored the implementation of cooperative learning by an expert 8th grade math teacher, identified as the project leader, and asked about the teacher's personal definition of "cooperative learning" and implementation of cooperative learning, and found that the project leader's conceptualization of cooperative learning was consistent with a research-based model, that the project leader used personal techniques to implement cooperative learning and that implementation involved the integration of cooperative learning with the project leader's previous basic lesson plan.

A qualitative study by Swan (2006) examined the effects of learning GCSE mathematics through discussion and reflection on students ages sixteen to twenty-one and found that when group discussions, along with reflection, were implemented in the classroom, students' existing knowledge and misunderstandings were brought to the surface. Swan (2006) examined the effects of learning GCSE mathematics through discussion and reflection, with a sample of 834 students, ages sixteen to twenty-one in GCSE retake course. The teachers in the study were from forty-four sixth-form colleges in England. One teacher from each school attended three workshops while other teachers from the same schools were invited to be part of a control group and report information on their "normal ways of working." The teachers in the experimental group were given a set of resources for teaching algebra lesson including classroom materials, illustrative video clips and questions for teachers to use for reflection and discussion. The theoretical principles underpinning the design of the resources may be summarized as follows:

* Lessons were conducted in supportive social contexts,
* Lessons consisted of rich, challenging tasks.

* Students were encouraged to make mistakes and learn from them
* Teaching emphasized methods and reasons rather than just answers
* Students created links between mathematical topics
* The purpose of each lesson was communicated clearly to students
* Appropriate use was made of technology.

Both the teachers and their students were asked to report on their classroom practices. The students were given a list of fourteen practices and asked to rank on a scale of 1 (low) to 5 (high) how frequently their teachers engaged in these practices. The statements were categorized as either teacher-centered (T) or student-centered (S). The student-centered statements included several statements about whether students worked in cooperative groups such as, the teacher "asks us to compare different methods for doing questions", " expects us to learn through discussing our ideas," and "asks us to work in pairs or small groups." These statements suggest that cooperative learning is key to a student-centered approach, compared to teacher-centered statement "the teacher expects us to work mostly on our own." The researcher then developed a scale of teacher-centeredness by reversing the scores of the student-centered practices and adding them to the scores from the teacher-centered practices. In classes where "many" of the activities from the resources were used, the teacher-centered statements received lower scores than they did in classes where none or only a few of the activities were used, and similarly the student-centered statements received higher scores in the same classes.

An algebra test was designed to assess a range of elementary algebra concepts and skills such as evaluating expressions involving numbers, interpreting an expression set in a simple everyday context and rearranging formulas. Meanwhile, individual teachers were first rank ordered according to the mean teacher-centeredness scale ratings given by their students and split into three groups based on whether they used “many,” “few,” or “none” of the activities from the curriculum. Then each group was split at the median rating so that samples of roughly equal size were obtained.

A two-way MANOVA was conducted on the results of the test. The mean changes showed that within each group a student-centered approach resulted in a greater improvement in performance than a more teacher-centered approach. The greatest mean improvement (9 marks, 15%) was therefore observed with the 59 students who were taught many of the activities in student-centered ways. The results show statistically significant gains ($p<0.001$) from pre- to post-course algebra results overall, with significant interactions between the number of discussion lessons used from pre- to post-course algebra results, and for the student- or teacher-centeredness from pre- to post-course algebra results ($p<0.001$). Thus it does appear that the greater the number of discussion lessons used and the more student-centered the teaching style, then the greater the gains in algebra learning. These gains are not very large, however, and this reflects the difficulty of the test for these students.

This study confirmed much of what researchers already knew about (GCSE) math classes. The teaching methods were mostly transmission-oriented and often unsuccessful in terms of the students learning algebra.

However, the resources used by teachers appeared to encourage and support the implementation of collaborative, discussion-based approaches to learning algebra. There was evidence to suggest that learning was enhanced, particularly when teachers used the materials and resources in student-centered ways.

While this study did not show significant improvements in test scores for students in the classes, it did show evidence that many math classes focus on teacher-oriented approaches to instruction, and that these approaches do not help students learn. Additionally, this study showed a lot about what practices might be common in traditional mathematics classrooms, and therefore is likely reflective of some of the other classrooms observed in the studies featured in this paper.. Finally, as this study focused on GSCE retake-classes, many of the students in the classes were students struggling to meet basic standards. On the one hand, this means that the data gathered does not represent students who leave school rather than finish their GSCE math test, or those who manage to pass on the first try, so the results may not apply to higher or lower achieving students. On the other hand, it means that the results of the study suggest what we can do to help the students who want it, and who need it the most.

A longitudinal, multiple case quantitative study by Boaler and Staples (2008) investigated how mathematics can be taught effectively in a variety of different settings and circumstances. The researchers studied roughly 700 students in three California schools: Railside, an urban high school with an ethnically, linguistically and economically diverse student population, Greendale, situated in a coastal community with a more homogeneous, primarily White student body, and Hilltop, a rural high school with primarily White and Latino/a students. The schools were chosen because they were roughly the same size, but allowed researchers to examine three different mathematics-teaching approaches: Greendale and Hilltop offered families a choice between a traditional math curriculum and an integrated sequence of courses. Railside used a reform-oriented approach that did not offer students and their families choice. Data was collected and analyzed to inform an understanding of each of the teaching approaches.

Railside, the focus of this study, was a large school located in an urban setting and had a diverse student population with a variety of ethic and cultural backgrounds. At Railside, before the study started, students were performing poorly in math compared to those at the other two schools. Researchers collected both quantitative and qualitative data to inform their understanding of both the teaching approaches taking place and the interactions in the classroom. The researchers videotaped over 600 hours of lesson and analyzed them in three ways. (1) They produced thick descriptions of the teaching and learning in different classes based on observations, including identifying focal teachers. (2) They conducted a quantitative analysis of the allocation of time in each of the classes. (3) Finally, they performed a detailed analysis of the questions teachers asked students divided their questions into such categories as probing, extending and orienting.

The researchers found that most of the teachers at Hilltop and Greendale High School used traditional methods, as described by teachers, of mathematics instruction. Students sat individually and worked on short, closed problems while teachers presented information through lecture. The time coding showed that teachers at these schools spent about 21% of class time lecturing, 15% questioning students in a whole-class format and 48% of class time working on short examples independently. Less than 1% of classroom time was spent with students presenting their work.

In contrast, at Railside teachers only spent about 4% of class time lecturing and students were taught in small, heterogeneous groups. Teachers spent about 9% of class time questioning students as a whole group, and students spent 72%, of their class time working in groups while the teacher circulated the rooms and helped small groups. Additionally, students spent about 9% of class time presenting their work.

At the beginning of high school, the researchers gave all students in the three schools that were starting Algebra classes, a test on middle school mathematics. At Railside, all incoming students were placed in Algebra. Comparisons of mean scores indicated that at the beginning of Year 1, the students at Railside were achieving at significantly lower levels than students at Greendale and Hilltop ($t = -9.141$, $p < 0.001$, $n = 658$). At the end of Year 1, the researchers gave all students a test of algebra to measure students' progress over the year. The difference in means (1.8) showed that the scores of students in the two approaches were now very similar (traditional = 23.9, Railside = 22.1), a difference significant at the 0.04 level ($t = -2.04$, $p = 0.04$, $n = 637$). The Railside students' scores were drawing near comparable levels after a year of algebra teaching. At the end of the second year, the researchers gave students a test of algebra and geometry, to reflect the content the students had been taught over the first two years of school. By the end of Year 2, Railside students were significantly outperforming the students in the schools using the traditional approach ($t = -8.304$, $p < 0.001$, $n = 512$).

After one year of working a mixed-ability, cooperative class, as opposed to the traditional math classes taught at the other two schools (grouped by similar abilities, primarily independent work) the students at Railside were performing as well as their peers from other schools, and after two years the students at Railside were significantly outperforming students at the other schools on standardized math tests.

The researchers also gathered data at the end of the third year, and concluded that it did not show significant differences between the other two schools. However, the researchers also noted that the classes taught during that year were taught by new teachers, and that the Railside curriculum had not been developed as much during that time.

One of the most notable strengths of this study is that it compared student progress between schools. The researchers stated that students at Railside had started out performing worse than students at other schools, but were out-performing them after two years. Boaler and Staples (2008) noted that "Railside school has an unusual mathematics department" where the teachers work collaboratively with one another. One potential implication of this is that a teacher who is trying to implement collaborative learning in his classroom, but who does not have other teachers to collaborate with himself may have difficulty getting results.

In a quantitative study, Chui (2004) explored how teacher interventions (TI) conducted during cooperative learning tasks affect students' subsequent time on-task (TOT) and studied two female, non-Hispanic, White teachers and their six classes, which were made up of a total 220 students in the 9th grade, and showed that Post-TI TOT generally exceeded pre-TI levels, particularly when student initiated a TI with a specific question, while greater problem-solving content tended to reduce post-TI TOT, supporting the claim that teacher responses involving small levels of content foster student autonomy and interaction. In groups showing understanding of the problem situation, greater percentages of teacher commands reduced post-TI TOT, while, TOT correlated with higher overall scores/completeness of the problem.

The subjects for this study were two female, non-Hispanic, White teachers with four and five years of teaching experience, each teaching three classes. Both had Bachelors degrees in math as well as teaching certificates. The six classes were made up of a total 220 9th grade students, who formed 55 groups. The students were mostly from lower to middle class neighborhoods and attended a large urban public high school. 114 students (52%) were girls and 106 (48%) were boys. 8% were Asian, 21% Black, 38% Latino and 33% White. In this study, teachers introduced problems to students working in groups of four. Students were instructed to ask their group members for help first, but otherwise teachers responded freely to questions. Teachers also displayed a poster that showed four prompts: “understand the problem”, “propose solution ideas”, “listen to others”, “give reasons for or against the proposal,” and ask others to clarify.” Teachers monitored students as they worked in groups. A student from each group was then selected to randomly present his or her groups’ solution to the problem. Each group of students and each teacher were videotaped and research assistants created summary logs of the videotapes.

Research assistants (RAs) coded time-off task and time-on-task. Chui (2004) defined on-task as “engaging in student behaviors conducive to problem solving, including suggesting or executing solution proposals, encouraging others to work on the problem, listening to solution proposals, and discussing solutions” (373). The RAs gave each student a TOT rating, computed by observing a student for one minute and dividing the TOT seconds by 60 seconds. This information was recorded for groups both before teacher-interactions (pre-TI) and afterward (post-TI).

The researcher then used mean difference t-tests to compare whether post-TI TOT differed significantly from pre-TI TOT (post- TI TOT minus pre-TI TOT). The 108 recorded TIs ranged from fourteen to twenty per class. Students rarely sought help from the teacher, initiating a TI with a specific question twenty-two times and with a general question six times.

The study showed that Post-TI TOT generally exceeded pre-TI levels, particularly when a student initiated a TI with a specific question. Greater problem-solving content however tended to reduce post-TI TOT, supporting the claim that as students work in cooperative groups, teacher responses involving small levels of content foster student autonomy and interaction.

The RAs recorded information about all lengths of TIs and found that TIs ranged in length from nine to 175 words. This included interactions such as a simple “get back to work” as well as longer interactions in which a teacher would ask the students what information they needed to find and help them come up with a plan to find it. This is a strength of the study as it took into account a wide range of interactions between students and teachers.

One of the major shortcomings of the results of this study, which was noted by Chui, was that teacher-initiated TIs happened more frequently when student were off-task. Therefore, one would expect pre-TI TOT to be low compared to the average TOT. Additionally, student-initiated TOT was initiated by students who were already engaged in the task, but may have had higher of-task time as they waited for their teacher, but who were more likely to be on-task once they had interacted with the teacher. It is therefore hard to draw conclusions about the effect of TIs on students who may have appeared to be on task, but were hesitant to ask for help.

A qualitative case study by Clarke (1997) focused on two teachers and their sixth grade students from a variety of SES backgrounds to record what happens when teachers use an innovative mathematics curriculum. Clarke observed how the role of the teacher changed when a unit of instruction based largely on non routine problems was used, and found that one teacher demonstrated little change while the other demonstrated increased comfort with posing non-routine problems to students.

Prior to the study, the consideration of the role of the teacher in a reformed classroom led to the development of a six-component categorization of the teacher's role. "Role" was taken to encompass not just what the teacher did in the classroom in terms of decision-making, interaction, and organization, but also the choices, decisions, and commitments made by the teacher in preparing for instruction. The researcher summarized his conceptual framework and the key components of the role of the teacher under two headings: what the teacher does and his (or her) related beliefs about the learning and teaching of mathematics.

A case-study approach was used in studying two sixth grade math teachers, given the pseudonyms of Mr. Martin and Mrs. Bartlett, from a school in a mid-western town in the United States. The students represented a range of socio economic levels. Minority students comprised about 22%of the school population. The school emphasized a strong sense of community, which was physically represented by being "a school without walls." Grade-level groups were separated into different areas of the school. Classrooms within a particular grade-level were separated by movable partitions, providing a feeling of connectedness, demonstrated by occasions when a teacher would change his or her lesson plan, after having observed or over heard a colleague, using what he or she thought was a richer or more fitting approach. Both teachers followed in the study had been teaching for 20 years, mostly at the sixth grade level.

Martin and Bartlett both received support, including trainings, to implement an “innovative curriculum” that had students working in groups and placed an emphasis on open-ended questions and student justification. The researcher observed each of the teachers in their classrooms on thirty-six occasions, as well as observed them in meetings and professional development trainings. The researcher also conducted regular interviews with each teacher, which included asking for each teacher’s insights on the changing role of the other teacher.

The researcher took notes on various components of what he considered to be an Innovative Teacher’s roles in the classroom. The key components, for the sake of this paper, were as follows:

> 1.The use of non-routine problems as the starting point and focus of instruction, without the provision of procedures for their solution.
>
> 2. The adaptation of materials and instruction according to local contexts and the teacher's knowledge of students' interests and needs.
>
> 3. The use of a variety of classroom organizational styles (individual, small-group, whole-class).
>
> 4. The development of a "mathematical discourse community," with the teacher as "fellow player" who builds on students' solutions and methods

The researcher found that at the beginning of the study, Bartlett engaged in several of these components in her class, at least partially. The students almost always worked in groups. Bartlett regularly adapted ideas from her colleagues to make problems more relevant, and also often provided students with open-ended tasks that they could struggle with and discuss, which made the tasks group-worthy.

Martin meanwhile usually had his students working in groups but would often provide them with instructions to complete the mathematical tasks they worked on. He did also use his colleagues' ideas to adapt problems to make them more relevant to students, as this was part of the culture at the school, but as he often told students how to approach the problems, he played less of a role of "fellow player" and more the role of an instructor. In this way, the group-work done in Martin's classes before he had implemented the new curriculum was not overly group-worthy.

By the end of the year, Bartlett showed little change in her teaching practices, though she stated that she wished to continue to move toward a curriculum in which these components were more fully adapted. Martin meanwhile showed significant increases in his use of open-ended questions and implementing the components in his classroom. He engaged in the practices more frequently than Bartlett. Martin stated that the journey had been difficult for him, but that he thought it was worthwhile. Both teachers expressed they thought that a classroom revolving around group-work and mathematical discussion was something they wished to engage in, and a community that was worth the struggle.

One of the shortcomings of this study was that there was not a lot of data to show what was meant when the researcher said that something was done more frequently by one teacher than by another, or to support the claim that Martin's role had changed a lot while Bartlett's had not changed much. On the other hand, this is one of the few studies in which the researcher interviewed teachers and reported the teacher's impressions of the curriculum. As both teachers in the study supported the innovative, cooperation and discussion based curriculum, it appears that this or a similar curriculum would be something that a teacher could adopt, even if, like Martin, the teacher had some initial difficulties.

A qualitative study by Kotsopoulos (2010) explored the effects of 'self-surveillance' in a collaborative learning environment. The researcher videotaped students in an 8^{th} grade classroom in a large urban setting, located in a region with a high concentration of families with high socio-economic status and found that over time, self-surveillance generated normalized ways of collaborating.

The researcher videotaped collaborative lessons spanning two or more double periods of mathematics (140+ minutes). Students worked on open-ended problems while the teacher rotated through the groups and reminded students about various aspects of collaboration that the students had discussed in class. The researcher viewed the videos at least twice and transcribed them. The researcher also conducted interviews and focus groups. The study focused on students in an 8^{th} grade classroom in a large urban setting, located in a region with a high concentration of families with high socioeconomic status. The students were all thirteen or fourteen years of age. Their teacher had ten years of experience.

The researcher found that initially, students in the collaborative groups took on two primary roles: the foreman, who tended to direct the progress of the group and did not always engage in the activities, and the laborers who did most if not all of the calculations and constructions. The researcher reported that the videotapes showed that students were not working together, and that the groups sometimes excluded certain members. Some students felt silenced by more vocal members of their groups, while others reported hostility.

Students then viewed the video from their group interactions. Many of the more vocally active students were surprised to see themselves dominating the conversation. Some even defended their actions. Meanwhile many of the students who had been predominantly silenced spoke of not being at all surprised by the videos.

The non-collaborative learning sent a message of incompetence and exclusion to certain students who were not assertive and were overshadowed by their foreman classmates. The teacher's efforts proved inconsequential, as did the video camera recording their collaborative learning and the task. The only observable variances occurred when students had the opportunity to view themselves on video and talk about the interactions.

After watching videos of themselves, the students showed an increased awareness of the dynamics in their groups. One girl, who was the foreman of her group, watched a video of herself excluding another student from the group. According to Kotsopoulos (2010), the foreman girl stated, "I felt really bad after because I didn't realize I was saying that… " (137). In videos taken after the self-surveillance session, the same girl was seen trying to ensure that her lower-status peer was able to partake in the activities. The self-surveillance helped to generate new normalized means of collaboration. The power within the group became more evenly distributed, rather than isolated to one or two students. This suggests that while cooperative learning may initially consist of a few high-status students excluding their peers, teachers can help students equalize status within their groups by helping students monitor their own behavior.

This was one of the few studies that focused on detailed descriptions of the conversations that students had while working in groups, as well as the students' own reactions to seeing themselves. Unlike the other studies in this section, which focused on teachers' practices, this case study focused on the ways that students interacted in a classroom. Because the researcher showed the students the videos of themselves, and then saw changes in behavior, the results suggest that a teacher may have success equalizing status within groups by having her students use self-surveillance.

One of the shortcomings of this study, as mentioned by the author, was that it was obviously impossible to see how the students interacted when they were not being observed. Therefore, the researchers could only drawn conclusions about how the students behaved when being observed—it is possible that if the students were not being watched, they would monitor themselves less and fall back into their previous roles within their groups.

A qualitative study by Siegel (1998) investigated how a particular, expert math teacher implemented cooperative learning in class, and how the teacher's personally definition of cooperative learning affected the implementation of cooperative learning. Siegel found that the teacher, referred to in the study as "the project leader" used personal techniques to implement cooperative learning and that implementation involved the integration of cooperative learning with previous basic lesson plans.

The subject of this study was an 8^{th} grade math teacher who had been chosen by his district to serve as a project leader for a cooperative learning initiative that his district was implementing in classroom. The teacher's job as project leader was to attend trainings and to educate his colleagues on ways to implement cooperative learning.

The researchers interviewed the teacher several times to obtain information about his personal definition of cooperative learning. They also conducted observations of his classroom. They observed three groups of students from a general mathematics class and three from an advanced mathematics class.

Transcripts of interviews with the project leader revealed that his concept of cooperative learning reflected a model by Johnson and Johnson (1983), the model in which he was trained. He defined cooperative learning as instruction that contained five crucial elements identified by Johnson and Johnson: (1) face-to-face interaction, (2) individual account- ability, (3) positive group interdependence, (4) social skills instruction, and (5) debriefing.

According to Siegel, classroom observation data on a particular occasion revealed that the enactment of cooperative learning in the teacher's class involved integrating the five elements with his basic lesson plans. The lesson plan included review of previously learned material, introduction of new material, and practice of new skills. The teacher used cooperative learning primarily for practicing newly acquired skills and occasionally for reviewing previously learned material.

The researchers also reviewed narrative notes for further analysis of differences between general and accelerated mathematics students' approaches to disagreements. In the general mathematics classes, the student discussion leader's method for solving a problem was typically viewed as correct and adopted by the group. On the few occasions when these students disagreed about the solution process, they adopted another member's method and solution as correct, referred to the textbook, or asked for help from the teacher. In contrast, in the accelerated math class, students in groups often disagreed with the discussion leader's methods for problem solving. The accelerated students suggested alternative solutions and discussed different possible solutions within their groups. Often, more than one solution yielded the correct answer. Individual group members adopted different methods, but all agreed on one answer.

When the researchers asked the teacher about how and when he decided to use cooperative learning, he responded that often used cooperative learning in his accelerated classes for review activities, because he was confident that at least one student in a group would understand the procedure or content. He also admitted that he was less confident in the ability of students in the general math classes and suggested that he felt a need to use teacher-centered instruction methods for review in those classes. Finally, he stated that he had only now, after twenty years of teaching been taught how to implement cooperative learning, and that if he had been taught earlier in his career, he might have felt differently.

This study is a valuable contribution to literature on the topic of cooperative learning because unlike others reviewed in this paper, the study focused on just one teacher and his methods. It provided detailed descriptions of what the teacher did in his classroom, as well as a teacher's perspective on why he often chose not to use cooperative learning in his general classes, stating that he was concerned that students might struggle. The study also however had shortcomings. There were no cases to compare it to. It did not show how another teacher might implement the same framework, which could be useful to see. It also did not explain why this particular teacher was chosen as the best person to receive the training and be the project leader—it suggested that he had been teaching for many years, and that the district judged him to be a good teacher, but said nothing about the criteria that had been used to decide who would be the project leader.

The six studies featured in this section in focused on the ways that a variety of teachers implemented cooperative learning in their own classrooms, and the effects of cooperative learning in those environments. Three of these studies showed the success of individual teachers who had used cooperative learning in their own classes. Swan (2006) investigated the effects of learning GCSE mathematics through discussion and reflection, and found that when one teacher implemented discussion and reflection in the classroom, students' existing knowledge and misunderstandings were brought to the surface.

Meanwhile, Chui (2004) examined how one teacher's interventions conducted during cooperative learning tasks affected students' subsequent time on-task and showed that Post-TI TOT generally exceeded pre-TI levels, particularly when a student initiated a TI with a specific question, while greater problem-solving content tended to reduce post-TI TOT. Additionally, Kotsopoulos (2010) studied the effects of "self-surveillance" in a collaborative learning environment by videotaping students in an 8^{th} grade classroom in a large urban setting, and found that over time, self-surveillance generated normalized ways of collaborating.

Similarly, an additional study showed the success that multiple teachers had when their school adopted a cooperative learning curriculum for mathematics. Boaler and Staples (2008) studied how mathematics could be taught effectively in three California schools. They found that when cooperative learning was implemented at Railside, previously the lowest performing of the three schools, students started performing as well as their peers from other schools after one year, and after two years those students were significantly outperforming students at the other schools on standardized math tests.

The other two studies featured in this section examined the changing roles of secondary mathematics teachers in cooperative classrooms. Clarke (1997) focused on two teachers and their 6th grade students from a variety of SES backgrounds, the teachers' use of innovative mathematics and how the role of the teacher changed when a cooperative unit of instruction based largely on non routine problems was used, and found that over time, one teacher demonstrated little change while the other demonstrated increased comfort with posing non-routine problems to students. Additionally, Siegel (1998) explored the implementation of cooperative learning by an expert 8th grade teacher and asked about the teacher's personal definition of "cooperative learning" and implementation of cooperative learning, and found that the project leader's conceptualization of cooperative learning was consistent with a research-based model, that the project leader used personal techniques to implement cooperative learning and that implementation involved the integration of cooperative learning with the project leader's previous basic lesson plan, showing that he had built a cooperative curriculum by making adjustments to his previous lessons.

Overall, these studies showed that individual teachers, and schools, can be successful in implementing cooperative learning in their classrooms. Some teachers may find this easier to do than others. However, making adaptations to the previous curriculum can ease the transition.

Summary

Chapter two reviewed 30 studies on cooperative learning in mathematics and other classrooms. Of these studies, eight compared test scores between students who worked in cooperative groups and those who worked in independent learning situations. Ten studies examined the relationship between student behaviors and cooperative learning, and the effects of cooperative learning on student interactions in the classroom, while six studies showed the potential effects of cooperative learning on students' attitudes toward mathematics and toward their math classes. Finally, six studies focused on specific teachers and school that had implemented cooperative learning in math classes and discussed the impact on students as well as the way that teachers approached cooperative learning. Overall the studies found that cooperative learning had a positive impact on students' academic achievement and behaviors in the classroom, and appeared to have a positive impact on students' attitudes toward math class.

Eight studies in this chapter compared test scores between students who had worked in cooperative groups and those who worked independently or in whole class settings. The studies done by Zakaria, Lu Chung & Daud (2010), Sherman and Thomas (1986), Harskamp and Ding (2006) and Madrid, Canas, and Ortega-Medina (2007) found that students working in cooperative groups had higher test scores than students working in individual or whole-class settings. Meanwhile, Leonard (2001) found that low and middle achieving students received significantly higher test scores after working in groups than they had previously. The remaining studies suggested that cooperative learning alone might not have an effect on student achievement on tests, but that when combined with other techniques, cooperative learning had a positive impact on student achievement. Kramarski and Mevarich (2003) found that of four instructional methods, cooperative learning with metacognitive training produced the best results. Retnowati, Ayers, and Sweller (2010) found that group setting did not appear to have any statistically significant impact on student achievement. Finally, Linchevski and Kutscher (1998) found that in five out of twelve schools, students gained more by being hypothetically part of a lower ability group than of the next higher one, while for seven schools the opposite was true. This suggests that the benefit to a student of working in a group setting depends on the particular group. No study discussed in this paper showed that cooperative learning had a negative effect on student's test scores.

Ten studies examined the relationship between student behaviors and cooperative learning, and the effects of cooperative learning on student interactions in the classroom. Four of these studies, done by Esmonde (2009), Galton, Hargreaves and Pell (2009), Shachar and Sharan (1994) and Liekin and Zaslavsky (1997) showed that cooperative learning lead to a higher level of student engagement in class than did individual or whole class instruction. Additionally, three studies, done by Bentz and Fuchs (1996), Ross (1995) and Webb and Cullian (1983) found that cooperative learning was most effective when the students were given guidance about their interactions. Finally, three studies showed that cooperative learning was beneficial to students who were not typically high achievers. Jacques, Wilton, and Townsend (1998) found that a cooperative learning program yielded gains in social acceptance from classmates of students with mild intellectual disability. Pijls, Dekker and Van Hout-Wolters (2006) showed that a middle achieving student had benefited significantly more from cooperative interactions than a higher achiever did. Finally, Webb and Farivar (1994) found that when helping skills were included in the curriculum, minority students gave and received more elaboration on their answers to questions than they had previously. Overall the cooperative learning environment led to higher levels of engagement, particularly for low and middle achieving students, and particularly when students were given instruction about how to work in cooperative groups.

Six studies examined the effects of cooperative learning on students' attitudes toward mathematics and toward their math classes. The results of these studies varied. Three studies, done by Bilican, Demirtasli and Kilmen (2011), Vaughan (2002) and Ifamuyiwa and Akinsola (2008) found that students who worked in cooperative groups had more positive attitudes toward mathematics than students who learned through whole-class instruction. Additionally, the study done by Ke & Grabowski (2007) found that cooperative game-playing resulted in more positive attitudes for economically disadvantaged students, but showed no significant differences for economically normal students. Two of the studies had ambiguous results regarding students' attitudes toward mathematics. Owens and Barnes (1982) found that that senior high school students expressed a much greater preference both for more cooperative and more competitive social contact in learning than did first year high school students, indicating that older students expressed a preference for cooperative learning, but showed little information about the impact of cooperative learning on students' attitudes. Additionally, Moskowitz, Malvin, Schaeffer and Schaps (1983) found that jigsaw had been implemented in five of six classes but that only three of the classes were showing exemplary results, with little information about what was meant by "exemplary" results. However, no studies indicated that cooperative learning had a negative effect on students' attitudes about mathematics.

Finally, six studies focused on the ways that a variety of teachers implemented cooperative learning in their own classrooms, and the effects of cooperative learning in those environments. Three of these studies, done by Swan (2006), Chui (2004) and Kotsopoulos (2010) showed the success of individual teachers who had used cooperative learning in their own classes. Similarly, an additional study by Boaler and Staples (2008) showed the success that multiple teachers had when Railside School adopted a cooperative learning curriculum for mathematics. The other two studies featured in this section examined the changing roles of secondary mathematics teachers in cooperative classrooms. Clarke (1997) found that over time, one teacher demonstrated little change while the other demonstrated increased comfort with posing non-routine problems to students, while Siegel (1998) found that one teacher used personal techniques to implement cooperative learning, which involved the integration of cooperative learning with the teacher's previous basic lesson plan, showing that he had built a cooperative curriculum by making adjustments to his previous lessons. Overall, these studies showed that individual teachers, and schools, can be successful in implementing cooperative learning in their classrooms and that the transition can be eased by making adaptations to the previous curriculum.

These studies showed that cooperative learning had a generally positive effect on students in the secondary mathematics classroom. Cooperative learning resulted in higher test scores than whole class or individual learning.

Cooperative learning also resulted in positive behaviors and interactions among students, and appeared to have a positive effect on students' attitudes about mathematics. Finally, cooperative learning appears to be a technique that individual teachers and schools can implement as part of the mathematics curriculum and use to help students be more successful in school.

CHAPTER THREE: CONCLUSION

Introduction

It was made clear in Chapter one, that there is a need for reform in the way mathematics is taught in this country. Historically, math in the United States has been taught in an individualistic manner, where the teacher lectures and provides guided practice for students, a method that is failing to serve the needs of many students. Therefore, a movement toward different methods of teaching mathematics is a necessity.

The literature and research on cooperative learning is clearly in favor of the teaching strategy. This paper asked the question, “What are the effects of a cooperative learning environment on students in mathematics classes?” Chapter two critically reviewed the literature on the effects of cooperative learning in the classroom. First and foremost, the literature showed that cooperative learning has had a positive impact on students’ academic achievement in mathematics classes, as well as their attitudes toward mathematics. Additionally, the literature showed that cooperative learning has had a positive impact on students’ behaviors, both when interacting with the material, and when interacting with other students. Finally, the literature showed a variety of ways that teachers have implemented cooperative learning and examined their effectiveness.

Summary of Findings

Overall, the research on cooperative learning supports a cooperative learning environment in the classroom. The literature showed that cooperative learning has had a positive impact on students' academic achievement in the classroom, producing higher test scores for students than whole-class instruction. Cooperative learning also fostered positive changes in behavior among students within the classroom, particularly when paired with feedback from teachers or other sources. Finally cooperative learning has the potential to have a positive impact on students' attitudes toward mathematics.

Impact on student achievement on tests in mathematics

Studies across the board showed that cooperative learning had a positive impact on students' academic achievement. This was determined primarily based on students' performance on tests due to the fact that test scores are the easiest way to measure and compare student achievement. One study done by Retnowati, Ayers, and Sweller (2010), showed no significant effects of cooperative learning, while seven studies showed a positive effect. Sherman and Thomas (1986), Harskamp and Ding (2006) and Zakaria, Lu Chung & Daud (2010) all showed that students who had worked in cooperative groups outscored their peers who had worked independently and/or in whole-class instruction on tests. Additionally, Kramarski and Mevarich (2003) showed that when paired with meta-cognitive training, students working in cooperative groups could significantly outperform their peers. Finally, Leonard (2001) showed that cooperative learning was effective for low and middle-achieving students. None of the studies in this review showed a negative effect as a result of cooperative learning.

Impact on student behavior

Group-work fosters positive interactions among students. When paired with feedback and instruction on how to work in a group, group-work can help students become more independent as learners and more accepting of their peers.

Esmonde (2009), Galton, Hargreaves and Pell (2009), Liekin and Zaslavsky (1997), Ross (1995) and Shachar and Sharan (1994) all found that students who worked in cooperative groups participated in more activities in which they were actively engaged in learning than did their peers who were taught as a whole-class. Webb and Farivar (1994) and Jacques, Wilton, and Townsend (1998) found that cooperative learning also had a positive impact on the social acceptance and engagement of students who had previously been disadvantaged. Meanwhile, Bentz, J.L. and Fuchs, L.S. (1996) emphasized the importance of teaching students cooperative behaviors, as students who had been trained had more success asking for and giving help than their peers who had not been trained.

Impact on students' attitudes towards mathematics

The studies on the effects of cooperative learning on students' attitudes toward mathematics show varying results. Ke & Grabowski (2007) and Moskowitz, Malvin, Schaeffer and Schaps (1983) suggested that cooperative learning had no impact on students' attitudes toward mathematics. Vaughan

(2002) suggested that cooperative learning had a positive impact on the attitudes of students of color toward mathematics. Meanwhile, Owens and Barnes (1982) showed that girls preferred a strictly cooperative environment in math classrooms, while boys preferred a competitive one.

Some of these findings are obviously stronger than others. In many of the articles about student behavior, only a few classes were observed. Additionally, most of the classes were intact groups who might behave differently from a random group of strangers, or from another group. Finally, it is difficult to determine how students will behave if they are not being observed. As all of the students in the studies were being observed at the time, their behaviors may have been reactions to being watched. However, the consistent results among the studies on student behavior are evidence that these studies are representative of the population as a whole.

Additionally, as the studies on the effects of cooperative learning on students' attitudes are inconsistent, it follows that some of these studies may not be reliable. The study done on the largest number of subjects did not actually show a correlation between cooperative learning and students' attitudes, but rather just suggested that there might be a correlation. Further, many of the other studies did not perform a student attitude pre-assessment, so the evidence that cooperative learning had anything to do with students' attitudes was minimal.

Trends and Patterns in the Research

Most of the studies done on the effects of cooperative learning are quantitative. It was easy to find articles that had statistical analysis in them. It was more difficult to find articles that had qualitative descriptions. With the exception of the Pijls, Dekker and Van Hout-Wolters (2006) study, all of the studies used as "qualitative" studies were studies that had both qualitative and quantitative findings, and the qualitative findings were often difficult to sort through among the quantitative analysis.

Virtually all of the quantitative studies presented in this review used either ANOVA or MANCOVA. In the academic achievement and behavior categories, the two tests turned out very similar results, supporting the claim that cooperative learning has a positive impact on both academic achievement and student behavior.

Finally, these studies were done on fairly diverse student populations. Many of the classes were described as racially, economically and culturally heterogeneous. One study focused on students of color. Another focused on students with mild intellectual disability, though it was the only study that examined populations of students with disabilities.

Implications for Teaching

The research showed that students working in small cooperative groups tended to have higher test scores than their peers who worked independently or in whole-class settings. In particular, many minority students as well as low and middle-achieving students seemed to benefit from a cooperative learning environment. As the public schools in our country become more diverse, and achievement gaps become more of a concern, it appears that cooperative groups have the potential to help decrease the achievement gap.

Cooperative learning also has the potential to teach students important interpersonal skills. However, those skills do not come without practice. The teachers whose classes benefited the most from cooperative learning were those where students were given feedback on their behavior as they interacted within their groups. A teacher who wishes to use cooperative learning in the classroom must be prepared to teach her students how to work as a group in addition to teaching the material. This may include giving feedback to students as they perform in a desirable manner, or spending time with the class each week discussing the group behaviors that take place in the classroom.

Finally, it is important for a teacher to keep his students' attitudes about the class in mind. The evidence supporting cooperative learning as a way to improve student attitudes toward mathematics is not as strong as the evidence supporting cooperative learning as a way to raise test scores. Still, there is evidence that a movement toward cooperative learning as a primary means of instruction in math may improve students' attitudes in the long run.

Suggestions for Further Research

While most of the literature on cooperative learning advocates for cooperative learning, it remains the case that many teachers continue to use whole-class instruction in mathematics classes. Additionally, most of the teachers who partook in the studies covered by this review were eager to learn how to implement cooperative learning, and wanted to use cooperative learning in their classrooms, so there was little information on teachers who opted not to use cooperative learning as a method of instruction. Therefore, an obvious topic for further research would be to explore why many teachers choose not to use cooperative learning. This could include case studies of individual teachers who have tried cooperative learning and had unsuccessful results. It could also include interviews with teachers and an analysis of common reasons that teachers choose not to teach cooperative lessons. If teachers made claims that could be tested, such as "cooperative lessons take longer to plan," those claims could serve to form questions for additional further research.

The other major gap in the research done in these studies is that most of the studies were done over the course of less than a year, with many studies being done over only a few weeks. This means that there is little information about the long-term effects of cooperative learning on students in mathematics classrooms. Therefore, in order to get a more complete picture of the impact of cooperative learning, more longitudinal studies should be done on the topic. Studies done over the course of three, five or even ten or more years could help to determine the lasting effects of cooperative learning.

Conclusion

We live in a society that requires us to produce citizen who possess higher-order problem solving skills. Therefore, we live in a society in which the role of mathematics education is becoming more and more prominent. As more people in our society will need mathematical reasoning in order to be successful, we must consider the direction in which our country is currently heading when it comes to mathematics education. With an increased push to raise test scores, many people are concerned about the most effective way to teach mathematics in our schools. This should include the suggestion of cooperative learning as a model of teaching that can support students in a variety of aspects of education.

The literature shows that students who have learned in cooperative groups have had higher test-scores than their peers who have worked independently. Therefore, even if our main concern in education is how students perform on tests, cooperative learning produces better results than whole-class instruction. The literature also shows that cooperative learning has had a generally positive effect on the behavior of students in the classroom, and potentially on the students' attitudes toward mathematics and school. Students who worked in cooperative groups learned important interpersonal skills, as well as how to ask questions that could help them further their own learning, and engaged in more higher-order thinking than students who worked on their own. Additionally, students who had worked in cooperative groups reported increased levels of comfort and confidence with mathematics than their peers, and more students who worked in cooperative groups reported liking math.

It appears that cooperative learning may be just what mathematics education in this country needs.

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