

EXPLORING EQUITY IN A MATHEMATICS CLASSROOM
THROUGH COOPERATIVE TASK DESIGN

by

Katie E. Gates

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Katie E. Gates

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by

Sara Sunshine Campbell, Ph.D

Member of the Faculty

ABSTRACT

This action research study examined how the design of cooperative math tasks was related to students' opportunities to learn mathematics. It was conducted by a teacher-researcher during her first student teaching experience for the purpose of improving her ability to select and design cooperative math tasks in ways that support equitable student learning. Analysis of student work and classroom video across different tasks found that the task's evaluation criteria played an important role in providing students access to each others' ideas as well as creating opportunities for rich, mathematical discussion. The study also found that the way in which the task was launched, or delivered to students was important for their ability to access the mathematical ideas in the task, as was being given sufficient opportunities for personal think-time. The implications of these findings are specific to this particular classroom context, but were also found to be consistent with the current literature about effective practice in mathematics instruction.

Keywords: cooperative learning, equity, cooperative math tasks, task design, task launch, action-research

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CHAPTER 1: INTRODUCTION

Perspectives on Cooperative Learning

Cooperative learning and group tasks are becoming increasingly common in reform-oriented mathematics classrooms. There is a large body of educational research supporting this trend. Studies have repeatedly found cooperative learning to be highly effective for the development of students' mathematical understanding, thinking, and reasoning (Boaler, 2006; Cohen & Lotan, 2014; Cohen, 1994; Featherstone, 2011; Slavin, 1983; Walle, Karp, & Bay-Williams, 2012). Additionally, researchers have found cooperative learning to be associated with a variety of other positive outcomes including higher student achievement (Boaler, 1998; Slavin, 1983), positive identities and dispositions for students (Alexander, Chizhik, Chizhik, & Goodman, 2009; Boaler, 1998; Usher, 2009; Slavin, 1983), positive student relationships and social benefits (Alexander et al., 2009; Boaler, 2008a; Slavin, 1983), and increased student discourse and math-related discussion (Featherstone, 2011; Hufferd-Ackles, Fuson, & Sherin, 2004; Khisty & Chval, 2002).

Despite the body of research highlighting the numerous benefits of cooperative learning, there are also notable critics. Randall (1999) identified three major weaknesses or flaws in cooperative learning: (1) that groupwork lends itself to fact-based activities rather than encouraging thinking, (2) that it is unfair to hold students accountable for each others' learning, and (3) that typical ways of organizing groups does not support the learning or participation of all group members. Another common critique of cooperative learning—perhaps heard more frequently from students rather than from teachers or researchers—is that it is problematic to require the participation of students who prefer not to work collaboratively. Unfortunately, there is not a strong body of research looking specifically at students' own perceptions of cooperative learning, but it

is not uncommon for students to resist working together on collaborative activities, particularly in middle and high school (Race & Powell, 2000). In her popular book *Quiet*, Cain (2012) echoes this concern, criticizing the trend toward group activities and teamwork mentalities in schools and arguing for the importance of individual work and classrooms supporting the needs of introverts. This issue has not yet been taken up by the research community, but these are common criticisms worth noting in any discussion of cooperative learning.

Randall's (1999) first criticism regarding the nature of cooperative activities has been addressed by many researchers in the past decade, whose findings contradicted her claim that cooperative learning does not lend itself to higher order thinking. On the contrary, several studies have examined the kinds of tasks that are effective in cooperative settings, and found rich, open-ended, and cognitively demanding tasks to be very important for successful and collaborative groupwork (Cohen & Lotan, 2014; Henningsen & Stein, 1997; Lotan, 2002; Smith & Stein, 1998).

Randall's (1999) second criticism about holding students accountable for each others' learning remains controversial. Particularly contentious is the classroom practice of evaluating students as a pair or as a group rather than individually (partner quizzes, group products, etc.) or the practice of assigning participation grades (Cain, 2012; Gillies & Boyle, 2010). Many research studies have found that the most successful groupwork has an element of *positive interdependence*, meaning that students are required to work together in some way to complete the task (Alexander et al., 2009; Cohen, 1994; Johnson & Johnson, 1987; Lotan, 2002; Stein et al., 1996). These studies found that tasks and structures utilizing positive interdependence were associated with more productive and collaborative groupwork. These researchers are not, however, necessarily advocating for practices such as group evaluation, which, according to Randall (1999), places "too great a burden on students". They emphasize the

importance of group *and* individual accountability, for example, students writing up an individual solution as a result of the collaborative process. Cohen, Lotan, Scarloss, Schultz, & Abram's 2002 study supports this practice, which found that the quality of students' individual work was highly correlated with the quality of the group's discussions and collaborative interactions.

Equity and Cooperative Mathematics Learning

Randall's third concern about inequitable participation and learning opportunities during groupwork is perhaps the most important issue in this research project. This is a well-documented problem, and is often framed as an issue of equity. The question of how to implement groupwork equitably continues to drive educational research on cooperative learning. This action research study follows the same line of inquiry. In particular, I am interested in better understanding the ways that my students can be barred access to the mathematical learning during cooperative tasks. What barriers exist for students that prevent them from entering into and benefiting from the collaborative process?

While cooperative learning has been studied for many decades, the study of equity in cooperative mathematics learning is a relatively new field. Researchers in this field have yet to come to a consensus about a working definition of equity in this context. When we discuss equitable mathematics learning, do we mean equal opportunities for students to learn, or do we mean equal achievement and learning outcomes? Are we interested in equity within the classroom or on a broad, national level? What role does equitable participation and interaction play in these issues? Certainly, all of these aspects of equity matter for students and for teachers. For the purposes of this study, it will be important for me to clarify what equity means in this context.

Because my study is limited to a single classroom and one particular group of

students, I did not examine issues of equity on a large scale. However, my study is situated within a larger educational context and so issues of equity on a national level are not separate from my classroom. Several studies have found that the students who benefit the least from cooperative tasks are often students from marginalized social, cultural, and linguistic backgrounds (Cohen & Lotan, 1995; Esmonde, 2009a; Featherstone, 2011). These inequities found in the classroom reflect broader patterns in the U.S. of unequal opportunities to learn mathematics. According to Boaler's research around math class and gender (2008b), "girls experience stereotyped attitudes and behaviors, contributing to their low interest and participation in math" (p. 140). Flores (2007) found specifically that "African American, Latino, and low-income students are less likely to have access to experienced and qualified teachers, more likely to face low expectations, and less likely to receive equitable per student funding" (p. 29). Whether a symptom of or a contributor to these larger patterns of inequity, teachers' attention to students' access to mathematics learning during cooperative tasks has been framed as a social justice issue in much of the current mathematics education research (Boaler, 2008a; Esmonde, 2009a; Featherstone, 2011; Gutiérrez, 2007; Khisty & Chval, 2002).

Elizabeth Cohen (Cohen & Lotan, 2014) and her colleagues at Stanford University attributed this problem of inequitable participation in groupwork to the *status* positions students have relative to one another (Cohen, 1994). They argue that students' unequal status positions often produce a hierarchy of perceived authority and competence within groups. This can be problematic for some students' opportunities to learn during cooperative tasks because it can elicit inequitable participation of group members and can cause some students' contributions to be overvalued and others undervalued (Alexander et al., 2009; Cohen & Lotan, 1995; Esmonde, 2009a; Featherstone, 2011). Other research has connected students' status positions in the classroom to a variety of factors, including popularity, prior academic achievement,

gender, race, etc. (Brown & Mistry, 2006; Cohen, 1994b; Holden, 1993).

In addition to status, another barrier for students can be the content of the task itself. Many researchers have found that the *type* of task matters for successful student collaboration (Alexander et al., 2009; Henningsen & Stein, 1997; Lotan, 2002). In her frequently-cited study about task design, Lotan (2002) identified several task characteristics that help make a task “groupworthy”, which have been found to support more equitable groupwork. Henningsen & Stein (1997) found that the level of cognitive demand was an important factor in facilitating successful and equitable group math tasks. Jackson et. al (2013) identified three features of a task which students must understand to access the mathematical learning: the context of the task, the mathematical ideas and relationships in the task, and the language in the task. In the same study, these researchers found that the ways in which teachers set up or launched cooperative tasks had a significant impact on whether or not students understood these task features.

In summary, this body of research suggests three potential barriers for student learning during cooperative tasks. First, status hierarchies can arise during groupwork which can cause some students to be shut out of the collaborative process. Second, the content of the task itself can present a barrier for students, either because they do not have the prerequisite mathematical knowledge to enter the problem, or because they are unfamiliar with the language or the context of the task. Third, the teacher can present the problem to students in ways that do not support students in accessing the content of the task or that take away opportunities for rigorous mathematical learning. The focus for my literature review, then, is as follows: What specifically can teachers do to help break down the barriers students may face during cooperative tasks, which in turn would provide more equitable learning opportunities for students?

CHAPTER 2: LITERATURE REVIEW

In my review of the relevant literature on cooperative mathematics learning, I have identified five important aspects of cooperative tasks that have been shown to increase students' access to the mathematical learning. These five aspects are as follows: (1) developing classroom norms around discourse and equitable groupwork, (2) grouping students heterogeneously, (3) designing tasks to be "groupworthy", (4) structuring the task to support equitable groupwork, and (5) launching the task with explicit discussion of task features without lowering the cognitive demand.

Developing Classroom Norms

Research has repeatedly found that how equitably a task is implemented depends not only on the task itself but also on the classroom environment, and the particular students in the classroom. Research has found that teachers play an important role in developing the classroom environment which can strongly affect student participation and nature of students' interactions during cooperative learning. These context factors include the degree to which discourse and interaction are encouraged in the classroom (Khisty & Chval, 2002; Walshaw & Anthony, 2008; Wells & Arauz, 2006), the classroom norms around collaboration and the nature of mathematics (Featherstone, 2011; Tatsis & Koleza, 2008; Yackel & Cobb, 1996), students' dispositions and beliefs about their own and others' competence in mathematics (Dweck, 2000; Esmonde, 2009a), and the way in which students are grouped during cooperative learning (Boaler, 2008a; Brown & Mistry, 2006; Cohen & Lotan, 2014).

Discourse and Interaction

The messages that a teacher communicates to students about how discourse is used in the classroom has important implications for cooperative learning and equity. Furthermore, the research reviewed in this study relies on constructivist theories of

learning, and the idea that discourse and interaction are an essential part of constructing knowledge. These theories have origins in Piaget's notion that children "have an active part in the process of knowing and even contribute to the form that knowledge takes. Cognitive humans actively select and interpret information in the environment. They do not passively soak up information to build a storehouse of knowledge" (Miller, 2011, p. 33). Constructivist theories of learning are prominent throughout educational research. Numerous studies have shown a strong link between student discourse and meaningful, retained learning (Chapin, O'Connor, & Anderson, 2009; Hufferd-Ackles, Fuson, & Sherin, 2004; Walle, Karp, & Bay-Williams, 2012; Wells & Arauz, 2006). Furthermore, the current mathematics reform movement has related participation in mathematical discourse to student learning. This helps to frame the problem of inequitable learning during groupwork. A constructivist understanding of learning implies that if not all students are comfortable or able to participate in the mathematical discourse, not all students have equal opportunities to learn during the task.

It is also well documented that during cooperative learning, the frequency of student interaction is less important than the quality or nature of the interaction (Chizhik, 2001; Cohen, 1994b). In her literature review, Esmonde (2009b) found that "some interactions are more likely than others to lead to meaningful mathematical learning, [such as] explaining one's thinking, getting a detailed response to one's questions, maintaining joint attention, and guiding a group's problem solving process" (p. 1024). In her research study, Esmonde (2009b) paid close attention to the work practices of students engaged in group tasks, and to the way that students positioned themselves and others during groupwork.

Importantly, she found that the students' work practices and the way they positioned themselves and others during the task influenced the opportunities to learn the mathematics involved in the task. She also found that the most equitable groups

tended to work collaboratively rather than individualistically. These findings were part of an in-depth case study on a single teacher's practice which limits their generalizability. However, because the researcher used rich, ethnographic data from many different sources over the course of an entire academic year, her findings regarding the link between students' work practices and learning were highly credible and transferrable beyond the particular context of her case study. Esmonde's classification of work practices and positioning is presented in Table 1.

Table 1.
*Classifying Work Practices and Positioning Within Groups*¹

Classifying Work Practices	Characterized by:
<i>Individualistic Work Practice</i>	Large periods of individual work before consulting each other Minimal interaction
<i>Collaborative Work Practice</i>	Group members put their ideas together Group members work together, and act as critical friends Group considers multiple ideas/strategies
<i>Helping Work Practice</i>	Math talk is asymmetrically organized by authority Uncritical uptake of ideas without considering other options
Identifying Positioning	Characterized by:
<i>Experts</i>	Group member who is frequently deferred to and often granted authority to decide whether work was correct
<i>Novice</i>	Student who defers to an expert and whose opinion is frequently passed over in discussion of mathematical controversies
<i>In-Betweens</i>	Neither
<i>Facilitator</i>	Students who orchestrate group activity and make sure that all group members are included in the discussion

This body of research paints a clear picture of the essential role that discourse, participation, and interaction play in cooperative tasks. Some researchers go so far as to argue that if students are not able to participate in mathematical discourse, then they will not learn the material (E. Cohen & Lotan, 2014; Esmonde, 2009a; Featherstone, 2011;

¹ Esmonde, I. (2009b). Mathematics Learning in Groups: Analyzing Equity in Two Cooperative Activity Structures. *The Journal of the Learning Sciences*, 18(2), 247–284.

Walshaw & Anthony, 2008). Cooperative tasks provide numerous opportunities for mathematical discourse, but if not all students are able or willing to engage in these interactions they may be barred access to the mathematical learning. This action research study relies heavily on Esmonde's (2009b) findings, and on the idea that participation, collaborative work practice, and high quality interaction are necessary for equitable learning during cooperative tasks.

Student identities and dispositions. Many teachers and researchers have noted the important relationship between students' identities and dispositions and the way that they approach learning and interaction with their peers. Students' dispositions can include the degree to which they perceive themselves as competent in a subject area, ideas about their own intelligence and ability, and their ideas about what it means to be "smart" in a subject. Dweck (2000) found that students' dispositions had important implications for learning and achievement. In her decades-long research, she found students' beliefs about the nature of their own intelligence to be strongly connected to how they responded to challenging tasks, as well as their achievement in school. More specifically, the study found that children who believed their intelligence was fixed gave up much more quickly on challenging tasks than students who believed their intelligence was malleable and could change with effort. A related finding was that the type of tasks students encountered in school influenced their dispositions. Importantly, she found that in mathematics, when students had experiences persevering through sufficiently challenging, genuine problems they were more likely to develop malleable theories of intelligence, as compared with students who had experienced less challenging, rote, procedural tasks.

In her review of the body of research around equity in mathematics learning, Esmonde (2009a) also found that issues of equity in cooperative classrooms were strongly related to students identities and dispositions. She argues that this is because

“students’ positional identities, especially with respect to their mathematical competence, influence the way that they interact with their peers. In diverse classrooms, girls, working-class students, students of color, and students who are generally considered low achieving by their teachers and peers may be marginalized and thus prevented from engaging in meaningful sense-making discussions with their groups” (p. 1024).

These are powerful findings with important implications for cooperative learning and equity. This body of research suggests that students’ perceptions of themselves and each other in relation to mathematics have a strong influence on their interactions during group tasks. They also suggest that group tasks should support students in persevering through challenging problems, and that teachers play an important role in fostering positive and productive student identities through their classroom practices and norms.

Cohen & Lotan (1995) identified some teacher interventions that can help students with low status or unproductive mathematical dispositions become positioned more competently during cooperative learning. One of these teacher moves is *assigning competence* to students with low status in a group. This status treatment is centered on drawing attention to the student’s intellectual contributions through public praise, validation, and discussion of these contributions, which can serve as a status booster (Cohen, 1994; Cohen & Lotan, 2014; Featherstone, 2011). These researchers found that over time, classrooms in which teachers more frequently assigned competence to low-status students were able to significantly diminish status problems during groupwork which resulted in more equitable groupwork.

These researchers also identified the importance of establishing classroom norms and selecting tasks which broaden students’ ideas of what it means to do mathematics. Cohen & Lotan (2014) argue that this practice provides students more opportunities to feel that their contributions to the group process are valuable, and to value the variety of strengths of their other group members. Several studies have also

found a link between students' perceptions of mathematics and the quality of student interactions. In her literature review, Esmonde (2009a) echoes Cohen & Lotan's (1995) findings when she notes that "in the long term, helping students to see that mathematics is more than just speedy calculation and that each group member has something important to offer the group can lay the groundwork for equity" (p. 1030).

Sociomathematical Norms

Esmonde (2009a) uses the phrase *social ecology of learning* to describe the classroom environment and social backdrop that shapes students' mathematical beliefs, dispositions, and learning interactions. Many advocates of cooperative tasks stress the importance of carefully cultivating a social ecology that supports the kinds of interactions and identities required for meaningful and equitable mathematical learning (Cohen, 1994b; Esmonde, 2009a; Kazemi & Stipek, 2001) and have found that unsuccessful or inequitable groupwork is more likely to occur in classrooms without an intentionally cultivated classroom environment. A critical component of the classroom environment are the norms that are established around what constitutes doing mathematics in that classroom.

Yackel & Cobb's (1996) term *sociomathematical norms* has been widely adopted in the literature to describe "normative aspects of mathematical discussions that are specific to students' mathematical activity" (p. 458). Research has found that teachers play a crucial role in establishing sociomathematical norms in their classrooms through the nature and setup of learning activities, interactions with students around the mathematics, and the extent to which they emphasize explanation, justification, and argumentation (Featherstone, 2011; Kazemi & Stipek, 2001; Tatsis & Koleza, 2008; Yackel & Cobb, 1996).

Classroom norms have been found to have important connections to student participation and the nature of interaction during groupwork. One way of explaining this relationship between norms and interaction is through the ways in which the norms shape students' dispositions, identities, and beliefs around mathematics (Featherstone, 2011; Kazemi & Stipek, 2001; Tatsis & Koleza, 2008; Yackel & Cobb, 1996). Esmonde (2009a) found that students' beliefs about mathematics underlie the nature of their interaction during groupwork. She concluded through her research that students who understood mathematics to be about quickly obtaining answers were much more likely to be engaged in helping work practice or individualistic work practice (see Table 1), which as we have noted do not optimally support equitable interaction. Conversely, Esmonde discovered that students who understood mathematics as a process of reasoning, conjecturing, explaining, and justifying were much more likely to be engaged in collaborative work practice and more equitable interactions.

Kazemi & Stipek (2001) found that the following sociomathematical norms were highly associated with student dispositions and beliefs around mathematical reasoning and justification: (1) An explanation consists of a mathematical argument, not simply a procedural description, (2) mathematical thinking involves understanding relationships among multiple strategies, (3) errors provide opportunities to reconceptualize a problem, explore contradictions in solutions, and pursue alternative strategies, and (4) collaborative work involves individual accountability and reaching consensus through mathematical argumentation. They also found these norms to be important in helping students develop conceptual understanding and reasoning abilities.

The teaching practices that these researchers found to be most highly associated with the development of these norms were characterized as teachers' *high-pressure for learning* (Kazemi & Stipek, 2001). This meant that teachers emphasized student effort, focused on learning and understanding, supported student autonomy, pressed students

for explanation and justification, and emphasized reasoning over producing correct answers. Teachers also influence these kinds of sociomathematical norms by clearly deciding and communicating what counts as mathematical justification, argumentation, sufficient explanation, mathematical competence, and different solutions (Featherstone, 2011; Kazemi & Stipek, 2001; Yackel & Cobb, 1996).

In addition to establishing sociomathematical norms, researchers have found that it is also helpful for teachers to intentionally set down social norms around what cooperative and respectful groupwork looks, feels, and sounds like (Cohen & Lotan, 2014; Featherstone, 2011). In her literature review Cohen (1994) found that a common frustration among teachers attempting to implement groupwork was that their students did not know how to work together. She notes that teachers should not expect them to, and that learning to work together productively and respectfully is a skill that must be explicitly taught. Group training, participation quizzes, and roles are some of the ways that teachers do this (Cohen, 2014, Cohen & Lotan, 1994; Esmonde, 2009a; Featherstone, 2011). Esmonde (2009a) echoes these findings, noting that “training, roles, and scripts can each be seen as moves toward equity because they provide scaffolding for all students to (a) participate in the valued discourse practices of academic mathematics and (b) support competent identities for all group members” (p. 1027).

Heterogeneous Student Grouping

Another important equity-related factor in cooperative learning is the composition of classrooms and groups. Researchers Boaler (2006) and Burris, Heubert, and Levin, (2006) argue that tracking and grouping students based on achievement or ability (e.g., remedial, support, accelerated, or gifted) exacerbates the so-called “achievement gap” and makes access to high quality mathematics education even more inequitable.

Instead, proponents of equitable groupwork (Boaler, 2006; Burris et al., 2006; Cohen & Lotan, 1995; Featherstone, 2011) advocate for a heterogeneous or mixed-ability approach both to classrooms and groupwork. Burris (2006) found that de-tracking math classes and grouping students heterogeneously greatly improved student achievement for both initial low-achievers and high-achievers. The study showed impressively that under these reforms, “probability of completion of advanced math courses increased significantly and markedly in all groups, including minority students, students of low socioeconomic status, and students at all initial achievement levels. Also the performance of initial high achievers did not differ statistically in heterogeneous classes relative to previous homogenous grouping, and rates of participation in advanced placement calculus and test scores improved” (p. 105). Boaler’s work (2006) further supports the use of de-tracked, heterogeneous classrooms by noting the ways that they supported *relational equity*, a term she uses to describe the ways that “students learned to appreciate the contributions of students from different cultural groups, genders, and attainment levels” (p. 40).

Through this sequence of research findings, I have suggested that the way that teachers develop the classroom environment can affect student participation and nature of students’ interactions during cooperative learning. This then impacts students’ opportunities to learn in this setting. Therefore, when utilizing cooperative tasks in mathematics classrooms, this social ecology of learning is essential for supporting equitable participation and high-quality interaction.

Designing Tasks

Many researchers have found that a powerful way for teachers to impact student participation and interaction during cooperative learning is through the task itself. Lotan (2002) provides a useful and well-cited theoretical framework for determining a task’s

“group-worthiness”. This framework guided my research about task design and I have modified Lotan’s task characteristics slightly to specifically reflect mathematics tasks. These modified criteria are shown in Table 2. The following section of this literature review will explore how each of these task design features has been shown to support more equitable student learning.

Table 2.
*Group-Worthy Task Features*²

Task Features	Characteristics
Open	<ul style="list-style-type: none"> • Genuine dilemmas and authentic problems with real-life uncertainties and ambiguities • Many possible solutions • Accessible problem solving context
Multidimensional	<ul style="list-style-type: none"> • Many ways to approach the problem • Many ways to model the problem (picture, table, graph, etc.) • Requires a variety skills and mathematical competencies to solve the problem • Many ways to demonstrate mathematical competence or “smartness”
High Cognitive Demand	<ul style="list-style-type: none"> • Meets criteria for high cognitive demand tasks (i.e. <i>procedures with connections</i> tasks or <i>doing mathematics</i> tasks, see Table 3) • Highlights a big mathematical idea • Opportunities for justification and reasoning
Positive Interdependence	<ul style="list-style-type: none"> • Requires students to work together to complete the task • Group and individual accountability • Multiple abilities and mathematical competencies needed
Clear Evaluation Criteria	<ul style="list-style-type: none"> • Clearly stated outcomes or product • Clear deliverables • Could use a rubric or performance assessment

Open Tasks

Tasks are usually described in the literature as *open* if they allow students multiple entry points, invite multiple solution paths, and have more than one possible solution. These are in contrast to *closed* tasks, which funnel students toward one

² Adapted from “Group-Worthy Tasks: Carefully constructed group learning activities can foster students’ academic and social growth and help close the achievement gap,” Lotan, R. A., 2002, *Educational Leadership*, 60(6), 72–75.

particular line of reasoning, solution path, or single answer. Open tasks have been linked to increased student participation and higher quality interaction (Esmonde, 2009b), positive mathematical identities (Boaler, 1998; Esmonde, 2009a), diminished status hierarchies or gaps in achievement (Alexander et al., 2009; Boaler, 1998; Chizhik, 2001), higher likelihood of student math-talk and productive discourse (Chizhik, 2001; Hufferd-Ackles et al., 2004), and deepened mathematical learning and understanding (Boaler, 1998; Cohen, 1994a; Featherstone, 2011).

Researchers have noted a variety of possible explanations for the positive effects of open tasks on student participation and quality of interaction. Esmonde (2009b) found that open tasks were more likely to invite quality and cooperative interactions where multiple strategies were discussed (instead of eliciting independent work or a group “expert” explaining a single solution). Open tasks can also serve to help broaden the ideas and shape norms in the classroom about what it means to do and be smart in mathematics (Featherstone, 2011; Walle et al., 2012), which in turn provides students more opportunities to feel that theirs and others’ intellectual contributions are valuable. Usher (2009) found that open tasks also helped students to develop persistence in solving challenging problems, which was found to be important in building student’s self-efficacy. Efficacy and persistence have also been highly associated with the development of students’ positive mathematical identities as competent math learners (Dweck, 2000; Esmonde, 2009a). Research conducted by Chizhik (2001) and Boaler (2008a) also found that open tasks used in groupwork settings were correlated with higher student achievement and virtually erased differences in achievement between students of different racial groups. All of these outcomes were found to be important in helping to break down status hierarchies in the classroom, and positively affected student interaction by influencing students’ perceptions of each other across perceived

differences and fostering positive relationships between group members (Alexander et al., 2009; Boaler, 2008a; Slavin, 1983).

Multidimensional Tasks

In addition to being open-ended and inviting multiple solution paths, tasks that require a wide variety of mathematical abilities and competencies have been associated with more equitable participation and interaction during cooperative learning (Cohen & Lotan, 1995; Lotan, 2002). Researchers often refer to these kinds of tasks as *multidimensional* (Boaler, 2006; Lotan, 2002). Lotan (2002) suggested that one explanation for this correlation between multidimensional tasks and improved participation and interaction is that these kinds of tasks “allow more students to make significant contributions to the group effort by using their various talents, intellectual competencies, and diverse repertoires of problem solving strategies” (p. 73). These tasks also provide students more opportunities to demonstrate their intellectual competence and thus can boost their status among their peers.

High Cognitive Demand

Many researchers have noted that in order to help all students develop strong conceptual understanding and reasoning abilities, they must be provided with opportunities to engage with challenging and cognitively demanding mathematical tasks (Henningsen & Stein, 1997; Hiebert & Stigler, 2004; Jackson et al., 2013; Stein et al., 1996; Walle et al., 2012). Many members of the math educator community have recently begun referring to these kinds of tasks as “low floor, high ceiling” tasks, referring to students’ ability to easily enter the problem while also engage them in higher order thinking. Several studies have found these types of tasks to be an important component of productive and equitable groupwork (Chizhik, 2001; Cohen & Lotan, 2014; Henningsen & Stein, 1997; Jackson et al., 2013; Stein et al., 1996).

Stein, Grover, and Henningsen (1996) conducted an investigation of mathematical tasks as important vehicles for building students' capacity for mathematical thinking and reasoning. From their study they developed a framework for determining the level of cognitive demand for a task, which has been widely adopted in

Table 3.

*Classifying Mathematical Tasks by Level of Cognitive Demand*³

Low Cognitive Demand Tasks and Characteristics

Memorization Tasks:

- Reproducing or memorizing facts, rules, formulas, or definitions
- Cannot be solved using procedures
- Non-ambiguous, involves exact reproduction of previously seen material
- No connection to the concepts or meaning that underlie procedure being used

Procedures Without Connections Tasks:

- Entirely algorithmic. Use of procedure is called for or evident from prior instruction
 - Limited cognitive demand, little ambiguity about what needs to be done and how
 - No connection to the concepts or meaning that underlie procedure being used
 - Focused on producing correct answers
 - Requires no explanations or explanations focus on describing procedure that was used
-

High Cognitive Demand Tasks and Characteristics

Procedures With Connections Tasks

- Focuses attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
- Suggests pathways to follow that have close connections to underlying conceptual ideas instead of narrow algorithms
- Uses multiple representations
- Requires some degree of cognitive effort, procedures cannot be followed mindlessly
- Students need to engage with conceptual ideas and deep content understandings

Doing Mathematics Tasks

- Requires complex and non-algorithmic thinking, conjecturing, justifying, and reasoning
 - Requires students to explore and understand the nature of mathematical concepts, processes, or relationships
 - Demands self-monitoring or self-regulation of one's own cognitive process
 - Requires students to access and decide on appropriate use of relevant knowledge
 - Requires analysis and interpretation of the task
 - Requires considerable cognitive effort
 - Solution process is unpredictable or ambiguous
-

³ Adapted from "Mathematical Tasks and Student Cognition: Classroom-Based Factors That Support and Inhibit High-Level Mathematical Thinking and Reasoning," Henningsen, M., & Stein, M.K., 1997, *Journal for Research in Mathematics Education*, 28(5), 524–549.

more recent studies of cognitively-demanding tasks and is presented in Table 3. Their results were internally valid due to stratified random sampling methods, double coding practices, and high intercoder reliability. Their conclusions were also externally valid due to the wide variety of schools, teachers, and students that participated in the study.

Positive Interdependence

Another way that tasks have been found to encourage more equitable participation and collaboration is by requiring positive interdependence among group members (Featherstone, 2011; Johnson & Johnson, 1987; Lotan, 2002), meaning that students are in some way dependent on each other to complete the task. Positive interdependence can take multiple forms as noted by Johnson & Johnson (1987), as mutual goals, division of labor, division of materials or resources, assigned roles, and joint rewards. Whichever the form of interdependence, many researchers have argued that if students understand that they will need each other to complete the task and that no single student can solve the task alone then they will be more likely to engage in a collaborative process (Cohen, 1994a; Featherstone, 2011; Lotan, 2002).

Clear Evaluation Criteria

The final important task design feature in a group-worthy task is that it includes clear criteria for how the group product will be evaluated (Lotan, 2002). A study conducted by (Cohen et al., 2002), found that groups without clear evaluation criteria were much less focused on the task than groups with clear criteria, which were often in the form of a rubric or clear indication of the ways in which individuals and groups would be held accountable for their work. Their research was not specific to mathematics, but their findings that “evaluation criteria were a motivational tool helping groups to be more self-critical and increasing their effort to turn out a superior group product” is certainly applicable to mathematics groupwork (p. 1064).

Connections to Prior Knowledge

In addition to carefully designing or adapting math tasks to meet Lotan's (2002) criteria for group-worthy tasks, there are other important aspects of tasks that teachers can attend to prior to implementation which support more successful and equitable groupwork. Henningsen & Stein (1997) found that a common reason for a lack of student engagement and a decline in cognitive demand during task implementation was the degree to which the task content built upon students' prior knowledge. The authors thus emphasized the importance of making sure that the content of the task is connected to what students already know and can do, while also providing sufficient mathematical challenge.

Structuring Tasks

Many reform-oriented classrooms and curricula utilize a three-phase lesson structure when using cooperative learning in their classrooms (Jackson et al., 2013; Walle et al., 2012), the three phases of which are usually *launch*, *explore*, and *summarize*. The launch phase refers to the ways in which teachers communicate the task or problem and instruct students to begin their work, the explore phase is the period during which groups are engaged in the task, and the summarize phase consists of a short, whole-class discussion highlighting the important ideas that students generated during the explore phase. Stein, Engle, Smith, & Hughes (2008), found that this three-phase format was a highly effective structure for supporting mathematical discussion in the classroom. According to Esmonde's (2009b) study, task structure has implications for equity because the way a task is structured affects students' work practices and positioning, which are closely connected to their opportunities to access the task and engage in mathematical learning.

Launching Tasks

While the three-phase model is generally effective for structuring cooperative tasks, even a perfectly designed task might lose its effectiveness when implemented. Both Jackson et al. (2013) and Stein et al. (2008) have noted that students' opportunities for reasoning, conceptual understanding, and cognitively demanding mathematical work can be lowered during any of the three-phase lesson stages. The launch phase is particularly challenging for teachers to implement in a way that helps students access the content of the task without lowering the level of cognitive demand (Jackson et al., 2013).

In a large-scale qualitative study of four large, urban school districts, Jackson et al. (2013) found that the ways in which teachers launched math tasks had a significant impact on students' ability to access the mathematical learning in the task. In a sample of 165 teachers, these researchers found that in 64% of the lessons that were studied, teachers lowered the cognitive demand of the task during its launch phase. The authors noted that this "is concerning given that the cognitive demand of an activity is a significant predictor of students' opportunities to learn" (p. 678). Henningsen & Stein (1997) found several factors responsible for this decline, including removal of the challenging aspects of the task, inappropriate amounts of time, and the use of tasks that were not appropriate for particular groups of students. These large scale studies clearly showed that teachers play a crucial role in the effectiveness of cooperative tasks (and thus students' access to the mathematics) both through the task design and through the launch of the task.

In their study, Jackson et al. (2013) found that in the most successful cooperative tasks the teacher facilitated discussion of three key aspects of the task during the launch. These key aspects are as follows: (1) key contextual features of the task scenario, (2) key mathematical ideas and relationships as represented in the task

statement, and (3) language describing contextual features, mathematical ideas and relationships, and any other vocabulary central to the task statement that might be confusing or unfamiliar to students. The researchers found that explicit discussion during the launch of these key task features helped to support student participation in complex cooperative math tasks.

Rationale for the Study

In my own experience trying to implement cooperative tasks I have witnessed many issues that I now understand to be equity issues. I have observed students dominating the group process, excluding group members, expressing confusion and quickly becoming disengaged, refusing to work together, and working independently instead of collaboratively. I have also observed students working together in equitable and powerful ways, and engaging in rich mathematical discussion with meaningful learning outcomes. In designing this research study, I wanted to better understand what factors and teaching practices result in the latter. More specifically, I wondered how teachers could better design and facilitate cooperative tasks in their classrooms to ensure that all students are able to access the mathematical learning and meet rigorous academic standards.

Through my literature review, I have identified a significant number of factors that can act as a barrier for students during cooperative learning activities. I also identified a plethora of research-based practices involved in making cooperative tasks more accessible to students, including planning and designing the task, structuring the task, setting up classroom norms, launching the task, monitoring student work on the task, interacting with groups, responding to and supporting students' mathematical thinking, attending to status dynamics, orchestrating whole class discussion, summarizing the task, and assessing student work on the task (Alexander et al., 2009; Cohen & Lotan,

2014; Esmonde, 2009b; Featherstone, 2011; Henningsen & Stein, 1997; Jackson et al., 2013; Stein et al., 2008). I believe it is important for teachers to learn and develop all of these practices in order to successfully and equitably implement groupwork in their classrooms. For the scope of this action research study however, I have narrowed my focus specifically on the design and launch of cooperative math tasks.

Task design and launch refers to the selection of the problem or task, the way it is organized, communicated, and structured, and how it is delivered to students. I chose this focus because this is the aspect of cooperative learning that I feel the most uncertain about in my own teaching practice. I do not yet have good intuition about what kinds of math tasks engage students in collaborative, equitable, and meaningful mathematical learning. My research question is as follows: *During cooperative math tasks, how can the task design and launch help more students gain access to the task?*

CHAPTER 3: METHODS AND ANALYSIS

Action Research

In my literature review, I identified several research-based practices related to the design and launch of cooperative tasks that have been found to increase student access to the task. These practices are summarized in Table 4. Through this action research project I will study my own implementation of these practices. Teacher-research, or action research is different from conventional education research in that it is performed by teachers in their

own classrooms in order to critically study and improve their own practice (Freeman, 1998). Through my own implementation and study of these practices in my own classroom, I hope to gain some insight into how the

Table 4. Action Research Practices

Task Design	Task Launch
Open-Ended	Discussing Context Features
Multidimensional	Discussing Mathematical Ideas & Relationships
High Cognitive Demand	Developing Common Language
Positive Interdependence	Maintaining Cognitive Demand of Task
Clear Evaluation Criteria	
Connection to Prior Knowledge	
Three-Phase Lesson Format	

design and launch of cooperative tasks can support greater access to the task for the students in my classroom. Further, this study will help me to critically examine my own practice and improve my ability to facilitate meaningful and equitable learning experiences for my students.

Participants and Study Setting

I conducted this action research during my first student teaching placement after having completed one academic year of a Master in Teaching program. The classroom I student-taught in during this study was a sixth grade accelerated math and science class

at Jackson Middle School⁴ in a small city in Washington State. Jackson is a three-grade middle school serving about 350 students from sixth to eighth grade. The demographics of Jackson Middle School are as follows: 2.5% African American, 0.5% Native American/Alaskan Native, 13.7% Asian/Pacific Islander, 65.8% Caucasian, 9.8% Hispanic, and 7.7% two or more races, with 42.4% of students qualifying for free or reduced-price meals and 19.7% of students qualifying for special education. 73% of the teachers at Jackson hold a master's degree, and the average experience of their teachers is 12.7 years.⁵ Jackson transitioned to the Common Core State Standards in the previous school year, as did much of Washington State. The year of my study was the first school year that the associated Smarter Balanced standardized assessments were implemented, although Jackson Middle School voluntarily piloted these assessments the year before.

I began student teaching at Jackson in the fall of 2014. At this time I had studied a substantial amount of mathematics pedagogy, education research, and progressive theories of learning though I had very little teaching experience. My academic background greatly impacted the lens through which I viewed interactions and behaviors in the classroom, as did my sociocultural lens. I am a white female, age 25, with a middle class, rural upbringing. Much of my schooling has been in progressive and alternative educational settings, which influences my beliefs about teaching and learning.

The accelerated program at Jackson is a rigorous and integrated math and science program that emphasizes imaginative and critical thinking. The program consists of two cohorts of sixth grade students who attend a three-period block of math and science every day. Students and families go through an application process for the

⁴ All of the names in this paper are pseudonyms to protect the identity of the school and participants

⁵ Data comes from the district's 2012 – 2013 School Performance Report

program and are selected through a lottery. Students' test scores on the Measurement of Student Progress (MSP) and Measures of Academic Progress (MAP) exams are also used to determine students' eligibility for the program.

In my student teaching experience, I worked closely and collaboratively with the classroom teacher Ms. Davis through a co-teaching model. In this model, Ms. Davis and I worked collaboratively and shared teaching responsibilities evenly, and did not emphasize our mentor/mentee relationship with the students. Ms. Davis had taught the accelerated math and science program at Jackson for three years prior to accepting me as a student teacher. She had therefore already established the overall curriculum for the program. While the larger picture of the program was pre-determined, Ms. Davis allowed me substantial professional freedom and responsibility in terms of planning, designing learning activities, teaching, and assessing.

It is important for the reader to note our approach to teaching mathematics and our shared pedagogical beliefs. Ms. Davis and I have highly constructivist views toward math teaching and learning, and believe that the best way for students to learn mathematics is through opportunities for problem solving and mathematical discussion. We emphasized reasoning, problem solving, and mathematical argumentation much more heavily than computation or procedural knowledge. We almost never told students whether their answers were correct, and supported them instead to make sense of their solutions. We rarely used direct instruction in our classroom, aside from vocabulary or teaching occasional mathematical procedures. Our approach to structuring a unit was to give students a problem too challenging on first day to motivate the unit's learning. Most of the unit was then spent on rich math tasks and problem solving. As procedures and problem solving strategies arose naturally in the students' work, we named and highlighted them in class discussion and supported students in practicing them. Toward

the end of the unit, students spent several days doing metacognition and practice on the unit's learning targets before their unit test.

During my time at Jackson, the sixth grade science topics were biology and diversity of life, and the math topics were ratios, fractions, and division. Ms. Davis and I gave an average of one group task per week. Group tasks consisted of a mathematically complex problem with multiple solution strategies, and students worked mostly in groups of four. Sometimes, the group tasks took two or three days for students to complete. I taught almost all of the lessons in this study involving cooperative tasks except for one that Ms. Davis taught. In addition to group tasks, students also did one or two partner tasks per week. Both group tasks and partner tasks usually included some personal thinking time and individual work. The rest of students' mathematical learning took place during a problem solving routine during which students could make choices about where in the room to work, whether to work alone or with others, and which problems to work on.

Data Collection and Analysis

The data collection period for this study took place over my ten weeks of student teaching. During this time we completed two five-week units, one about ratio and one about fractions and division. While I collected data from both classes, I decided to only analyze one of them. I chose this class for analysis because it was the larger of my two classes, more diverse in students' academic readiness, and more balanced in gender.

For each task, I collected two types of data. The first type of data captured information about the tasks themselves, including the task design features and how I launched the tasks. The second type of data related to how well students were able to access the task. To analyze this, I looked at two equity-related aspects of the lesson. First, I looked at how well students were able to begin problem solving after I launched

the task. Second, I looked at the nature of students' interactions during the task, and what might be motivating their interactions. The specific sources of data and methods for analysis are described in the following sections.

Lesson Plans and Task Materials

I collected data from 10 cooperative tasks. Four were about ratio, four were about fractions and division, and two involved concepts from both units. Each of these tasks was designed, structured, and set up slightly differently in order to explore how task design might support equitable mathematical learning. Although the structures of the tasks varied, all of them followed a general three-phase lesson format with a launch, explore phase, and summarizing discussion.

For each cooperative task I gave to students, I collected all versions of my lesson plans and task materials. These data were essential for analyzing and comparing the task features, and the ways that I structured the tasks. I used Lotan's (2002) framework for groupworthy tasks (See Table 2, Chapter 2) to analyze and compare the features of my tasks. As I engaged in the analysis process, some tasks emerged as more interesting than others in relation to my research question. In particular, I chose the following tasks for a more in-depth analysis: *Sub Sandwiches*, *Playgrounds*, *Painting Rooms*, *Pizza Night*, and *Perfect Purple Paint*. All of these tasks are provided in Appendix A. I taught the first four of these tasks, and Ms. Davis taught *Perfect Purple Paint*. I found that analyzing several of my own lessons alongside one from a much more skilled and experienced teacher provided me useful insight that I may not have gained had I only analyzed my own teaching.

Post-Lesson Reflections

Sometimes, my lesson plans and task materials did not accurately reflect my actual lessons due to the responsive nature of teaching. To keep a more accurate record

of my lessons, I kept a reflection journal immediately after teaching them. In this journal I took notes about the structure and progression of the lesson, my impressions and observations of each lesson, as well as students' participation, interaction, and work practices during the explore phase. This helped me to notice equity-related aspects of my lessons and made it easier to identify emerging patterns and changes in my classroom over time. This data has limitations in that it is subjective—it is my own unique perspective and experience as the classroom teacher—yet it is also an important and valuable perspective. It is valuable because it is informed by knowledge of the students and the classroom, and an understanding of the intentions behind the task design. Throughout the data analysis process I used these reflections to check that my findings were consistent with what I observed during the lesson, and to look for disconfirming evidence.

Student Work

When working on cooperative math tasks, students in our classroom were usually expected to record their individual thinking and problem solving process on their own personal recording sheet. Sometimes, we asked students to make an individual final draft of the solution after the groupwork. I collected and scanned these documents after most of the cooperative tasks I gave. This data provided useful information for analyzing students' developing mathematical understandings and the degree to which they were able to enter into the problem or task. For some of the cooperative tasks I gave, I asked students to work together to create a group poster of their solution. The expectations and the nature of these posters varied as I experimented with ways of designing and structuring group tasks. For each of these group tasks, I photographed or scanned these posters to analyze how equitably groups were working, and to examine

how the task design and structure was related to students' participation, interaction, and work practice.

I used qualitative coding practices to analyze student work. I did not enter into my analysis with pre-determined expectations or codes, but instead allowed patterns in the data to emerge naturally. As a teacher-researcher doing qualitative research about my own classroom, I was the primary instrument of analysis (Anderson et al., 2007) and simply looked for patterns in my data and tracked the development of my observations in a research journal.

Video Recordings

To look at and analyze students' work practices, I captured video recordings of the lessons I taught which involved cooperative tasks. These recordings provided rich and unbiased documentation of my task launches, and allowed me to analyze how the launch was connected to students' access to the ideas in the task. In viewing my video recordings I followed an observation protocol (Anderson et al., 2007, p. 186) and systematically created video logs and selected transcriptions. I coded these transcriptions according to the framework for task launches used in the 2013 study by Jackson et al. (see Table 4, Chapter 3). Video data has the important benefit of being reviewable any number of times with multiple questions in mind. The limitations of video recorded data are that it provides only an outside perspective of the classroom events, and does not capture the experiences of participants or the intentions behind classroom actions (Anderson et al., 2007).

Student Surveys and Reflections

Because of this limitation, I also collected data about students' experiences of the lessons through surveys and brief written responses. Students' perspectives served as member checks and contributed to the internal validity of the study (Mertens, 2009).

Unfortunately, I did not collect student reflections as frequently as I had hoped, because we often ran out of time at the end of class periods. Still, the surveys provided some valuable information about students' dispositions and preferences related to cooperative learning.

CHAPTER 4: FINDINGS AND IMPLICATIONS

This action-research study explored the relationship between cooperative task design and students' access to the learning of the task. The five tasks I selected for in-depth analysis were *Sub Sandwiches*, *Playgrounds*, *Painting Rooms*, *Pizza Night*, and *Perfect Purple Paint*. From the multiple data sources I collected about these tasks, I identified three themes around which my findings are organized. First, I identified an interesting relationship between a task's evaluation criteria and the quality of students' interactions. More specifically, I found that using evaluation criteria that required individual students to record more than one strategy was linked to higher quality student interaction and learning outcomes during cooperative tasks. My second finding was about the effectiveness of my task launches, and their relationship to students' learning outcomes. I found that before presenting the task, explicitly discussing the concepts and strategies necessary for the task supported more equitable learning outcomes. Both of these findings have important implications for my future teaching practice, which will be discussed in the following sections. My third finding was unexpected, and raised more questions than it provided insights. I noticed that personal think time seemed to play an important role in how well students were able to access the groupwork. In the following section, I also will pose some questions about these observations for further investigation.

Evaluation Criteria and Quality of Student Interaction

In looking at the student work for each task, one of the first things I noticed was that some of the final products exhibited more collaborative interaction around the mathematical ideas than others. In particular, I noticed that the products from the *Sub Sandwiches* and *Playgrounds* tasks exhibited relatively minimal collaboration and interaction, whereas the products from the *Painting Rooms* task and *Perfect Purple Paint*

task indicated much more collaborative and high quality work practices (See Table 1, Chapter 2). In Sub Sandwiches and Playgrounds, nearly all of the posters consisted of students simply transferring their individual solutions from their personal recording sheets onto one fourth of the group poster. For a representative example of this observation, see Figure 1 and Figure 2.

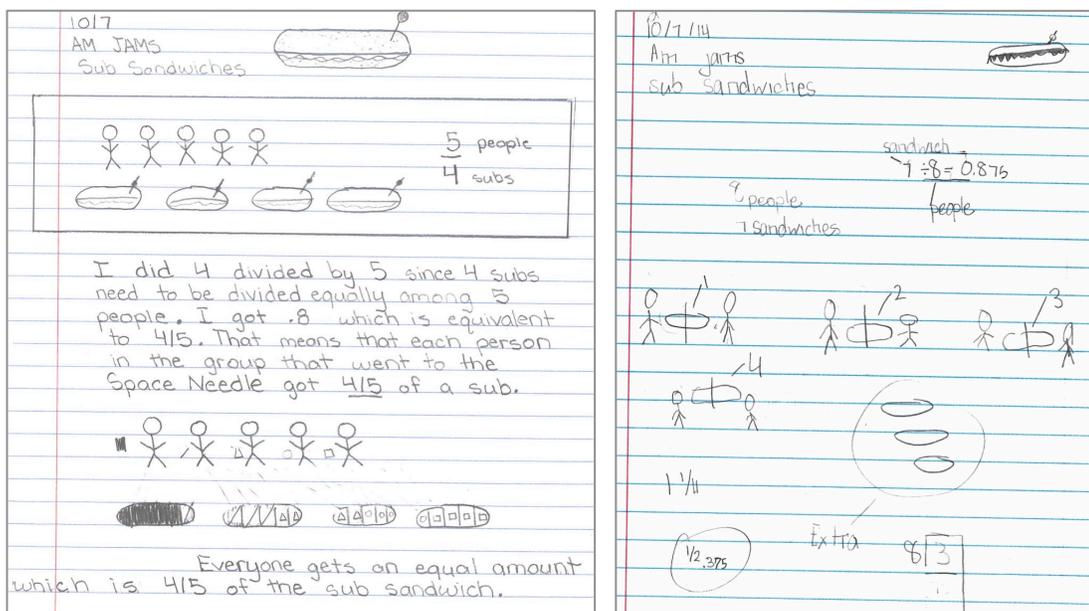


Figure 1. Personal recording sheets prior to groupwork from the Sub Sandwiches task.

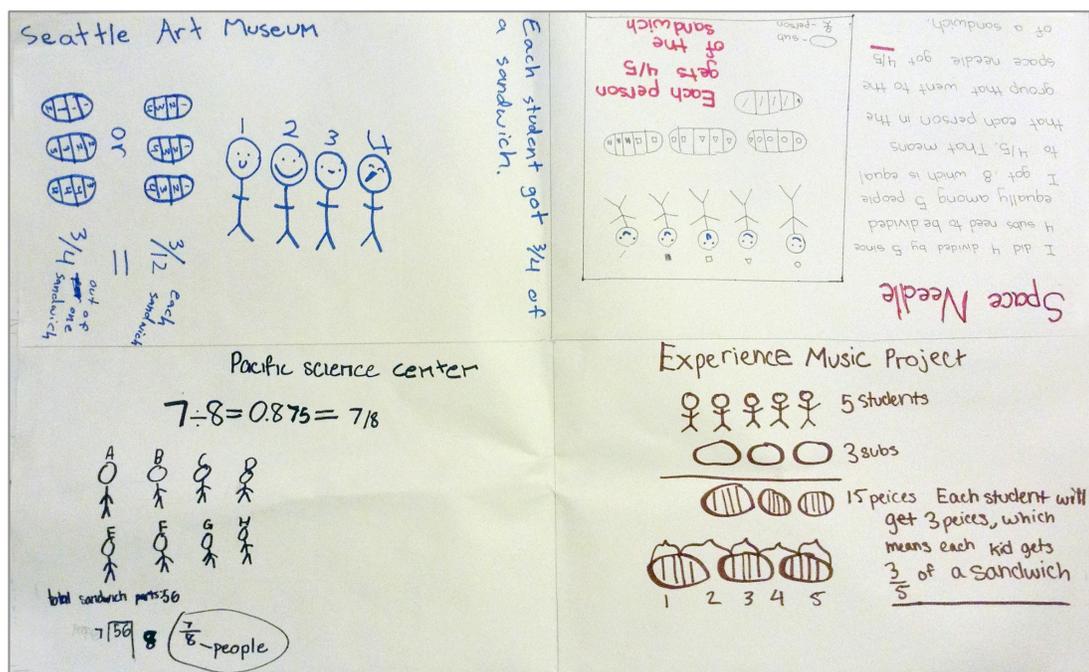


Figure 2. Group poster from the Sub Sandwiches task. The work shown in the upper right and lower left corners of this poster are from the same students as in Figure 1.

In these examples, it is evident from comparing students' personal recording sheets and their group poster that their mathematical ideas did not change or deepen as a result of creating the poster with their group. This pattern was highly prevalent across

all of the student work from Sub Sandwiches as well as the Playgrounds task which had an identical task structure to Sub Sandwiches.

The products from Sub Sandwiches and Playgrounds tasks were noticeably different in

nature than the products from the Painting Rooms and Perfect Purple Paint tasks. I noticed that in these later products there was clear evidence of students having collaborated to understand each others' mathematical thinking strategies. In Painting Rooms, students created small posters of their solutions in groups of two or three, and in Perfect Purple Paint, each student created an individual final draft of their solution. In both products, students

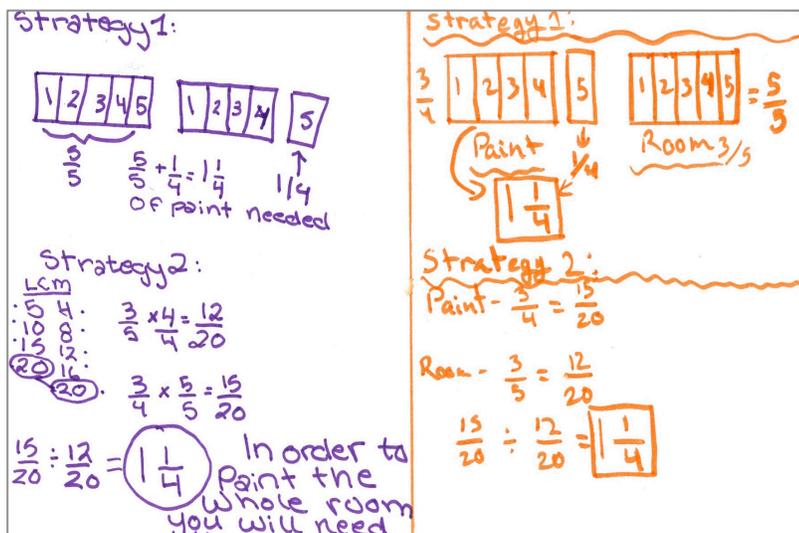


Figure 3. Partner Poster from the Painting Rooms Task.

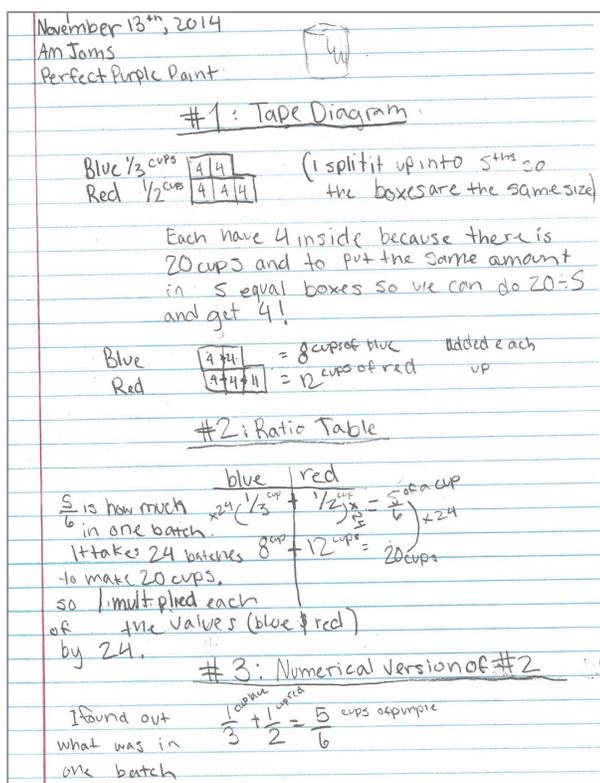


Figure 4. Example of an Individual Final Drafts from the Perfect Purple Paint Task.

frequently made connections between each others' ideas and most of them included multiple strategies on their final drafts. Evidence for this observation is provided in Figure 3 and Figure 4.

This observation led me to the question: What was it about the tasks themselves that had led to these different types of products and interactions? Sub Sandwiches and Playgrounds were very similarly structured. For those tasks, students were instructed to work on a part the problem for a short time on their own personal recording sheet. They then came together in a group of four to answer a group question and create a poster of their group's solution. These tasks culminated in a gallery walk during which students recorded the strategies they saw on each other's posters. The Painting Rooms and Perfect Purple Paint tasks were structured quite differently. These tasks also began with personal think time, but instead of thinking on paper, students began by problem-solving on the dry-erasable floor with whiteboard markers in groups of three (See Figure 5). Students were given no directions regarding how and when to collaborate. They were simply instructed, by the end of the time given, that each student was to individually write up a final draft of their solution on a piece of paper to turn in.

In addition to being structured differently, these two sets of tasks were also quite different in their evaluation criteria. Sub Sandwiches and Playgrounds had two evaluation criteria which were: (1) everybody's

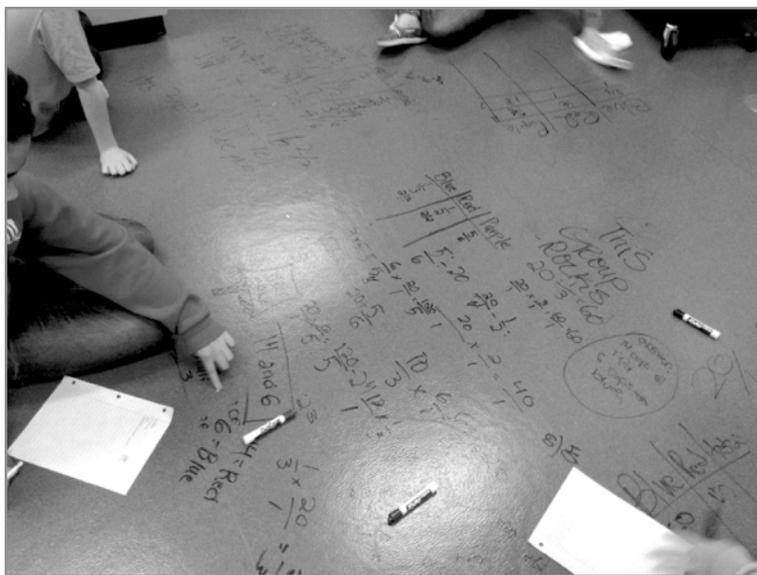


Figure 5. Students problem solving on the floor during the Perfect Purple Paint task.

thinking is represented with different colored markers, and (2) the group has shown why the answer makes sense using labeled pictures or diagrams. In Painting Rooms and Perfect Purple Paint, students were given our Problem Solving Rubric before engaging in the task. They were told to focus on the rubric categories of Process & Strategy and Communication, and challenged to “knock our socks off” by exceeding our highest rubric expectations that “at least 2 complete strategies are used that support the answer and each other” and “the work is clearly and thoroughly explained and presented logically.” Our full problem solving rubric can be found in Appendix B.

These observations led to my first action research finding: *Using evaluation criteria that required individual students to record more than one strategy was linked to higher quality student interaction and learning outcomes during cooperative tasks.* In revisiting Lotan’s (2002) groupworthy task criteria, I realized that what Sub Sandwiches and Playgrounds were lacking was an element of positive interdependence. During the groupwork phase of these tasks, students did not intellectually need each other to complete the group product. In fact, for Sub Sandwiches, I found from students’ personal recording sheets that 100% of them had completely solved their assigned portion of the problem by themselves during personal think time. This meant that the groups had very little to do or think about together beyond transferring their individual solutions onto a group poster. The task did not push or support them to deepen their understanding or discuss anything as a group.

In Painting Rooms and Perfect Purple Paint, the evaluation criteria from the rubric seemed to have added an element of positive interdependence. Specifically, I think that the requirement that students record at least two strategies on their individual final drafts was what motivated students’ high quality interactions. This outcome can be seen as high-equity, because so many students not only solved the problem in their own

way, but also had access to each others' ideas and strategies which then deepened their mathematical understanding.

After identifying this apparent relationship between evaluation criteria and student interaction, I looked across my other data sources to look for supporting or disconfirming evidence. I noticed that what my students were saying about their experiences with groupwork seemed to support my observation that the group products during Sub Sandwiches and Playgrounds did not enhance or deepen student learning. In a survey I gave students at the end of my student teaching, I asked students to rank our various classroom routines structures based on how well they helped students learn math. Only two students ranked "groups of four tasks" as the most helpful classroom routine for their math learning, and over half the students ranked groupwork among the least helpful routines for their math learning. Although I do not know exactly why students felt this way, my intuition tells me that during the tasks students were responding to in the survey, they were not supported in learning from each other. I imagine that students came away from these group tasks with the sense that they could have completed the task on their own.

Implications for Practice

This finding has important implications for my teaching practice and my future implementation of cooperative tasks. I am coming to better understand the importance of positive interdependence in helping students intellectually value each other in the classroom. Before embarking on this study, the idea of positive interdependence made sense to me as a theoretical concept but I did not realize what this looked like in practice until after I had analyzed my data. In fact, even immediately after giving tasks like Sub Sandwiches and Playgrounds I felt that I had done an excellent job supporting students in working together. I thought that these tasks did require positive interdependence

because I had assigned parts of the task to different group members, assuming that this would give each of them something to contribute to the groupwork which would help them gain access to the group process. In looking at my data, what I realized was that students did not need each other. They may have even resented having to work together on something that did not have apparent value to them. I worry that by not attending closely enough to positive interdependence, students may have learned that collaborative learning was a waste of time and not valuable for their learning. This is the opposite of what I had intended and a potentially damaging message for an equity-supportive classroom culture.

If I were to give the Sub Sandwiches or Playgrounds task again, I would add a greater level of challenge and complexity to the group product to create more positive interdependence. I believe that since students were able to solve the problem on their own, they needed something new and more challenging to do together in their groups of four in order to benefit from the group process. For example, I could have added a set of extension and connection questions for students to come to consensus about after sharing their individual solutions. I could have put a protocol in place for students to share their thinking and understand each other's ideas. I needed something for students to *do* or *create* together that was intellectually challenging and important. I also needed to add an element of individual accountability by having each student produce a reflection or individual-write up of the groupwork, to create additional motivation for students to engage in the group process. These recommendations are consistent with Cohen and Lotan's (2014) description of effective practice in *Designing Groupwork* and with Lotan's (2002) criteria for groupworthy tasks. Alternatively, as recommended by Featherstone et al. (2011), students could have been given less personal think time, just enough time to think of a strategy, so that they didn't have enough time to solve the problem on their own and needed to rely more on their group members.

Task Launches and Learning Outcomes

Another equity-related issue I wanted to explore was the effectiveness of the task launches at helping student access the ideas in the tasks. To analyze this, I transcribed the video recordings of each task launch and coded them using Jackson et al.'s (2013) framework (see Table 4, Chapter 3). I then coded students' personal recording sheets on each task, looking at how successful students were at problem solving during their personal think time. These were rich sources of data for analyzing student entry into the task, because immediately after the task launch students spent several minutes thinking about the problem on their own on their personal recording sheets. I also checked these results against my post-lesson reflections for each task, where I had noted my initial impressions about how well the class was able to start work on the problem.

In this analysis, a few of the tasks stood out to me. In the task Pizza Night, many students seemed to have trouble understanding the ideas in the task. Nine out of the 29 students gave an incomplete or incorrect solution on their personal recording sheets. I wondered what the issue had been for these students, and how it may have been connected to the task design or launch. The problem simply showed two different sized tables with different numbers of seats and pizzas and students were asked to find how much pizza a person at each table would get, and which table they would rather sit at. Upon closer examination of their work, I found that their misconceptions were consistently about naming the quantities in the problem as

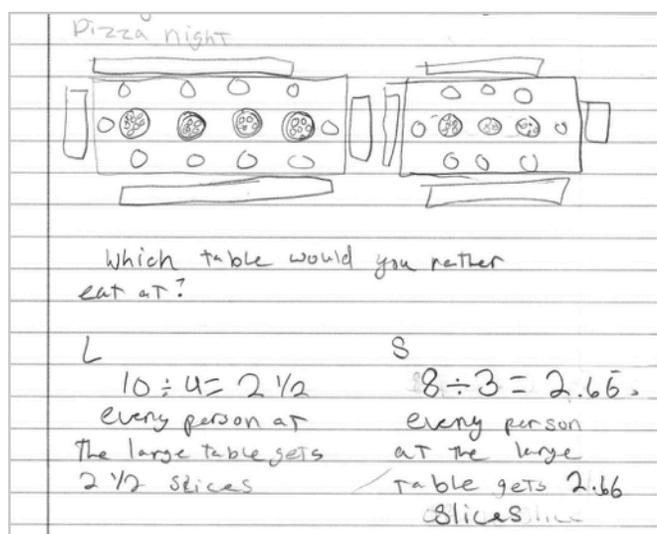


Figure 6. Example of student misconceptions about the quantities in the Pizza Night task.

fractions of a pizza rather than in the number of slices. The issue seemed to be about a lack of attention to fraction concepts in the task. An example of this is shown in Figure 6, where the student incorrectly divided the number of people by the number of pizzas, and came up with an answer in slices.

In contrast to Pizza Night, the two tasks I coded with the highest entry (meaning that students had strategies to begin thinking about the problem right away) were Perfect Purple Paint and Sub Sandwiches, both of which I discussed in the previous section. On Sub Sandwiches, every single student completed the problem on their personal recording sheets during personal think time, and 27 out of 29 students showed evidence of correct understanding of the mathematical ideas. On Perfect Purple Paint, 27 out of 29 students had correct solutions and most of them used multiple strategies to show their answers made sense. The different outcomes on these three tasks made me wonder: What about the task launch could have accounted for these different degrees of student access?

To investigate this question, I looked to the transcriptions of the task launches. To remind the reader, Pizza Night and Sub Sandwiches were lessons that I taught, and my mentor teacher Ms. Davis taught Perfect Purple Paint. In revisiting Jackson et al.'s (2013) task launch framework, I noticed that I spent most of my launches attending to the discussion of the context features and language in the task. Sometimes I did this through storytelling, as a way of engaging students in the problem context, as in this excerpt from my Sub Sandwiches task launch:

Ms. Gates: I worked with 8th graders last year in Seattle because I used to live in Seattle. And we took a field trip. We went to the EMP which is that really big weird shiny building, the Pacific Science Center, Space Needle, and Seattle Art Museum. It was a really fun field trip. We ordered sub sandwiches for the kids, because they can feed a lot of people. Have you

seen them? They are huge, sub sandwiches are like this big [shows with hands]. So we gave this group three, [draws different numbers of sub sandwiches next to each group], we gave this group four... So that's how we divvied them up on these different field trips. And do you know what they said when they came back? When they came back I thought they'd be talking about how much fun they had on the field trip but instead they said, "Ms Gates it wasn't fair!" So the question, to return to the math here is: were the 8th graders right? What do you think? Was it fair?

In Pizza Night, I attended to the context features by involving students in a discussion of what they noticed in the picture and asking them to put themselves in the problem situation.

Ms. Gates: How many of you like pizza?

[Students raise hands]

Ms. Gates: How many of you have been to summer camp?

[Students raise hands]

Ms. Gates: Some of you. At this summer camp in this problem, there are two different size tables. This is a large table, and this is a small table. What do you notice? Rachel⁶.

Rachel: That one table has four and the other has three.

Ms. Gates: Michael what do you notice?

Michael: The people at the large table get more pizza.

Ms. Gates: That's exactly what this problem is going to be about. Also, I want you to assume that everybody at the table is going to share the pizzas equally. Sara what do you notice?

Sara: The big table has 10 plates and the small table has eight.

⁶ All names are pseudonyms

Ms. Gates: The question I want you to think about is which table would you want to sit at? I know what table Michael wants to sit at.

I also noticed that in these task launches I never explicitly discussed the key mathematical ideas or relationships in the task, which is one of Jackson et al.'s (2013) criteria for a high quality launch. In reflecting on this, I realized this was because I did not know how to do this without giving away how to solve the problem, or steering students in a particular direction and lowering the cognitive demand of the task. However, I felt that this is precisely what the stumbling block had been for students on the Pizza Night task—they had not understood the key mathematical ideas and relationships in the task. I was left with the question: what did it look and sound like to discuss these aspects of the tasks with students to provide them greater access without giving away how to solve the problem?

The transcript of Ms. Davis' Perfect Purple Paint task launch shed some light on this question. She did something quite different than what I did. I noticed that she spent most of her launch discussing the mathematical ideas and relationships required for the task, and didn't spend any time at all on the context features. Furthermore, she facilitated this discussion before students knew what the problem was, and she did it without hinting at a problem solving direction or lowering the cognitive demand. The following is an excerpt from Ms. Davis' task launch:

Ms. Davis: What strategies do you remember for solving problems about ratios?

Claire what do you remember?

Claire: Ratio tape diagrams... [inaudible explaining]

Ms. Davis: Hold on slow down, you're going really fast. I'm just going to draw a little tape diagram. What ratio does this represent, everybody?

Whole Class: 2 to 3!

Ms. Davis: What's the whole? Show me with your fingers. [Students hold up fingers]. Some of you are saying 5 and some of you are saying 3, can someone make a case for 5 or 3. Emily.

Emily: It's a part-to-part ratio.

Ms. Davis: If it's 2 to 3, and there's 2 and there's three, these are both parts. So what's the whole, if these are both parts? Show with your fingers.

[Students hold up five fingers]. Yeah. It's a convention of a tape diagram to have the boxes represent the whole. Claire, did you also say tables? I'm going to draw a table and see if other people have some ideas.

[Draws table]. Lets put 2 and 3 in the table. We don't know what these things are, but we know the ratio is 2 to 3. Can somebody name another ratio that could fit in that ratio table? Go ahead.

Student: 4:6

Ms. Davis: Can someone name one more?

Student: 1:1/2

Ms. Davis: Ooh, you're using your fraction number sense already. Were there other strategies you used? What else do you remember Cory?

Cory: You could also convert them into fractions.

Ms. Davis: Ohhh, okay. Can you name a fraction that would go with this tape diagram?

Cory: 2/3, or 2/5.

Ms. Davis: I would say you could argue for 2/3 if it were in a sentence that said, lets say for example these are white eggs and brown eggs. You could say there are 2/3 as many white eggs as brown eggs. Or you could say that 2/5 of the eggs are brown. Okay. You may use any of these strategies and others that you learned when you are exploring and solving this

problem about perfect purple paint. You'll be doing thinking on the floor and a final draft on paper.

This example helped me begin to understand what it can look like and sound like to attend to the mathematical ideas and relationships in the task during the launch. Still, I was left with some confusion as to why Sub Sandwiches had been so successful, despite my not having attended to these features during the task launch. I came to the conclusion that Sub Sandwiches had been the first task of a new unit, and therefore was far more accessible and did not rely on prerequisite knowledge or prior concepts from the unit. In contrast, both Pizza Night and Perfect Purple Paint were tasks at the end of their respective units, and thus drew from students' understanding of prior concepts and strategies in the unit. This leads me to my second action-research finding: *Before presenting the task, explicitly discussing the concepts and strategies involved in the task supported more equitable learning outcomes without lowering the cognitive demand.*

Implications for Practice

This finding has useful implications for my future practice. I learned that I will need to attend more closely to the discussion of concepts and strategies in the task in my task launches, especially as the unit is coming to a close. My findings also suggest that these discussions are less important at the beginning of a unit, or if a task is highly accessible and does not require pre-requisite knowledge. I am reminded that I need to think more carefully in my planning process about what the necessary mathematical concepts and strategies are in a task, and use these to inform my task launch. These recommendations are consistent with Jackson et al.'s (2013) findings about the aspects of high quality task launches.

Unexpected Findings

During this study, I was struck by how important personal think time seemed to be for supporting equitable interactions. When I have seen cooperative tasks implemented in other teachers' classrooms, it was often the case that a group was given a task and expected to start together on it right away. I noticed that status hierarchies were prevalent in these teachers' classrooms, and that often the idea taken up by the group belonged to the student with the first (but not necessarily the best) idea. I also frequently saw some students taking charge of the group product while other group members were excluded.

Ms. Davis and I did not conduct our groupwork this way. After launching the problem, we usually gave students 5 to 10 minutes of personal think time to solve it on their own, or at least come up with some ideas and strategies. This seemed to be important for giving quieter students access to the group discussion and diminishing status problems. By allowing personal think time, I think that groups generated more ideas and strategies, because each member had a personal contribution and differing ways of thinking about the problem that could be shared and discussed. Groups were less likely to uncritically take up an idea from a student with high-status, because everyone had an opportunity to generate an idea before the group got started. The class affirmed this observation in their learning survey, ranking personal think time among the most useful classroom structures for their learning.

To reiterate a point from the discussion of my first finding, personal think time also has the potential to remove the element of positive interdependence from a cooperative task. I learned that if students can solve the problem on their own during personal think time, the problem may not be challenging enough, or there needs to be an additional task for groups to do that builds on or deepens the problem. In one or two cases, I felt that I had given too much personal think time, because some students who

did not have an idea became increasingly anxious as the personal think time went on, causing them to become upset and give up.

Areas for Further Research

I am left wondering about the role that personal think time can play in cooperative learning. Finding an effective balance between personal think time and groupwork could be an area for further study. The study helped me better understand what kinds of tasks and evaluation criteria can motivate high quality interaction, but I would like to explore this further. What are some other ways to encourage positive interdependence and high quality interaction besides evaluation criteria? I would also be very interested to learn whether these findings would extend to groups of students with different dispositions and demographics than the students I taught.

Limits of the Conclusions

As a teacher-researcher in my own classroom, I was both an actively involved participant in the study as well as the primary instrument of analysis and data-gathering tool (Anderson et al., 2007). As such, it is impossible to completely eliminate my own biases in the action-research process. One way I tried to minimize my own biases and assumptions was by analyzing data across multiple sources of data from multiple perspectives. Throughout this process, I discussed my ideas and findings with several other colleagues and researchers to gain alternative viewpoints on my work. I worked with insiders who were well versed in my field of study as well as outsiders with little knowledge of my work. This process lends validity to my analysis and findings.

The conclusions of this study have limits for transferability and generalizability as the study only looked at one group of 32 students. The particular population of students in the study also poses a threat to the transferability of its conclusions. The students were a high-achieving group in an accelerated and tracked program, and my data would

look quite different with different populations of students. However, issues of status play out no matter how homogenous a group may seem, and the problems with cooperative learning studied in this context would surely exist in any classroom, with any group of students. Furthermore, the features of groupworthy tasks and the methods of launching tasks that I investigated came from studies that involved a wide range of student populations, which lends some transferability and generalizability to my study's conclusions. I also tried to strengthen these quality indicators by providing a rich description of the study's contextual setting which allows the reader to decide if my methodology and conclusions could be transferred to other settings.

Another limitation of the study was the short data collection period of ten weeks. To strengthen the study's confirmability, I tried to provide the reader an audit trail, or clear path through my analysis process to my conclusions. I justified my conclusions with evidence and provided representative samples of my data. I used student surveys as member checks to help verify the patterns I was finding in the data, which also aided in the study's credibility and confirmability. Additionally, my conclusions were consistent with the body of research on cooperative learning reviewed in this paper, which aids in the dependability of my findings.

Additional Factors

There were several aspects of the classroom that likely played a role in the results of the study. The students' dispositions related to mathematics were productive and generally positive, as would be expected from a tracked, high-achieving group. In a disposition survey I gave students toward the end of my student teaching, nearly all of them indicated that they saw mathematics as useful outside of school, and important to their future careers. Needless to say, I might have had different outcomes had I

conducted this study with a group of students with negative orientations and identities around mathematics.

The sociomathematical norms in the classroom were also highly supportive of collaboration, reasoning, and mathematical discourse, which may have influenced the study's outcomes. Through our co-developed assessment tools and feedback structures, students were continually sent the message from myself and Ms. Davis that competency in mathematics meant explaining and justifying their reasoning, and making sense of problems.

Benefits and Challenges

As a new teacher and a new researcher, understanding the action-research process was difficult at times. I struggled to understand the nature and validity of qualitative data analysis methodology, but came to better understand these paradigms through my study of epistemology and different research paradigms, and through discussing these issues with my professors and peers. With the stressful demands of my first quarter of student teaching which included completing the state's Teacher Performance Assessment (edTPA), I was unable to devote the necessary time to my data analysis process while the data collection was still underway. Had I been able to do this, I might have made different choices about what kinds of data to collect and been able to explore my research question more deeply.

With the high demands of teaching, especially those in one's first quarter of student teaching, it is difficult to find the time to critically reflect on and analyze one's practice. This study provided an invaluable opportunity to examine aspects of my teaching that felt particularly challenging, and to better understand how to implement complex, research-based practices in my classroom. Without the action-research process, I may not have arrived at the same important insights until many years into my

teaching career. I also have become more comfortable with the process of locating research that relates to a troubling issue in my teaching, attempting research-based practices that address the problem, and reflecting on my implementation to continually improve my teaching.

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APPENDIX A: COOPERATIVE MATH TASKS

SUB SANDWICHES TASK:

Original Source: Young Mathematicians at Work

I used to live in Seattle, and I worked in an 8th grade classroom there. We took an 8th grade field trip where we took students to four different places around Seattle. I really liked living in Seattle because there are a lot of fun places to go in the city. On the field trip, the 8th graders were split up like this:

- Four students went to the Seattle Art Museum
- Five students went to the Space Needle
- Eight students went to the Pacific Science Center
- Five students went to the Experience Music Project (Music Museum)

We needed to provide lunch for the students on the field trip, so we bought 17 sub sandwiches for the kids for lunch (the adults brought their own lunch). We adults who planned the field trip thought that this would be plenty of sandwiches, and here's how we divided up the sandwiches between the different groups:

- The group going to Seattle Art Museum got 3 sandwiches
- The group going to the Space needle got 4 sandwiches
- The group going the Pacific Science Center got 7 sandwiches
- The group going to the Experience Music Project got the last 3 sandwiches.

Was it fair? How much did each student get?

PLAYGROUNDS TASK:

Original Source: Contexts for Learning Mathematics

The city of Olympia is thinking about building two new parks, one in Northeast Olympia and one in West Olympia. These neighborhoods have to write a proposal to the city to ask for money to build them. These two neighborhoods are competing for the funding because the city only has enough money in the budget to build one park. Both neighborhoods have an empty lot in mind to build the parks on, and both lots are about 50 yards by 100 yards, which is a typical size for an empty lot.

Here's what was decided in Northeast Olympia:

- $\frac{3}{4}$ of the lot will be a playground. $\frac{2}{5}$ of that playground will be paved with blacktop for playing ball.

Here's what was decided in West Olympia:

- $\frac{2}{5}$ of the lot will be a playground, and $\frac{3}{4}$ of that playground will be blacktopped.

Does one lot have more blacktop than the other, or do they have the same amount?

PAINTING ROOMS TASK:

Original Source: Young Mathematicians at Work

$\frac{3}{4}$ of a can of paint will cover $\frac{3}{5}$ of a room.
How much paint will you need to paint the whole room?

PERFECT PURPLE PAINT TASK:

Original Source: Progressions for the Common Core State Standards in Mathematics

You are working in a paint store. Your store is known for its perfect purple paint. The recipe for perfect purple paint is mixing $\frac{1}{2}$ cup blue paint with $\frac{1}{3}$ cup red paint. A customer comes in asking for 20 cups of perfect purple paint. What will you do?

APPENDIX B: PROBLEM SOLVING RUBRIC

	Concepts <i>What?</i> <i>Translating the concepts in the task into math ideas</i>	Process & Strategy <i>How?</i> <i>Choosing strategies that can work and then carrying out the chosen strategies</i>	Communication <i>Show Us!</i> <i>Using pictures, symbols, and/or words to show the path to the solution</i>	Justification <i>Defend!</i> <i>Taking a second look at the strategies and calculations and defending the solution</i>	Accuracy
5	KNOCKS	YOUR	SOCKS	OFF	!!!!
4.5	The task is translated into thoroughly developed math ideas that work and enhance each other.	At least 2 complete strategies are used that support the answer and each other.	The work is clearly and thoroughly explained and presented logically.	The review is complete and defends the solution with a generalized mathematical argument.	The answer is correct and the work supports it.
4	The task is translated into thoroughly developed math ideas that work.	One complete strategy was used that supports the answer.	The work is shown and the path to the solution is clear.	The review is complete and supports the solution.	The answer is partially correct and the work mostly supports it.
3	The task is translated into partially developed math ideas that may not work.	A strategy is partially complete or may not work	Some work is shown but gaps in the solution path have to be inferred.	The review is partially complete or does not support the solution.	The answer is partially correct but the work doesn't support it.
2	Math ideas don't make sense in the context of the problem or are missing.	A strategy didn't work, wasn't completed, or didn't support the answer.	The work is not shown or the path to the solution is unclear.	The review doesn't make sense or is missing.	The answer is not correct and the work does not support it.