The Effect Constructivist and Traditional Teaching Methods on Students' Mathematical Achievement

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ABSTRACT

In recent years, there has been a debate in the public schools over the different approaches to how mathematics is taught in public schools. In the past, the schools believed that teaching mathematics the traditional way with direct instruction and route memorization of facts and procedures. In recent years, there has been a shift in the educational system in moving towards a more constructivist approach to the teaching of mathematics. School are beginning to adopt new mathematics curriculum with includes the use of manipulatives, small group work and developing cognitive thinking skills.

This paper seeks to investigate what effect different mathematical teachings had on student achievement. The research explores both traditional and constructivist teaching to include students with learning disabilities and mathematic assessments' around the world. In addition, it will investigate constructivist mathematics curriculum and the affect on student mathematical achievement.

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CHAPTER ONE: INTRODUCTION

Introduction

Mathematics has the ability to confuse, frighten, and frustrate learners of all ages. If a child has a negative experience in mathematics, that experience has the ability to affect his/her attitude toward mathematics as an adult. Every daily activity includes mathematic procedures, whether people are consciously aware of it or not. Possessing solid mathematical computational ability will help make additional skills come easier to the learner. The current debate in the school system is based on two different approaches to mathematics, constructivist or traditional. Constructivist teaching or constructivism is influenced by Jean Piaget and based upon the student being actively engaged in their learning process. Students build on their prior knowledge or schema, to aid in their learning. (Singer & Revenson, 1999; Marek & Cavalla 1997). Traditional mathematics instruction was influenced by Skinner, holds a scientific approach to learning. Rote memorization, drills, and recitation as the foundation for traditional teaching methods (Skinner, 2002). Traditional mathematics instruction (also referred to as direct instruction) emphasizes learning procedures and number facts and uses drill and recitation to reinforce and assess learning.

Rationale

The rationale for this project is to discern whether traditional teaching methods or constructivist teaching methods produce a better understanding of the concepts of mathematics in the student learner. This research investigates the effectiveness of both teaching methods and the effect that the methods have on the overall achievement of the student in relationship to instruction, teacher beliefs, and mathematics curriculum. A teacher who comprehends mathematical procedures and is able to teach it in a way that is effective for their students' learning is important. The purpose of this research is to find the rationale behind the change in the concepts of teaching mathematics. In addition, this research will also investigate the affect that behaviorist and constructivist mathematics teaching on the achievement and understanding of the student. In addition, the research will also explore the mathematical achievement of students with learning disabilities and the affect that constructivist or traditional instruction has on assessment. Furthermore, the research will evaluate constructivist mathematics curriculum in relation to student achievement. Finally, the research explores mathematics around the world and the different instructional styles in other countries in relation to student achievement.

Controversies

With the change toward teaching mathematics from a constructivist view of learning in recent years, much controversy has erupted. One of the main conflicts over the change to constructivist mathematics is the reluctance of those who have taught traditionally to change the way mathematics has been taught. Teachers who follow the traditional methods of teaching believe that rote memorization and drilling of facts is the only way that a child will learn the mathematical skills needed for higher-level thinking. They believe children will not be able to develop the skills to know how to do addition, subtraction, time tables or other mathematical calculations if they are not taught in a sequential and regimented way. They also believe a child will not be able to know how to calculate properly if the procedure is not at first shown to them on how to complete the problem followed by drills and practice of the problems.

However, those who embrace a constructivist view of learning believe that a child does not need route memorization to learn mathematic skills. If a child is given time and the proper guidance, then they will be able to formulate the answer on their own. Students will be able to come up with ideas on how to solve problems by themselves or with groups with guidance instead of pre-teaching from the teacher. The advantage of constructivist mathematics is that it allows students to develop critical thinking skills. "Problem-Based Learning in mathematics classes would provide young students more opportunities to think critically, represent their own creative ideas, and communicate with their peers mathematically" (Roh, 2003). Constructivist mathematics allows students a great range and a different way to come to have a greater mathematical understanding. Constructivist instruction “builds on the student’s existing knowledge base, extends the individuals repertoire of cognitive and meta-cognitive strategies, and corrects specific learning problems" (Gales & Yan, 2001, p. 9). Understanding the influences that teaching strategies in relation to the learner will be better prepare a teacher to ensure that students have the mathematical skills that will be needed through out their lifetime.

Definition of Terms

The two main terms that are used throughout the paper are traditional and constructivist. In this paper, traditional refers to the teacher directed learning style that can also be called behaviorist. Behaviorist or behaviorist approach can also be called traditional, transmission, instructivism, teacher-centered, direct instruction, static view or mechanistic view (Barakatas, 2005). Behaviorist theory is based on observable behavior. The behaviorist approach to mathematics focuses on route memorization, call and response, and drills. "In the behaviorist view, knowing is an organized accumulation of associations and components of skills. Learning is the process in which association and skills are acquired" (Gales, 2001, p. 1). Behaviorists believe that to achieve higher-level skills you have to build upon the lower level skills in a sequential order (p. 1-2). While behaviorists believe in basic skills and memorization as the essentials of mathematics, constructivism views mathematics as an active process.

Constructivism can also be referred to as, radical constructivism, social constructivism, cognitive constructivism, problem-based or inquiry learning. Constructivism and social constructivism involves the learner as an active participant in their environment (Morrone, Harkness, Ambrosio & Caulfield, 2004). This belief is grounded in the theories that for a child to expand their learning, they need to explore their environment and learn from hands on experience. Radical constructivism and cognitive constructivism refers to the cognitive process that occurs when learning takes place. The process of assimilation or adaptation that occurs when new information is being processed (This was developed by Jean Piaget and will be discussed in Chapter Two) (Singer and Revenson 1996). The main concept of constructivism that is important to remember is that teaching with constructivist methods in mind involves the learner being an active participant in the classroom.

Statement of Purpose

The change in the teaching methods in school systems from skills based to interactive learning and abstract thinking can affect the achievement of the student. Therefore, this paper examines the affect that constructivist and traditional teaching methods have on the mathematical achievement of students from elementary to high school while primarily focusing on elementary education students. In investigating the mathematic achievement of students, this paper also will examine specific curricula such as: Everyday Mathematics, Investigations in Numbers, Data and Space and Core-Plus Mathematics. In addition, both the mathematical achievement and progress of students of all ages in the United States as well as the results of studies from other countries will be reviewed.

Summary

The approach to teaching mathematics is constantly changing and has evolved over time. The two most prominent views of teaching mathematics are traditional mathematics and constructivist mathematics. The traditional ways of mathematics teaching involve route memorization and computational skills. Constructivist mathematical teachings involve the students being directly involved in their own learning and more hands on activities that involve the use of manipulative. “Constructivist instruction may falter…but when it works…it is exciting, energizing and productive” (Alsup, 2004, p. 15). The ways in which mathematics has been taught over time has changed many times. The history of mathematics in the ancient world, through the 18th and 19th to the evolution to the traditional (behaviorist) and constructivist mathematics practices in United States classrooms will be traced in chapter two.

CHAPTER TWO: HISTORICAL BACKGROUND

Introduction

This chapter examines the historical trend in the teaching and learning of students and the shifts that the educational system has made in relation to its instructional practices from the 18th century to the present. In addition, this chapter examines the historical changes from behaviorist to constructivist teaching strategies. How the shifts have been made, how effective the changes in teaching methodology were and the people or events that were important contributors will be highlighted.

Mathematics in the ancient world

The history of the ancient mathematical world begins with Pythagoras in Egypt and Mesopotamia. The history continued with Plato, and Aristotle in Greece (Bowen, 1972). During his travels to Egypt and Mesopotamia, Pythagoras established religious brotherhoods devoted to religious visions and the pursuit of knowledge. It was from these religious brotherhoods that he established that many were able to investigate the workings of mathematics. The religious brotherhoods were active for over twenty years and were disbanded due to riots shortly after Pythagoras death. Before his death, Pythagoras was able to discover harmonic notes in relation to mathematics. In addition, he was able to expand mathematical knowledge in relation to geometry. “From the evidence of geometrical figures, and the ratios found in number sequences, Pythagoras was led to the generalization that the entire universe is governed by the principal of harmony" (Bowen, 1972, p. 64). He believed that man had an inner harmony that revolved around his personal psyche. Through the contributions that Pythagoras made to the mathematical world, Plato expanded on what had already been obtained by Pythagoras.

Plato believed that learning occurred in a sequence of appetite and spirit and that educational training should follow that order. Furthermore, he believed that the youth should exhibit considerable effort to enhance their knowledge. "Arithmetic, geometry and other studies leading to dialectic should be introduced in childhood" (Bowen, 1972, p. 106). Leaning happened when the person was open to new possibilities and not experiencing any duress. During this time, after a child had finished their two-year military training, they were enrolled in the academy to learn the five Pythagorean sciences: arithmetic's, plane geometry, solid geometry, astrology, and harmonics. From this additional training a few were selected to continue training with five year instruction in the art of dialectic. By the age of thirty-five a person completed their training and was believed to have finally achieved enough knowledge to begin to know the truth (Bowen 1972). "Reason does not appear until the dawn of maturity, so that introduction to rational studies should begin only then" (Bowen, p. 107). Plato believed in developing the philosophers that were able to develop abstract thinking. This indicated the first movement towards constructivist teaching of philosophical teachers that were able to extract abstract ideals and relate them back to real world experiences.

While Plato believed in abstract mathematical thinking, Aristotle was the opposite. Aristotle believed that the mind was a logical structure that conveyed learning through the senses. Aristotle believed in the areas of facts and what is real and can be touched with your hands. “Behind all movement must rest a mover which is the cause itself, that is, beyond which no previous cause exists” (Bowen, p. 121). Pythagoras and Plato were two main influences in constructivist teaching and learning, the next section probes the influences during the 18th and 19th century.

Eighteenth and Nineteenth Century

The basis for constructivist teaching started in the 18th century by an Italian philosopher named Giambattista Vico who defined knowledge as a cognitive structure of a person so to know something is to know how to create. In 1781 another philosopher named Immanuel Kant adapted Vico's philosophical views into a book titled The critic of pure reason. He believes that the mind is actively involved in all areas of our lives (Bowen, 1972).

Social Constructivist Thinking in America

The social constructivism movement in America began at the beginning of the twentieth century. Before the change in education, it was believed that students would only learn through traditional methods or direct instruction. Schools would educate students by strictly following curriculum, teaching sets of skills instead of concepts, and used these skills to measure a student’s knowledge (Hirsch, 1987). Through traditional teaching, learning only occurs when students are presented by the teacher facts and rules that they memorize and apply (Bruner, 1996). With the shift of constructivist learning in America, the previous beliefs of teaching and learning changed. “Learning is an active process in which meaning is developed on the basis of experience” (Duffy & Jonassen, 1992 p. 21). The next section will investigate the important theorists that helped to influence the social constructivism movement in American.

The beginning of the social constructivist movement in America was partially influenced by philosopher John Dewey who wrote *Democracy and Education* in 1916 (Dewey 1997). He believed that education was the environment interacted on the individual to expand their body of knowledge. The main leader in constructivist thinking was Jean Piaget (Singer and Revenson 1996). He believed that learning is a continual process and reorganization in the mind. Piaget believed that learning happens in a sequential pattern like steps. Piaget believed that learning happens through a process of adaptation or accommodation of information into ones schema no matter the age of the learner. As children input new information they will be transforming "the input from the environment into their own mental structures. This will happen only if each child has a system that can make the required transformations" (Marek, 1997 p. 62). It was not until Piaget's ideas began to become accepted in America in the late 1970's that Vygotsky developed the zone of proximal development (Kim, 2005 p.8-9). The Zone of Proximal Development was explained as the difference between the actual developmental level of the student and their potential of development through problem solving and more capable peers. Vygotsky believed at a students learning was influenced by not just the classroom but by society and culture. Connections between culture and society and the influences that they represent are essential in understanding outside influences that affect student learning (Bruner, 1996). Vygotsky believed that “if learning can be influenced by social mediation, then conditions can be created in schools than can help students learn” (Vygotsky, 1978 p. 86).

Mathematics Educational Revolution, New Math

During the 1950's an important event occurred that began to change the educational system and teaching styles. In October 1957, Russia launched Sputnik, the first satellite. One event had the power to change the educational system forever. Because the Russian's beat the United State and were the first ones to reach outer space the United States began to panic. They were afraid that they were not preparing today's students in the mathematical and science skills that they need. This was the start of what was to later be called the space race, the need to beat the Russians. Where previously there has been no money to put into education, suddenly there were funds available to aid in reforming the mathematics and science systems (Spring, 2007). During this time the public was recognizing a need for greater mathematical and science skill for technology to make sure that the U.S. was still the world leader.

The change in the mathematical world shifted from a traditional and route memorization to a constructivist approach. The new math movement focused on strengthening students’ mathematical skills to enable them to expand their knowledge into the sciences. The goal of the new math movement was to allow students to develop their cognitive thinking abilities and to be able to relate mathematics to their everyday experiences. “One of the many problems in the New Math was that parents did not understand what was being done or why…’we don’t plug and chug anymore. We’re teaching them to think’” (Askey, 1999 p. 98). The new math movement lasted until the early 1970’s when the education system switched back to traditional instruction in education (Askey 1999).

Mathematics Educational Revolution, Back to Basics -- 1970's

The mathematics reform movement of the 1970’s and 1980’s focused on returning mathematics education back to more traditional and rote memorization. It was believed that students needed to follow the strict rules and complete practice drills to be able to gain knowledge in mathematics, “… it isn’t that American kids can’t calculate…it’s that they have trouble figuring out what calculations to use if they are not told…” (Roitman, 1999, p. 127). Many believed that using hand-on materials and other elements to relate their experiences to real life would inhibit the students learning. Students would not be able to relate the interactive mathematics to computational math (Askey 1999).

Mathematics Educational Revolution, NCTM Standards

During the 1980’s a new mathematical revolution began. Before 1989, it was deemed that the current mathematics curriculum was not able to meet the mathematical needs of the students. In response to this, the National Council of Teachers of Mathematics (NCTM) created and released new curriculum in 1989. The curriculum set a standard of mathematics curriculum in which all new curriculums must meet the standards set by the NCTM (Jacob, 1999).

"…textbooks have been written to conform to the NCTM standards and some of them are sorely lacking what is needed to improve mathematics education" (Askey, 1999, p. 97). The NCTM standards focus on the conceptual and cognitive understanding of student learning.

Summary

This chapter provided the background for the history of mathematics. Starting from mathematics in the ancient world, going forward to the mathematicians in ancient Greece and finally ending with the history of mathematics in America. The growth of mathematics in America begins in the 19th century and explored the change in the education system towards the constructivist methods of teaching and important theorists that were involved. The chapter continued by exploring the new math movement of the 1960's to the current mathematical debate over the standards set by the National Council for the Teachers of Mathematics. The next chapter will provide a critical analysis of the research literature that is relevant to how constructivist and traditional mathematics instructions effects student achievement.

CHAPTER THREE: CRITICAL REVIEW OF THE LITERATURE

Introduction

Chapter one addressed the differences between traditional mathematics and constructivist mathematics. Mathematics can be seen as a complicated and frightening subject and how it is taught can be an area of frustration and confusion for the learner. The different approaches in teaching mathematics to students can have an effect on their achievement in school. Chapter two discussed the historical aspect of Mathematics. It started with the 18th and 19th century in Europe and then begins to focus on mathematics in America. Mathematics in America starts with the 20th century and the social constructivist theorists and moves into the mathematics education revolution of new math of the 1950's. Mathematics changed again in the 1970's with the back to basics movement. Chapter two follows the movements of change in mathematics to present day with the NCTM standards and the current debate over to teach constructivist mathematics or to follow the traditional mathematics in schools. Chapter three organized the research in the following six sections: student achievement in relation to constructivist mathematics teaching methods, investigating the Everyday Mathematics curriculum, against Constructivist teachings in favor of traditional mathematics, investigates constructivism and traditional mathematical approaches, mathematics in relation to learning disabilities and student achievement and mathematics assessments' around the world. Although there are conflicting results in the research, the aim of this paper is to compare and contrast what the research reveals about the effects of mathematical teachings on student achievement.

Section One: Student Achievement In Relation to Instruction

This section investigates the mathematical achievement of students when given assessments. Anderson (1997); Gales and Yan (2001); and Morrone, Harkness, Ambrosio and Caulfield (2004) observed the participation of the students in the classroom and their ability to become active learners in their environment given the constructivist method of teaching. Clark (1997); Simon and Schifter (1997); Bay et at. (1999) recorded that students achieved a positive attitude for mathematics when they are instructed using constructivist methods of teaching. Alsup (2004) found that students experienced an increase in autonomy and a decrease in math anxiety when engaged in constructivist mathematics. Douglas and Olsen (2008) analyzed the assessment of students in relation to constructivist teachings.

Students Ability to Become an Active Participants in their Learning

Family interaction and involvement in the education of their own children plays a critical role in the development and achievement of children. Anderson (1997) conducted a study on families and mathematics and to determine what types of mathematics actually occur in the home and the nature in which the parent-child interaction occur that help to develop the child's mathematics learning. Anderson contextualized the research around three main perspectives: contextualized learning, social constructivism and mediated learning.

To investigate this, Anderson (1997) selected 21 parents from a middle-class background that all had children that were four years of age. Anderson selected the specific families by sending letters to the parents of four-year-old children who were enrolled in twelve different child-care centers. All of the child-care centers were located in a middle-class urban neighborhood. All of the parents selected for the study were well education and from a variety of ethnic backgrounds. Out of twelve centers, only eight different child-care centers were the final focus of the study with two families that were not enrolled in childcare at the time of the study. Anderson conducted twenty sessions in English and one session in Japanese translated into English. For the purpose of this study, Anderson focused on the age of the child rather than birth date and no distinction is made between ages and the age range of the children was from 4 years 1 months to 4 years 11 months in age. The family makeup of those involved in the study included: eight mother-daughter dyads, eight mother-son dyads, three father-daughter dyads, one mother-father-son triad and one mother-twin sons triad.

Anderson (1997) conducted the study over two days in which the families were able to choose four different 15 minute periods over the two days in which to engage their child in mathematics activities. The researchers provided each family with a kit to use when playing with their children. Each kit included: 50 to 60 multilink blocks, a children's book entitled One Snowy Night, a variety of blank paper that included some colored, white, lined, and unlined, and two preschool worksheets from a preschool mathematics workbook that was available at local stores at the time this investigation was conducted. In addition, the kit included a set of instructions in relation to data collection in the kit, but no other guidance on how the tasks were to be carried out was included in the kit. The researches collected the information by audio taping the information, collection of children's work and interviews with the parents. The researcher gave each parent an audiocassette recorder with 1 sixty-minute tape. Parents recorded as close as possible to the beginning of each activity and only record the first 15 minutes of each session. After the parent conducted all 4 fifteen minute sessions they gathered all materials and tape and place them back into the kit and complete a five minute exit interview with the researcher to record the parent's feelings about the experience. Anderson interviewed each parent and investigated the same four questions: who and what influenced what they did with the materials, what materials they would label as mathematical materials, and which sessions, if any, went beyond fifteen minutes, and how they and their children felt about the activities and any likes or dislikes.

Anderson (1997) focused this study around the verbal interactions between the parent and child when engaged in activities in the home that would have the potential to aid in the children's mathematical knowledge. In addition, Anderson hoped to answer two questions: what mathematics is evident in parent-child play and what, if any, are the observable trends from the specific parent-child interactions. In total Anderson observed a mathematical even that occurred in 82 session. With counting (55) and naming shapes (39) being the highest recorded and representation of shapes and numbers (1) and using fractions (1) being the lowest. In addition, Anderson broke down information to specific families and documented how many times a mathematical event occurred. The families had a range of five to twelve mathematical events that occurred with a mode of eight. In analyzing the types of activities that the parents and child were engaged in, most parents did not relate previous activities to current activity and kept each lesson separate. Furthermore, Anderson chose to determine if families started the lesson with mathematics as the focus or implemented mathematics into the activity. To catalog this information Anderson looked at the transcriptions of the audiotape and focused on the beginning of the tape to determine what type of interaction the parent and child had in the beginning.

From this study, Anderson (1997) concluded that parents and children posses the ability to construct activities at home which mathematical skills are used in a playful manner. In addition, Anderson believed that this research proves that parents who are not mathematics teachers are able to engage their children in mathematics and be able to assist in their learning. In relations to the three perspectives of contextualized learning, social constructivism and mediated learning Anderson concluded that the parent mediator was an essential element in the contextualized learning of mathematics by bringing out the mathematical language in the activity.

While this study focused on the positive impact of the parent-child interactions in relation to constructivist mathematics, it failed to note any negative connotations. Furthermore, the study only focused on one socio-economic status and did not take into consideration low-income families or families in which both parents work two jobs and would not have been able to so easily able to engage their child in a one on one activity. In addition, the study failed to note if there were other siblings in the residence of the children participating in the study. The research indicated that children from a middle to high class, two parent household, respond to learning mathematics from a constructivist approach when given one on one instruction from a parent.

In their study to calculate teachers' beliefs concerning student learning and the affect the instructional practices had in the classroom, Gales & Yan (2001), investigated the way in which educational systems, schools, and community affect the achievement of individual students. In their study, they examined the instructional experiences of 8^{th} grade students through questionnaire responses. The survey consisted of 527 teachers representing 10,970 students. In addition, the survey took into consideration the area in which the student lived in relation to how the student's environment might affect their achievement. Out of the 527 teachers participating, 75% taught in urban or suburban schools, 21.4% of the teachers taught in a rural area or village, and 3.4% of the teachers taught in an isolated area. The study aimed to determine if behaviorist (traditional) and constructivist teachers have different belief systems in regards to student learning. In addition, to determine if those teachers utilize different instructional practices and the effect that the different teaching styles has had on student learning and used The International Mathematics and Science Study (TIMMS) to calculate if behaviorist and constructivist teachers have different belief systems. TIMMS proved to be the most useful method in conducting the study because it is the largest international study of schools and students that has been conducted. In addition, TIMMS compared students learning of mathematics, educational systems, schools, and teaching. For the purpose of this study, the researchers chose to focus only on students within the United States.

To determine the learning experiences of 8^{th} grade students in relation to teachers' belief systems and instructional practices, Gales & Yan (2001) collected questionnaires from a sample of students and teachers. TIMMS used a sample Population of 2 in analyzing the results. In response to the questionnaire, 570 teachers answered in representation of 10,970 students. The TIMMS data system calculated the results of the survey by the sampling weight and design effect was also taken into consideration. The researchers used Houseweight (HOUWGT) to weigh the sample since the participants of the study included only students from the United States. In addition, HOUWGT took into consideration the disproportion of the sampling of groups due to non-response, and the selection probability of the student. To account for the design effect, the researchers implemented the jackknife repeated replication, (JRR) through WESVAR. The use of the JRR ensured that the questionnaire provided an unbiased estimate of the statistic of interest by selecting subsets of the sample from which to calculate the statistics. Gales & Yan (2001) weighted the different question differently depending on the type of question. The participants of the study had two to three possible responses in mark in response to each question that they were asked. From the preliminary results from TIMMS it indicated that the type of community the students live in effects the students' mathematical achievement.

Overall, Gales and Yan (2001) concluded that, teachers who believe in a constructivist approach to student learning believe that the students' in their classrooms should be able to think creatively, support their answers and find real world uses for mathematics. Teacher-directed student questioning had a positive effect on student achievement. In addition, female teachers with a high level of education had a significant positive effect on student learning. In addition, TIMMS found that students who work on projects in which there is no immediate correct answer were able to relate their learning to real world experiences and have higher achievement levels. While the study stated that teacher education had a significant effect on student learning, there was no data presented with the article to support this. The study did not take into account social factors, psychological factors, and organizational factors into the students learning and achievement. In addition, this study mainly focused on teacher beliefs and practices and failed to take into consideration the aspects of the schools in which they are teaching and any limitations that' the teachers might have on their instructional practices. In addition, there is no information available on student's previous achievement scores. In conclusion, the study found that students who received constructivist mathematics instruction had a higher achievement in school in relation to the teacher belief systems.

One controversy that surrounds constructivist mathematics is how students will learn if they are not given the proper, traditional ways in which to solve mathematical problems. Morrone, Harkness, Ambrosio and Caulfield (2004) sought to answer this question by investigating how instructional methods in a college level constructivist mathematics class influence the perception of achievement of the students. The subjects for this study were college students enrolled in the course, Mathematics for Elementary School Teachers attending a university in the Midwest United States in 1998. The course was taught using a constructivist approach to mathematics. Students completed comment sheets on specific aspects on the class at the end of the course. The results were organized using Patterns of Adaptive Learning Scales (PALS) and student comments were combined with the course description and objectives. Out of the comments that were received from the students over 90% of the comments were encouraging and supportive of the constructivist method of teaching. To analyze the transcripts of student discussions, the researchers used Observing Patterns of Adaptive Learning (OPAL), which focuses on six specific categories, that are: Task, Authority, and recognition, Grouping, Evaluation, and Time. In coding the transcripts, Morrone et al. looked for complexity in the teacher questioning and the ability of the students to understand the mathematics. The categories that was observed and recorded were: task-structure (9%), task-messages (3%), scaffolding (27%), and press for understanding (28%), higher-order thinking (2%), authority (16%), recognition (11%), grouping (1%), evaluation (1%) and time (1%). To account for coding discrepancies and to check the inter-reliability of the results the researcher coded the first third of the transcripts as a team. The next third were coded by two members of the research team (which two members were not listed) and given in inter-reliability rating of 0.87. With the inter-reliability score calculated, one researcher coded the remaining transcripts. To analyze the quantitative data from the transcripts QSR N5, version 5.0, software was used. The codes for the interaction between the students and the teachers were divided into three categories: task, authority, and recognition. The majority of the instances fell into the task category at 69% followed by authority at 16% and recognition at 11%. While students did engage in meaningful mathematical communication, Morrone et al. did not find any connection between positive reinforcement from the teacher and the students in the college classrooms and that additional research would need to be conducted in order to observe if there is any effect on students as they go through school.

Morrone et al. (2004) observed and analyzed both constructivist teaching and student behavior, however; because they were unable to answer the goal of the research is a weakness of the study. In addition, this study was focused on only one specific class and students, it does not provide any comparison results. While the researchers failed to compare the results with a traditional mathematics class, they concluded that 90% of the students preferred the constructivist method of teaching. There was no correlation made between the students preference or their mathematical achievement.

Student Perception and Attitude Towards Mathematics

In an initial study of students from 15 different teachers, Bay et. Al (1999) sought to investigate the reactions of the students in relation to the standards-based mathematics curriculum. Approximately 1,000 students respond to teacher-guided feedback with no specific numbers reported to the researchers. Students were between the 6^{th} and 7^{th} grades from five different school districts incorporating both large cities and small towns. The study took place between the 1995 and 1997 school year in which a new standards based mathematics curriculum has just been started. During the 1996-1997 school year, the teachers used the Connected Mathematics Project of the Six through Eight Mathematics curriculum.

After receiving the new curriculum instruction for one year the teachers asked the students to respond to their experience with the new curriculum resulting in over 1,000 letters. The researchers coded the letters by the class, teacher and by the curriculum project to look for any common themes between the letters. Some students had received only one year of the new instruction while others had received two. Not all teachers used technology such as graphing calculators, possibly due to lack of availability to account for some variances in the results.

In reviewing the responses from the students, Bay et al. (1999) found that the majority of the students preferred the new standards based curriculum to the previous traditional curriculum. The majority of the students experienced more enthusiasm for the new curriculum than the previous curriculum. The new standards based curriculum employed more hands on activities with real life applications that the students showed positive attitude for. The students showed a positive affirmation for the hands on activities, application to real life, activities, group work, and mathematics content. 25% of the students report having difficulty with understanding the curriculum. The researchers concluded that while the instructional methods of the lessons varied between teachers, the more time that the student had to become familiar with the materials the more they developed a positive attitude for it. This study had relatively little strengths. It did not give any specific information about who was chosen to participate in the study and no other specific information crucial to important research was given. The researchers concluded through student responses to instruction that most students prefer the constructivist method of teaching mathematics.

In a qualitative study on two sixth grade math teachers, Clark (1997) sought to examine the role of the teacher in the classroom and to see how their instructions might change and influences that might cause those changes. The study consisted of two teachers from a school in the Midwest United States. The student population of the school came from a range of socioeconomic status with a minority population of 22%. The teachers chosen in the study have been teaching in the elementary schools for over 20 years. In addition to the teachers in the case study, the mathematics coordination and project staff member were also interviewed. The researcher studied two teachers to observe the differences between individuals given the same situations and opportunities. During this time period, the teachers started to teaching a new curriculum that follows the constructivism methods of teaching.

To collect the needed information, Clark (1997) conducted 36 interviews with both teachers with the exception of one day when one of the teachers was absent. The researchers observed the teachers in the classroom, in their team meeting and at in-service sessions. In addition, the teachers participated in six interviews at the request of the researchers. The interviews consisted of three interviews with the mathematics coordinator and thee interviews with the project staff member. All interviews were approximately 30 to 60 minutes in length and were all were audio taped. The data collected period spanned over seven months. The researchers established trends and patterns from the results according to specific teacher beliefs.

At the end of the research session, Clark (1997) concluded that both teachers appeared to have positive attitude about the new mathematics curriculum. Teacher number one did not change the way in which they were teaching, but agreed that this way of teaching is the effective way to enhance students learning and the direction to be changing. Teacher number two changed much of what he had originally been teaching as he found the new method to be more effective. In addition, he was already changing his lesson plans for the following year to reflect the new methods. It also should be noted that during this year teacher number one had additional struggles with personal health, time constraints in the classroom and students with reading difficulties that teacher number two did not experience. Through the coding process, the researcher was able to identify 12 main reasons that influenced the reasons in which teachers would change their teaching methods. Those reasons were: 1. The reform movement in general, 2. The principal and school community, 3. Internal support, 4. The spirit of collegiality, collaboration and experimentations, 5. Internal support personnel, 6. Innovative curriculum materials, 7. The In-service program, 8.

External support personnel, 9. The researcher as audience and critical friend, 10. Outcomes valued by the teacher, 11. Day-to-day conditions under which the teachers work and 12. Teacher knowledge.

In conclusion, Clark determined that providing teachers with a curriculum in which they could adapt it would be more effective than having no basic curriculum. Furthermore, teachers having a lack of knowledge in the reform curriculum will impact student engagement. This study would have been stronger if quantitative data was presented and if there was a larger case study size. The researchers found that the teachers preferred the constructivist method of teaching and observed more student engagement (and thus possibly better student achievement) with the new constructivist mathematics curriculum.

In a qualitative and quantitative Simon and Schifter (1993) completed a study on elementary students from 4th through the 6th grade to determine the effectiveness than an in-service program would have on the student's beliefs and attitudes towards mathematics. Prior to the study, teachers were engaged in a two week in service, Educational Leaders in Mathematics (ELM), program aimed at giving teachers instruction in how to teach a constructivist mathematics curriculum. Thirty-five teacher responses are also included.

Simon and Schifter (1993) collected a series of surveys to include: Pre-Program surveys (171 from elementary students, 295 from secondary students), and post-program surveys (179 from elementary students and 303 from secondary students) and a response survey from 35 teachers. In addition to the surveys, the researchers collected standardized tests and teachers' reports of student change. The surveys and test were included for only classes of teachers who taught the same course two years in a row. A two-tailed T-test compared the pre and post-test scores. The tests and surveys were gathered between 1985 and 1988. The student's surveys were not the same from one year to the next.

Simon and Schifter (1997) concluded that there was a statistically significant increase ($p < 0.001$) in the elementary students' attitude towards mathematics. While the instruction of students changed, no significant change was seen in the students standardized test scores. The results of the standardized test could not tell if the students were able to develop a stronger conceptual understanding. While Simon and Schifter (1997) showed strength in the variety of research methods they were weak in their selection of a sample size. In addition, the teacher survey was by mail in response only, so many could have chosen not to respond. While the researchers found that the students recorded had a more positive attitude to mathematics in relation to constructivist mathematics teaching, no change was seen in their achievement on standardized tests.

In a comparison of constructivist and tradition mathematics Alsup (2004) conducted a study to discover which instruction practice on elementary education student teachers would prove to be the most beneficial. In an initial population of study of 4,000 students from a Liberal Arts College in the Midwest, 27 students enrolled in a required mathematics class for pre-service teachers, participated in this study during the fall of 2001. Students were divided into three groups (two experimental groups and one control group). The three classes in the study were math concepts I and math concepts II, with two classes one math concepts I being taught (one control, N = 17) and one experimental group, N = 27) and one class of math concepts II (experimental group, N = 17). The same instructor taught all three classes. Students were chosen at random as to which group (experimental or control) they would be in. All classes were taught on campus in a regular classroom setting.

Alsup (2004) used an abbreviated version of the Mathematics Anxiety Rating Scale (AMARS) to calculate the results of this study. The scale uses a 98-item Likert survey; each item on the survey represented something that might cause anxiety with the subject. In addition to AMARS, The Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) to enable to researcher to measure the confidence to teachers to teach mathematics and the teachers' expectations of student achievement. Students were given the AMARS, MTEBI, and autonomy survey on the first day of class and on the next-to-last day of class. The study included results from students who completed both sets of examinations. In addition to the examinations, students also participated in a 30 minute taped interview with the instructor and agreed to keep an observational journal during the course time focusing on their learning process and the instructional processes of the teacher.

Alsup (2004) used the ANCOVA in the analysis of the covariance to reduce the differences between group differences in relation to treatment effect. While the experimental group had a higher mean score in relation to math anxiety (M = 56.5) and Autonomy (M = 7.8) the control group had a higher mean score in relation to teaching efficacy, (M = 79.3). However, the differences between the control and experimental group in relation to math anxiety were not statistically significant. T-tests ($p < 0.05$) were used in analyzing the differences between the three groups. While the scores were relatively the same in most areas there was a significant difference in relation to the autonomy scale and all three math groups. In addition, the results showed a significant difference between the two experimental groups, Math Concepts I and Math Concepts II and there was no difference between the control group and Math Concepts II. Surprisingly, the students in the Math Concepts II class had a higher level of math anxiety and the lowest level of teacher efficacy. Comparing the results of the AMARS and the MTEBI, the ANCOVA scale found that there were no significant differences. The largest increase from the pre and post test scores was found in the students' sense of autonomy (Pre-tests Mean: 4.8, Post-test Mean: 6.6, t-value: 2.8). No p-value results were included to show significance.

Alsup (2004) concluded that while students only showed a large increase in their sense of autonomy, the course did allow for students to decrease their math anxiety and at the end of the course the students had a greater belief in their ability to teach mathematics. In addition, while the constructivist approach to mathematics had a positive impact on student learning in the Math Concepts I experimental class, it did not have the same effect on the Math Concepts II class, this could be the result of the students being less familiar with the material and not being as comfortable to try the problems on their own. A weakness of this study is in the use of two experimental groups and only one control group. It would have provided a greater comparison to have an equal amount of control and experimental groups in each math class. Strength of the study is that the researcher used several different methods to analyze the results with three separate tests to create a broader range of understanding the subjects in the study. The study found that the students in the class had the largest decrease in their anxiety towards mathematics. No significant changes in the students' mathematical achievement were reported.

Student Assessments

In a quantitative study Douglas, Burton and Reese-Durham (2008) sought to investigate what effect Multiple Intelligence instruction and direct instruction has on the academic achievement of eighth grade mathematics students. Students (N = 57) for this study were selected from a public middle school in North Carolina. The students were divided into two groups, an experimental group (N = 28) and a control group (N = 29). The gender ratio between the groups consisted of 15 (54%) males and 13 (46%) females in the experimental group and 14 (48%) males and 15 (55%) females in the control group. The study was comprised of students from all ethnic heritages, no specific percentages were given.

The researcher of this study developed a pre and posttest for the students based on the curriculum and instruction that they were being taught. Both groups were given the pre and post tests to determine the students' mathematical abilities before and after the experimentation. The test consisted of written responses to mathematical questions. Instruction included either the Multiple

Intelligences design, or direct instruction for one semester. Using a t-test analysis to calculate the change in student achievement all students completed a posttest once the instruction period was over. All tests were identical and covered the same material, used the same textbook and completed the same homework assignments.

Douglas et al. (2008) used the results from the two groups to compare their achievement they found statistically significant ($\propto$ .05) differences between the two groups. The students who were taught using the Multiple Intelligences instruction achieved higher math scores that the direct instruction students. The researchers calculated that the multiple intelligence students scored 25.48 points higher than the direct instruction students who only scored 17.25 points higher. In conclusion, students who received constructivist mathematics instruction scored higher than the students who received traditional instruction.

To develop and understanding between classroom assessments and the standards set forth by the National Council for Teachers of Mathematics (NCTM) Ohlsen (2007) investigated to find if there is any correlation between classroom assessments in mathematics classrooms and high-stake assessment programs. Simple random samples of 668 NCTM members from nine states were selected to participate. A mail response survey was mailed to those selected. The survey had a response rate of 41.6% (278), however, out of the 278 that were returned, only 262 met the specific criteria. The criteria had three parts, 1. Had a current fulltime mathematics teaching position, 2. Were employed in a public high school and 3. Were current members of NCTM. Out of the 262 response 37.8% were male (99) and 62.2% were female (163). The median year for teaching experience was 20 years.

Ohlsen (2007) used the survey to collect information was adapted from The Secondary Teachers' Classroom Assessment Methodologies Survey with permission from the author. The survey included 41 closed – form items and three open –form questions. There were 11 survey items that focused on the teacher methods on student assessment and analyzed using the 6-point Likert Scale.

Ohlsen (2007) concluded that the survey respondents relied on traditional methods to evaluate student assessment. The main choice for assessment relied on quizzes or exams but was not exclusive. Three-quarters of the survey participants reported that they preferred to create their own assessments over the ones in the text, which supports a more constructivist approach to assessment. Out of the participants, forty to fifty percent of the respondents reported that they used projects, performance based assessment and team projects as methods of assessment very little to not at all. While the teachers appeared to employ a variety of assessment methods, they rarely used the assessments set forth by the NCTM of a more performance based assessment. The larger and random sampling size of this study showed its strength. However, with the survey being an optional mail in survey that resulted in a less than 50% return rate is a weakness. The researchers conclude that while instructors prefer to create their own assessments, they use more of a traditional based assessment rather than a more constructivist approach to assessment base of performance.

Section one researched student academic achievement in mathematics in relation to the type of instruction that the student received. Gales and Yan (2001) found that the beliefs of the teacher had a direct impact on the achievement of the student and that the students had better achievement when they were able to relate mathematics to real world situations. The researchers did not present any specific information and failed to take into account social factors, psychological factors, and organization factors into student achievement. Anderson (1997) concluded that children, when given the opportunity, are able to learn mathematics in the constructivist methods when they are able to learn the mathematics in context to experiences. Morrone et al. (2004) surveys college mathematics students and found that 90% of the students preferred constructivist methods of teaching but did no compare the results to students who received traditional mathematics instruction. Bay et al. (1999) also reviewed student responses to the type of instruction that they received, like Morrone, the students preferred the constructivist methods of teaching, in addition, Bay used two different curriculum for comparison with the students and the majority of the students preferred the constructivist curriculum. Both Clark (1997) and Simon and Schifter (1993) found that student and teachers beliefs and attitude toward mathematics has an effect on students mathematical achievement. In addition, the teacher must believe in the constructivist method of teaching as well as believe that it is important for it to be effective for the students. Alsup (2004) found that constructivist method of teaching mathematics greatly reduced a student math anxiety in the classroom. The next section explores the differences between several mathematics curriculums.

Section Two: Investigating The Everyday Mathematics curriculum

This section investigates the response to the Everyday Mathematics curriculum that was developed in Chicago. Carroll (1997) and Fuson, Carroll and Drueck (2000) investigated the Everyday Mathematics Curriculum and found that students achieved higher test results than with traditional mathematics curriculum. Sood and Jitendra (2007) investigated four different mathematics textbooks in addition to the Everyday Mathematics curriculum with mixed results. Cramer, Post and DelMas (2002); and Huntley, Rasmussen, Villarubi, and Sangtong (2000) investigated both traditional and constructivist curriculum.

The Everyday Mathematics Curriculum

In recent years, in response to the standards set by the NCTM, there have been a few changes in the curriculum of mathematics towards constructivist teaching methods. The University of Chicago School of Mathematics Project developed an elementary mathematics curriculum called Everyday Mathematics. The goal of this curriculum is for students to work in small groups and to investigate real-world context with the use of manipulative, calculators and other tools to solve problems. In addition, almost all the problems are application-based with less time spend focused on computation and number skills. In order to understand how student achievement is measured on standardized tests with students who have more experience in the constructivist teaching methods, Carroll (1997) developed a study to examine the mathematical achievement of third grade students in Illinois who used the Everyday Mathematics curriculum.

The study selected only public schools, and only schools that had been using the Everyday Mathematics curriculum. Twenty-six schools in nine districts in the greater Chicago metropolitan area were selected. In total, 1885 students participated in the study. Fourteen of twenty-six schools that were selected included students who had been using the curriculum since kindergarten; the remaining schools had adopted the curriculum sometime between 1992 and 1993. The standardized test used IGAP to score the students achievement. The test scored for competency levels in addition to recording a school score on a scale from 0 to 500. To allow for comparison between each school and district and scores from different years, the test also provided a confidence band. The confidence band is +/- two standard errors around the mean.

From the scores on the IGAP, Carroll (1997) noted that the results suggest that the Everyday Mathematics curriculum has had a positive longitudinal effect on the mathematics achievement of the students. The scores of the third-grade classroom ranged from 276-423 with a mean class score of 332 and a mean school score of 337. All schools scored above the mean state score of 268. The 14 schools that have been using the curriculum since kindergarten achieved higher test scores, ranging from 310 to 423 with a median score of 343 and a mean school score of 351. Furthermore, Carroll compared the socio-economic status of the students in these schools and found that the low-income students had math scores were well above the state standards. Carroll also compared the overall achievement of the students with the students of the schools that has been using the Everyday mathematics curriculum since kindergarten. Of those schools, 54% exceeded to meet state goals with 2% that failed to meet state goals. For the years between 1992 and 1993, 31% of the students showed an increase in achievement scores, 15% showed a decrease 54% showed no change in achievement. However, in the schools in which the curriculum was used in the classrooms since kindergarten, the results showed a 42% increase from the previous year.

There are several weaknesses in the study conducted by Carroll (1997); the study did not take into consideration the achievement scores to of students using the traditional curriculum, other than to state that the constructivist curriculum did better than three of the four other districts with no actual number and/or scores reported. In addition, when Carroll compared the subtests of the test, the areas of the subtest that he observed were not recorded and only the scores of geometry and algebra were listed with no connection to the other 4 subtests. In addition, the author of this study was involved in some way with the development of the Everyday Mathematics curriculum and could be biased as to the results in order to show the positive effect of the curriculum so that the new curriculum will be adopted. This study indicated that students achieved higher scores in mathematics when using constructivist methods. However, this varied depending on how long the students has been learning mathematics in this way and little evidence was given on students who were learning mathematics from traditional methods.

While the previous study was conducted in 1997, Fuson, Carroll and Drueck (2000) conducted another study on the achievement results of third grade mathematics in relation to the Everyday Mathematics Curriculum (EM). The researchers combined the results of the achievement of second and third graders into two different studies. In the first study, the researchers followed the progress of second graders who have been using the EM curriculum. They chose to focus on the students' ability in mathematics related to whole numbers and multi-digit computation. Study number two involved the progress of third graders who have been using the EM curriculum as well. The researchers chose to investigate the students' ability to perform computations with whole number, in relation to a mathematical topic, and the areas of geometry and measurement. In their investigation of the EM curriculum, the researchers conducted two separate studies to compare the results.

The investigation for the first study included the Northwestern University Longitudinal Study to determine student achievement, 392 second graders from 22 classes and 11 schools. The schools consisted of a variety of urban, suburban, rural and small-town schools. Students completed the tests in a whole class setting and had all the time needed to be able to finish. Each test involved a set of 45 questions and took approximately 60 minutes for students to finish. Out of the 392 students in this study, 344 of them had also been in the original longitudinal study of first grade achievement. In addition, the socio-economic status of the students ranged from low income to affluent. The researchers used a significancant level of .01 in scoring the tests. To compare the results of the students that participated in their study, the researchers also sent test questions to students in San Francisco and Japan. The participants of that test consisted of 29 U.S. Second graders in San Francisco from an upper to middle class families and 33 Japanese students attending a middle class public school in Japan. Fuson et al. (2000), stated that while the two tests given to the San Francisco and Japanese students as a comparison, that they were biased against the EM curriculum. The test results comprised of different analysis areas: Numbers-Sense test, Mathematics Achievement Test and Additional items.

In evaluating the test results on the number-sense test, Fuson et al.(2000), concluded that the EM students did better than the San Francisco students on two items and lower than both the San Francisco students and Japanese students on one item. Three questions (i.e. Which number is closer to 28: 31 or 32? How many numbers are between 2 and 6, and What number comes 9 after 999?) were evaluated with the Chi-Square test, however; The Chi-square test was only used on the test results from the San Francisco students and the Japanese students. No reasons for only using the Chi-square test for the San Francisco and Japanese students is given. The mean results for the students were as follows: EM (n = 343) = 54, U.S. (n = 29) = 52, and Japanese (n = 30) = 50.

The results on the Mathematics Achievement portion of test showed a significant difference from the previous test, in which the Japanese students scored at the top of the test, EM students scoring in the middle, and the San Francisco students scoring below that. Furthermore, the Japanese students achieved a score that was significantly higher than the EM students on the top six questions that included the subjects of patterns, addition, and subtraction of tens. Furthermore, on the same six questions, the EM students scored higher than the San Francisco students. Eight questions from these tests were evaluated using the Chi-square test. No specific results reported. As in the previous section, only the scores from the San Francisco and Japanese students were compared using the chi-square test. No reasoning as to why those answers were scored differently.

The second study by Fuson et al. (2000), focused on the student's ability to understand and comprehend both symbolic and contextualized computation problem in addition to questions in geometry, data and reasoning. To fit the purpose of the study the researchers gathered questions from two different tests; a third-Grade level of the fourth NAEP tests and from cognitively based tests (Wood and Cobb) from the third grade (developed in 1989). The test was comprised of a total 64 questions, with 22 of the questions coming from the fourth NAEP and nine of the questions from the Wood and Cobb test. Fuson et al. believed that these two tests were important for comparison because they incorporated the essential elements from both new (constructivist) and traditional mathematics. Additional questions were taken from the third-grade Everyday Mathematics curriculum (EM). This study was comprised of 620 third grade students from 29 different classed and from the same heterogeneous groupings from the same school districts as in study number one. Due to a shifting of students and new schools, this test did not include all of the same students as the previous study. Out of the 620 students, 236 of those were from the previous first grade research group in 1997. Fuson et al. used the results of those 236 students for the focus of the second study. The test was administered in whole class sessions by a researcher from the Northwest University Longitudinal study. The students answered 33 questions and the test took the students approximately 50 minutes to complete. Students completed one portion of the test questions orally, that portion included the questions taken from the Wood and Cobb test. For the remaining test items the students worked independently to complete the questions. The researchers divided 22 questions from the NAEP into two subtests: a number concepts and computation subtest, and a geometry, data and reasoning subtests. The researchers used a significant level of .01 to evaluate the results.

Fuson et al. (2000) compared the test results from the NAEP students and EM students. On the numbers and computation subtest, the EM students scored higher than the NAEP students with a mean of 65% and 52% respectively. Overall, the EM students outscored the NAEP students on tests that involved mathematics that was conceptual. The results from the nine questions from Wood and Cobb indicated that the EM students scored roughly 20% higher on the questions. The questions included six number stories, a numerical problem, and addition problem involving the use of base-ten blocks. Fuson, et al. further compared the mathematical achievement of the students by repeating three questions that the second grade EM students to the third grade students. The mean results for the number and computation subtests are as follows: EM (Third Grade) (n=107 to 119) =65, NAEP (Fourth Grade) (n=1,800) = 52.

The geometry, data and reasoning, subtest focused on three main areas: geometry, data and graphing, and reasoning. The EM students outperformed the NAEP students on four of the geometry questions, half of the data and graphing items and the reasoning items. Fuson et al. (2000) concluded that the EM students were able to do better on the mathematics problems that involved computation and problem solving. Furthermore, the Elementary Mathematics students were able to out perform other students in the United States and underperformed compared to the students in Japan. The EM third grade students scored higher on areas of mathematics that involved place value, numeration, reasoning, geometry, data and story problems. Furthermore, this research supports the change of educational curriculum that is more constructivists in its teaching and involves hands on curriculum and cognitive thinking processes.

Although Fuson et al. (2000) collected a good heterogeneous grouping from a variety of students from their first sample, they failed to find similar heterogeneous groupings in subsequent studies and comparison tests. Furthermore, the sample results from the different levels of tests were different and would further adjust the mean results. In evaluating the results of the tests, they chose to use a chi-square test for some of the results but not all of them and gave no reasoning as to why this was chosen and what effect this would have on the results. When Fuson et al. (2000) conducted the second study; they chose to verbally administer part of the test instead of typing and combining it with the other test questions. This is a weakness in their testing and answer research because not all students are able to process the information; this way of testing is only helpful for students with auditory learners and not for students who are visual learner. Overall, the researchers found that the students using the constructivist EM curriculum outscored students who were using a traditional mathematics curriculum.

Comparing Different Curricula

Sood and Jitendra (2007) chose to analyze three different first grade mathematics textbooks (Math Trailblazers HB, Investigations in Numbers, Data and Space HF, and third title not listed, but it is published by Scott Foresman, SF), and one reform based textbook (Everyday Mathematics). Unlike the previous studies, neither Sood nor Jitendra had any part in the development of the curriculum. This research did not have any specific subjects and focused on a statistical analysis of the textbooks. The lessons that included number sense lessons ranged from 6 to 11 (M = 8.67).

The researchers analyzed both the teachers' manual and the textbook for the specific lessons that involved number sense; each lesson was then tallied and evaluated for effectiveness in instruction. The lessons were evaluated for effective instruction on big ideas such as number relationships, one and two more, and one and two less, anchors or benchmarks, part-part-whole relationships, real-world connections in addition to conspicuous instruction, mediated scaffolding, and judicious review. Conspicuous instruction indicated a more traditional approach to instructional methods where the instruction is explicitly stated and students are not encourage to come up with their own development. The researchers deemed that scaffolding is an important aspect of a students learning, and they believe that scaffolding happens only with teacher guidance, materials or task. The review of material must be adequate for students to be able to apply the knowledge that the students have obtained.

The reliability of the results was calculates and the mean interrater was found to be 96% with a range from 96% to 100%. The research showed that the traditional textbooks provide students with more opportunities for exploration with big ideas in number relationships (Mean = 10.27). However, the Everyday Mathematics Curriculum provided more emphasis on real- world connections (1.38). In examining the three traditional textbooks, the SF textbook had the largest percentage (13.11) of number relationship tasks, followed by HM (9.33) and HB (8.45). In addition, the researchers found that the vocabulary for main concept terms was not consistent across the textbooks. The researches found that none of the instructions from the Everyday Mathematics Curriculum of in the SF curriculum contained explicit instruction followed by teacher modeling, compared to 64% of the lessons in the HB textbook and 67% in the HM textbook. The traditional textbooks provided student with feedback 93% of the time compared to 50% in the Everyday Mathematics Curriculum. In addition, the traditional textbooks provided students with more opportunities to review the lessons. Overall, the researchers concluded that the Everyday Mathematics Curriculum provided students with real-world connections, promoting understanding, spatial relationships, instructional scaffolding, and review in subsequent lessons over the traditional text. This study was thorough in its analysis of the four different textbooks, however it would have added to their study if they had included some student assessments in relation to the text. The results of this study are mixed. While traditional textbooks provide students with feedback and more practice problems, the constructivist textbooks relate the text back to real world applications and one lesson builds on the other.

Huntley et al. (2000) sought to analyze the current Core-Plus Mathematics Project. In an attempt to get multiple responses to the survey, the researchers sent letters to teachers from 36 different high schools that are considered field test-sties for the Core-Plus Mathematics project (CPMP) asking for teachers to volunteer to be a part of their study. Out of the 36 schools, a total of 6 schools responded and agreed to participate: one in the Northwest, one in the South, two in the Midwest and two in the Southeast. Each site for the research consisted of two CPMP teachers and up to three control teachers. The number of student participants varied, but it ranged from 90 to 180 per site. The way in which students were group varied by site, some were randomly chosen other students were paired based on ability.

Huntley et al. (2000) collected both qualitative and quantitative data. The researchers designed and implemented survey questions for the teachers to get a better understanding of the teachers' instructional practices and curriculum. In addition, they develop a variety of procedures to assess the student's mathematical skill, and problem-solving abilities. The researchers conducted the study and collected the data between April and May of 1997. In addition to the surveys, the researchers conducted interviews lasting about one hour in length. Students completed the assessments in three parts. Part one included students having 50 minutes to complete and had the ability to use scientific or graphing calculators. Part two of the test students had 20 minutes to complete and could not use calculators of any kind. For Part Three of the assessment students were paired and each pair was given 20 minutes to complete one problem. Students had the ability to use calculators, graph paper, and rulers. Each pair submitted one answer.

Huntley et al. (2000) found that the CPMP students (n = 320) outperformed the control (n = 273) in parts one ($t_{570} = -5.69$, $p < 0.001$) and part three ($t_{370} = -2.77$, $p < .01$). In relationship to algebraic symbol relationships, the control group scored higher than the CPMP group. When students had to use strict calculations with only paper and pencil to solve problems, the control group achieved higher results than the CPMP group. The researchers concluded that students learn more about the areas that are emphasized in the classroom, so the materials and texts that are provided do matter. The researchers found that students who are taught how to solve problems with the use of technology and quantitative problems are more able to solve problems that they might not be as familiar with. The researchers did not assess which way of mathematics teaching is the right way to teach, but concludes that there are pros and cons to both sides. While the in depth analysis of the CPMP curriculum was informative and useful, the research would have been useful if the researchers would have supported one side or the other. No formative conclusion was made as to the effect that this might have on student achievement. However, they did mention that is the material is made relevant to the students life than they would be more actively engage in the material, pointing towards a more constructivist approach to teaching.

In a comparative research project, Cramer et al. (2002) sought to compare Commercial Curriculum (CC) and the Rational Number Project fraction curriculum (RNP) to assess the achievement of students. A total of 66 classrooms from fourth and fifth grade in Minneapolis were used in the study. Out of 200 teachers that were asked to participate, 66 responded. Teachers that participated in the study were not allowed to teach fractions until the time the study started, received one graduate quarter credit from the University of Minnesota with a $75 stipend, or they had the choice of a $100 stipend with no graduate credit. Teachers using the new curriculum were given the prototype along with classroom sets of manipulatives to keep, teachers who were part of the control received the curriculum and classroom materials after the study was finished. Teachers were randomly assigned to which program they would teach. Half (n = 33) of the teachers assigned to teach the CC curriculum, 27 of the teachers used the 1989 curriculum by Addison-Wesley and 6 used the 1992 Harcourt-Brace series.

Huntley et al. (2000) used three types of analysis in comparing the curriculum. The researchers, used a posttest-only control group, otherwise called a Type 6 design, to assess student achievement and eliminate biases. The second analysis was done with interviews of both the teachers and some of the students. The interviews were to assess the student attitude about fractions and how they were able to manipulate them. In the third process the researchers triangulated the results from an earlier study, a large scale data in the current study and student interviews.

The researchers evaluated the test using MANOVA. The reliability (α 0.0083) for the posttest and retention test was .88 and .90. A statistically significant [$F(1, 62) = 8.74$, $p = 0.004$] improvement was found in grade five in which the RNP outperformed the CC group. No significant difference [$F(1, 62) = .33$, $p = 0.571$] in the fourth grade. The student interview response showed that students who received the RNP instruction believed they were better able to answer conceptually oriented responses than the students in the CC group. In conclusion the researcher believe that the RNP students were able to perform better was because of the type of instruction that they received from their teachers and from the structure of the materials with hands-on activities and the set up of the lessons. The research showed that students who received the constructivist mathematics instruction achieved higher test results.

Section Three: Traditional mathematics

This section investigated research in which the results show better student achievement with traditional instructional methods. Alsup and Sprigler (2003) explored the mathematics curriculum, Houghton Mifflin and Core Applied Mathematics and found that student had higher test results from the traditional, Houghton-Mifflin curriculum. Flores and Kaylor (2007) investigated traditional and constructivist mathematics instruction to determine which instructional methods would provide higher achievement for at-risk students. Carpenter, Franke, Jacobs, and Fennema (1998) conducted a long term study on elementary education students mathematical achievement and found that students performed better with traditional mathematics.

In a quantitative research of two mathematics curricula, Alsup and Sprigler (2003) explored the two mathematic curricula (Houghton Mifflin and Core Applied Mathematics) and the mathematical achievement of 8th grade students on their SAT scores. The study included one classroom teacher and 335 8th grade students from a rural farming community in the Western United States. The students were of a variety of socio-economic status. The goal of the research was to determine any significant difference in the Sat scores between the two programs over the three years that were studied.

Student SAT test scores from the 8th grade mathematics students from the years 1997 to 2000 were collected and analyzed. The Houghton-Mifflin curriculum (traditional mathematics) was used during the 1997-1998 school year, Core Applied Mathematics was used during the 1998-1999 school year and both curricula were used during the 1999-2000 school year. The three school years were divided into three groups: group 1 ($n = 113$), group 2 ($n = 86$) and group 3 ($n = 88$). The test scores were analyzed with ANOVA with a 0.05 significance level, looking specifically for differences in the scores between the three years.

Alsup and Sprigler (2003) found that in comparing the SAT scores over the three years that no significant difference was found, [$F(2, 224) = 1.86$, $p > 0.05$]. The highest mean came from group number 3 ($M = 48.80$), then group

1 (M = 48.25) and group 2 (M = 45.30). When analyzing just the procedure section of the SAT test, group 1 had a significantly ($F[2, 284]= 11.72$, $p < 0.05$) higher mean score, (M = 17.18) when compared to group 2 (M = 13.38) and group 3 (M = 16.18). The researchers concluded that the test scores do not show any significant improvement in achievement when all areas of the test where compared. In addition, the traditional mathematics program of Houghton-Mifflin was found to have a significant improvement in achievement in relation to mathematical procedures when compared to the more constructivist Core Applied Math curriculum. Alsup and Sprigler showed strength in their research recording results over a period of years and comparing two specific areas of the test. However, with limiting their study to just one school, teacher, and school it limits their scores. If the researchers have chosen to expand the search to other schools that use the same curriculum would have provided a more reliable result. The researchers concluded that students who used the traditional Houghton-Mifflin curriculum had more significant improvement in achievement scores than the students who used the constructivist curriculum.

Flores and Kaylor (2007) conducted an exploratory study to investigate the how direct instruction effects the mathematical achievement of at-risk students. None of the students in the study had been diagnosed with a mathematics learning disability, but were considered to be at risk for failure. The study of seventh grade students from a rural city in the Southwest United States took place over the course of seven weeks. The students attended math class in addition to the regular mathematics instruction. All students in the study had failed the annual state-designated assessment at least two times. The participants in the study (N= 3 0) ranged in age from 12 to 14 with 11 females and 19 males. The majority of the students were Hispanic (n = 18) followed by White (n = 6) and African American (n = 6). The instruction that the students received occurred in three sections of the general remedial math course in a period over seven weeks. The instructors used the text *Corrective mathematics, basic fractions* published in 2005. All students received the direct instruction method two days a week and traditional instruction method for two days a week.

Flores and Kaylor (2007) found that the results of the pre-test showed range of 0-57% (M = 20%). The researchers did not give the results were to the students. The post-test results had a much higher range of 36-100% (M = 77%) with the majority of the students scoring above a 75% on the posttest. The researchers used a t-test to compare the overall data from the students. The results showed a significantly difference (t-score = 16.224, p = 0.005) between the pre and post test scores showing a remarkable improvement in the students mathematical achievement given the traditional teaching methods. Many of the students were able to improve their score by 50% or more. In addition, the researchers were able to observe the students behavior during both traditional and direct instruction. The researchers noted that the students appeared to be more engaged and more on task in the learning during direct instruction. The researchers concluded that at-risk students benefit from the traditional instruction and curriculum and improved their test scores from previous assessments using other instructional methods.

To investigate the long term results children's mathematical achievement, Carpenter et al. (1998) conduct a three year study that focused on the comprehension in elementary mathematics students in regards to multi-digit number concepts and operations. The three year study followed students from first grade to third grade to assess their conceptual understanding through a series of interviews. More specifically, the goal of the research was to observe and distinctions between students who use a traditional method to solve problems or students who use a more constructivst approach (inventive strategies) to solve problems. The original study started with 120 students and with moving and district transfers there was a final number of 82 students. The students chosen came from three different schools and 10 different first grade classrooms. Out of the three schools, the first school was rural with a 99% white population, the second school had a 70% white population (location not stated) and the third school had a 91% white population (location not stated). The teachers of the students that were in the study were all participants in a three-year intervention study to help teachers develop the skills needed to enhance children's mathematical abilities. The teachers received no specific instructions on how to create their lesson plans. Many of the lessons were based around word problems and students were able to use a variety of methods to solve problems, including using base ten blocks.

In conducting the interviews, Carpenter et al. (1998) looked for four main components: 1. Student's knowledge in base ten concepts, 2. Students strategies for solving addition and subtraction problems, 3. The abilities that students have developed in using invented strategies, and 4. Students to were able to extend the procedures for addition and subtraction problems easily. When the interviews were conducted, the students were interviewed individually and it lasted approximately an hour. Each child was verbally given a mathematical problem to solve and as much time as necessary to solve the problem. The results of the testing was put into categories and analyzed. The results were categorized into four groups: Cumulative use of invented strategy, percentage of students that used inventive strategies, performance of each group in base ten problems and percentage of students using Buggy algorithms.

Carpenter et al. (1998), interviewed the children (n=82) once when they were in grade 1, twice in second grade and twice in third grade. With the first interview that was conducted, the children used inventive strategies to solve problems 29% of the time for addition problems and 0% of the time for subtraction problems. By the end of the third grade with the last interview, the students had progressed to using inventive strategies for problem solving 88% of the time with addition problems and 68% with subtraction problems. In addition, the largest progression from standard algorithms to inventive strategies was seen during the second grade year between the fall and spring interviews. To be able to compare the ways in which students answer mathematical problems the group was divided into three sub groups: invented (n = 27), invented addition (n = 33) and algorithm (n = 18). The researchers categorized the information that they collected into categories such as: used inventive strategies, used inventive strategies during some interviews, used inventive strategies by fall of second grade, used algorithms by the fall of second grade and used inventive strategies in third grade interview. The invented strategy group (n = 27) used the inventive strategy method for 70% of the time when solving problems by the end of the second grade. In addition, when solving algorithms, they would continue solving with inventive strategies. By the end of the third grade, students would only use inventive strategies when a paper and pencil were not available.

The groups of students in the algorithm group (n = 18) relied primarily the algorithm methods and 94% of the students did not use inventive strategies at all by the end of the third year. In addition, many students were unable to use inventive strategies when they were asked to. When compared to the inventive strategy group, the invented-addition strategy group (n = 33), the invented-addition strategy group most students began with using invented strategies, but by the end would gravitate more towards the algorithmic method of solving problems over invented strategies.

Carpenter et al. (1998) compared the differences between the two invented-strategy group and the algorithm group in investigation the student's knowledge of base-ten number concepts and extension problems. The comparisons of the three groups were analyzed with a 0.05 significance level that was split among the three groups with each group tested at 0.017. Students in the invented strategy groups showed a significant (α .05) number of students were able to better understand the base ten concepts when compared to the algorithm group. The invented strategy groups obtained significantly different results in relation to the base ten number concepts than the algorithm group.

($p < 0.05$). Most students in the invented strategy group were able to demonstrate knowledge in base ten blocks, only one-third of the students in the algorithm group were able to do so.

Carpenter (1998) compiled the results of how often students used faulty algorithms, also know as buggy algorithms, when attempting to solve a problem. It was found that there was more faulty buggy algorithms were found with subtraction algorithms than addition algorithms by 30%. In addition, students in the algorithm group had significantly more faulty buggy algorithms in more than one interview compared to the inventive strategy group. In addition, the students had buggy algorithms on many of the same problems. While Carpenteret al. concluded that there is no sequence in which students used inventive strategies to solve addition problems. In addition, children had a harder time using inventive strategies for subtraction problems. Finally, that students who engage in the inventive strategies, had an easier time to connect their prior learning to the problems and they showed fewer errors than the groups of students the relied on algorithms to solve mathematical problems. Carpenter et al. showed strength in the length of time that they took to complete the study to fully be able to observe the effects over time that certain teacher practices has on student learning. However, their lack of providing any guidelines for the teacher show a weakness in how do they know what and how the students were being instructed during the week. Students who used constructivist methods to solving algebraic problems had an easier time completing problems with fewer errors in their calculations which would lead to higher mathematical achievement.

This section investigated both traditional and constructivist mathematics instruction, with the main focus being on traditional mathematics instruction. The main focus of the researchers of these studies aimed to prove that traditional mathematics instructions yielded better results. The results of the findings had surprising results with only one study that proved that traditional mathematics instruction improves student achievement. Flores and Kaylor (2007) found that at risk students perform better, have higher student achievement and are more attentive in class when given direct instruction with traditional teaching methods. Alsup and Springer (2003) presented strong results with comparison of student achievement and results over a two year period comparing both tradition and constructivist mathematics text books. The researchers sought to prove traditional teachings achieved higher SAT scores, however they were unable to provide any significant differences between the two curriculum. Carpenter (1998) found that real world connections had a positive impact on student learning.

Section Four: Constructivism and Traditional mathematical approaches

The research in this section analyzes both traditional and constructivist mathematics instructional methods. In several different investigations, Arra and Bahr (2005); Chung (2004); Clark (2001) ; Elliot, Oty, McArthur, and Clark (2001) and Hollar and Norwood (1999) did not find any significant differences in the assessment results with either constructivist or traditional mathematics. Jennings and Pawat (1997) investigated the teacher practices of both traditional and constructivist teachers and found most teachers favored the traditional methods in which students learn from the teacher.

In a study on the effects of graphing calculators, Hollar and Norwood (1999), extend a previous study completed by O'Callaghan (1998) that compares computer intensive algebra and process-object to investigate the effects that TI-82 graphing calculator has on a student's understanding of intermediate algebra. The participants in this study were enrolled in the intermediate algebra. Out of a sample of 28,000 students who scored the lowest on the mathematics placement exam, 90 students participated in this study. The students were split into two groups, treatment group (N = 46) and control group (N = 44).

All students used the college text Intermediate Algebra: A graphing approach published in 1995 along with the TI-82 graphing calculators. The students were able to use the graphing calculators in class and when completing homework, but not on the O'Callaghan Function Tests or the traditional final examination. All students took the O'Callaghan Function tests as both a pre-test and post-test to assess conceptual knowledge in these four areas: modeling a real-world situation, interpreting a function in terms of a realistic situation, translating among different representations of function, and reifying functions. All students received a 50-question test of conventional algebra skills to evaluate students on their traditional mathematical skills. The researchers also used the Revised Mathematics Attitude Scale to evaluate the students' attitudes toward mathematics with both a pre-test and a post-test. The experimental and treatment group compared randomly selected students from both morning and afternoon classes (N = 46). In addition, students in the experimental section had the option to switch to the control group, however; none chose to do so. Class profiles provided the researchers with data about the students' gender, ages, mathematical background, ability in mathematics and GPA. The ANOVA test calculated the characteristics of the students. In addition, the function test calculated the results using a MANOVA on the four scores and total score.

The researchers found that when analyzing the student results on the O'Callaghan Function test using MANOVA, that the experimental group had a significantly ($F[4,69] = 4.68$, $\alpha = 0.01$) higher mean ($M = 21.02$, $SD = 5.87$) than the control group ($M = 15.62$, $SD = 4.70$) in regards to their comprehension of functions. When the test results were analyzed with the ANOVA scoring system, there was a significant difference ($\alpha = 0.05$) in favor of allowing students the use of graphing calculators in enhancing their conceptual abilities of mathematics. The test results showed no significant differences in the test results in regards to the final examination between traditional final and experimental and control groups. In regards to the student attitudes on mathematics, there was no significant difference in regards to attitude between instructors or genders on the classroom. However, the students in the experimental group showed a slightly positive attitude in regards to mathematics and their mathematical abilities. Hollar and Norwood (1999) concluded that students that were comfortable and able to use the graphing calculator were better able to relate the information to real world situations. In addition, graphing calculators would be a practical application in the classroom because they would help make sense of mathematical computations that would be long and drawn out when computed by hand. Hollar and Norwood exhibited strength in their ability to combine several tests to analyze the students' results. However, there was a weakness in that one of the tests required that no graphing calculator be used when the students had been instructed on how to complete mathematical problems using the calculator. Their research would have been stronger if there was further collection and evaluation of test results from other ethno-graphic areas. No significant differences found between students who received traditional or constructivist mathematics instruction in relationship to their mathematical achievement.

A quantitative study on the acceptability of mathematics, Arra and Bahr (2005) compared three main teaching approaches in mathematics, which they defined as cognitive, behavioral, and traditional. The research involved teacher candidates and fourth grade students in an effort to compare the teacher candidates' beliefs about mathematics and the students' beliefs. All of the teacher candidates selected were seeking elementary education endorsement and had only the student teaching portion of their education remaining. There were 18 candidates in all, comprised of 16 women and two men ranging in age from 18 to 39. The ethnic background of the group was a white majority, with 16 white and 2 African American teacher candidates. The students that were selected to participate were a part of the practicum experience for the teacher candidates. A total of 55 students participated (27 male, 28 female). The students that participated were from low to middle socio-economic status and were all fluent in English. The ethnic makeup of the students included white (82%), African Americans (15%), Hispanic (2%) and Native Americans (2%).

During this study, the teacher candidates instructed groups of three students in mathematics. The research process included 4 survey conditions: Teacher candidate demographic questionnaire, Teacher Candidate questionnaire, student questionnaire, and to ensure accuracy within the research the treatment integrity measures was used. The total length of the study until completion was eight weeks. At the beginning of the study all teacher candidates filled out the demographic questionnaire as well as the teacher candidate questionnaire. At the completion of the study, the teacher candidates repeated the questionnaire. The researchers used a modified version of the Intervention Rating Profile (IRP) with a 20 likert – type items with a 6 point scale to measure the acceptability rating on the questionnaire. Different Conbach's Alphas were used for the three types of instruction, Behavioral (.72), Cognitive (.76) and traditional (.92). In assessing the questionnaire from the students, a modified version of the Children's Interventions Rating Profile (CIRP) was used. The survey consisted of a 7-item scale to assess the student's acceptability to the specific teaching approaches in mathematics. The researchers found three ranges for Cronbach Alpha according to instructional type used, behavioral (0.79), cognitive (0.71) and traditional (0.55). In addition, two different integrity treatments were used to ensure the accuracy of the instruction, one the teacher candidates filled out themselves, and the other the researcher filled out while observing the teacher candidate for 10 out of the 216 sessions. Before the intervention session on each topic the teacher candidates received two hours of instruction. The researchers trained the teacher candidates on the specific instruction methods as well to include cognitive, behavioral and traditional. For both of the IRP and the CIRP, the individual item scores compiled to create one total score. To analyze the data collected the researchers examined the means and standard deviations of all items from the IRP and CIRP for any possible patterns. Secondly, all scores from the IRP and CIRP compiled to create a total score to be used as a unit of analysis for group differences.

Using the analysis of variance (ANOVA) showed a nonsignificant ($F[2,13] = 0.408$, $p = 0.673$, Cohen's $f = 0.059$) three-way analysis. This analysis shows that the teacher candidates view all of the instructional practices to be equally acceptable with no preference to any one way. The student responses on the CIRP again showed a nonsignificant ($F[2,49] = 0.784$, $p = 0.462$, Cohen's $f = 0.071$) three-way analysis. This analysis concluded that the students have a highly positive attitude towards mathematics and had a positive attitude for all three instructional approaches. While the study did not present any specific finding towards one mathematics instruction over another, it did present the subjects with a variety of methods to get acquainted with the different types of mathematics. This study would have been able to provide stronger evidence if it had been conduced for longer than eight week and in more than one grade band. The researchers found that there was no significant difference over which preferred instructional method students preferred.

In a study on the effect of algebra and science on student's problem solving skills, Elliot et al. (2001) conducted a quantitative research on eight college level algebra classes (75 subjects in college algebra and 68 in algebra for the sciences). The study included 143 students (125 female, 86 male). The students were chosen at random from the class roster on the first day of class using a random number generator. Students that were chosen to participate in the algebra for sciences course were unaware of the change and believed it to be that same as the college algebra course and the instructors continued to refer to the course as college algebra. Caucasian students (82%) made up the majority of the students followed by, Native Americans (11%), African Americans (4%), Hispanic (2%) and Asian (1%). The mean age of the students was 21.4.

Elliot et al. (2001) choose to measure three areas of student achievement in mathematics: problem-solving skills, critical thinking skills and attitudes toward mathematics. To measure the student's problem solving skills, the researchers gave the students an exam at the end of the course that focused on common questions to solve from college algebra. A t-test analysis compared the results of the two classes. The Watson-Glaser Critical Thinking Appraisal (WGCTA) analyzed the student's critical thinking skills, consisting of 80 multiple-choice questions. Each student completed the test in 40 to 50 minutes. Independent T-tests analyzed the differences between the courses. Student evaluations at the end of the quarter analyzed their attitudes towards mathematics. A chi-Square test determined the differences in attitudes between the two classes.

Elliot et al. (2001) found two significant differences in problem-solving skills at the 0.10 levels but not at the 0.05 levels. The college algebra students outperformed ($p = 0.0752$) the Algebra for sciences students in solving exponential equations. In solving logarithmic equations, the Algebra for sciences

($p = 0.0883$) outperformed the college algebra students. However, the researchers noted that this difference could be the results of a Type I error and that little significance in the differences could be inconsequential. The algebra for sciences students had a significantly higher ($p = 0.0492$) difference in relations to the critical thinking scores. In relation to the students attitude towards mathematics, the researchers reported that the algebra for sciences students had a significantly higher attitude towards mathematics, in that they thought math was more interesting ($p < 0.005$), practical ($p < 0.005$) and had more positive attitude at the end of the course ($p < 0.05$). Elliot et al. had a strong sample size with a variety of students. It is interesting to note to wonder the reasoning about not letting the students know that they were going to be instructed in a different manner and any effect that this might have on their achievement and perception on this course. The algebra for science students had higher scores on their achievement in relation to critical thinking. However, the researchers failed to note any differences between instruction in the two classes and if it was taught using traditional or constructivist mathematics.

In a comparative study of traditional and constructivist mathematics, Chung (2004) sought to determine which instructional method would have the greatest impact on student achievement. The 71 students that participated in the study were from two public schools from the St. Louis Public School System. The study consisted of 42 female students and 29 male students. The public schools included students from both the middle to low class socio-economic status.

The researchers divided the students into two groups, in which they would receive two sets of instruction. The first class of students received instruction using the constructivist method for teaching mathematics, the second class received instruction using traditional mathematics methods. A quasi-experimental, 2 x 2 factorial, pretest and post-test evaluated the students achievement. In addition: the Stanford Diagnostic Mathematics test, KeyMath: Diagnostic Inventory of Essential Mathematics and a Researcher-made test consisting of 10 open ended questions determined the students results. The test score results were ANAOVA analysis of variance ($p < 0.05$).

Chung (2004) found that all students improved their comprehension in multiplication skills and multiplications concepts. There was no significant differences between the three test: Stanford Diagnostic Test ($F[1,38] = 2.59$, $p > 0.05$), Key-Math Revised ($F [1,38] = .87$, $p > 0.05$) and Multiplication Survey ($F [1,38] = 1.99$, $p > 0.05$). While no significant change was found, the researcher believed that teacher effectiveness in being able to teach using constructivist methods directly impacts student achievement. Chung exhibited strength in choosing a range of students from different socio-economic status. Extending the quantitative study to include how children transfer knowledge could have strengthened the study. In addition, repeating the study with a larger sample size would offer more credibility to the results. Furthermore, specific training for the teachers on how to teach mathematics constructively would be beneficial to the study. The researcher found no significant changes in student achievement. However, not all of the teachers instructed the students effectively using constructivist teaching methods which the researchers believed could have a direct impact on student achievement.

Teacher case study on mathematical beliefs

In a five year longitudinal study, Jennings and Pawat (1997) recorded their results of teacher practices and beliefs of two upper elementary school teachers in California. The two teachers chosen as the focus for the study were a small part of a larger study of 24 teachers in California. The study focused on the educational reform movement in California towards a more constructivist mathematics curriculum. Both of the teachers had years of teaching experience (Scott = 18 years, Hill = 25 years). Hill teaches in an upper to middle class, mostly white school district in southern California. Scott taught in a school district about one-quarter the size of Hill, upper class, mostly white school district in California.

Between 1988 and 1992 Jennings and Pawat (1997) conducted interviews and observations twice a year. During the 1993 to 1994 school year they conducted one visit. For each visit, the researchers observed the instructors teaching two math lessons and then followed up with interview questions. In addition, Jennings and Pawat collected audio recordings of classroom discussions during instructional periods. The researchers transcribed the lessons and observations, focusing on the flow of the lesson with the changing of activities and mathematical topics. To guide the observations Jennings and Pawat used a set of analytical questions focusing on classroom instruction focusing on the changes from traditional to constructivist mathematical teaching.

In addition to the observations, the researchers completed detailed interviews with the instructors. Each interview was roughly two to three hours in length and each interview was transcribed verbatim. The focus of the interview was around the teachers beliefs in response to the mathematics reform movement in California.

During the initial interview, Jennings and Pawat (1997) concluded that both Scott and Hill had similar beliefs about how student learn mathematics. They both believed that students learn mathematics through traditional measures and memorization. Through the four years of the study, both teachers slowly implemented more of the constructivist design, however; they both showed reluctance to change. Both strongly believed that their way of teaching is what is best to fit the students' needs. While it was stated that these two teachers were part of a larger collaboration, no other information was provided to provide comparison. In addition, both of the teachers in the study appeared to have similar views so it did not provide any contrast to instructional beliefs. Through the results of the case study, the researchers concluded that the teachers believed that students would have higher mathematical achievement using traditional mathematics teachings.

In one case study conducted by Jennings and Pawat (1997) the researchers recorded and observed both constructivist and traditional mathematics teachings. The study showed that the teachers believed that traditional mathematics teachings resulted in better student achievement. The participants in the case study attempted constructivist teaching methods but showed resistance to change and did not believe it to be more effective. Hollar and Norwood (1999) sought to find if student achieved higher results in algebra using a graphing calculator. They found that students in the experimental group that used the calculators in math had a higher positive attitude towards mathematics and their abilities. However, no difference in their academic achievement was found. Arra and Bahr (2005) found no differences between which instructional method, constructivist or traditional, that students preferred. In researching two algebra classes in a regular college algebra class and a algebra for science class Elliot et al. (2001) found that the college algebra students achieved higher results in solving exponential equations and algebra for science students had higher achievement in critical thinking. No information on how the students received instruction was given. Chung (2004) believed that students had better achievement when instructed using constructivist methods, however, their results proved to show no significant changes in student achievement. Chung believed that this was a result of teacher error in not teaching constructivist mathematics effectively.

Section Five: Mathematics In Relation to Learning Disabilities and Student Achievement

This section analyzes the effect that different mathematics instructions has on student achievment. Groebecker (1999) examined the learning styles if two student concluded that one student would benefit from constructivist teachjng methods while the one would benefit from traditional instruction. Three different studies Geary, Hoard, Bryd-Craven, Nuget and Numtree (2007); Mazzocco and

Thompson (2005); and Witzer, Mercers and Miller (2003); found that there was no significant differences between students with a mathematics learning disability and regular attending students on assessment scores.

In a study of two students with learning disabilities, Grobecker, B. (1999) examined the differences between information processing and constructivist teaching in relation to knowledge and the interaction of the learner during the assessment process. The study was comprised of two 6th grade students, (one male one female) considered to have a learning disability. The male student has received instructional support in addition to the regular 6th grade curriculum. The female student received separate mathematics instruction that replaces the regular 6th grade math curriculum. At the time of the study she was working on multiplication with two digit multipliers and long division with one digit divisors. Both students were being taught with a traditional mathematics approach and receive extra help in mathematics daily. No other information regarding the students was given.

Grobecker, B. (1999) gave both students a series of story problems to complete that were about flowers. Both students completed the same questions with time as needed to solve the problems. Both students completed the tests individually instead of working together.

In analyzing the word problems results Grobecker, B. (1999) concluded the ways in which each student solved the problem and what type of instruction would benefit the student. The research showed that the female student used life experiences and prior knowledge to help her approach how to solve a math problem. The researcher determined that she would benefit by working with a peer who is at the same mathematical level as her to expand her logical-mathematical structures. The male student used the trial and error approach and the researcher felt that he would benefit from activities that help to enhance his logical thinking skills. In addition, he needed to expand his knowledge on two-step problems and to be able to relate the problems to his life experiences to bring meaning to the problems.

While the study was strong in its historical background and the reasoning behind completing the study, it was lacking in specific information about the students and why they were chosen. In addition, it did not state where the students were from, their socio-economic status or ethnicity. In addition, there is no scientific backing to the study and is limited to the two students in the study and the two questions that were asked. The results of the study were mixed. The female student in the study would benefit and have higher achievement from constructivist teaching methods while the male student would benefit and have higher achievement from traditional teaching methods.

In an attempt to evaluate which algebra mathematics curriculum best fits students with learning disabilities, Witzel et al. (2003) analyzed the effectiveness of a new algebra curriculum that presents traditional mathematics curriculum versus the hands-on constructivist curriculum. In an initial study that included twelve classrooms and 10 teachers from the Southeastern United States, 358 6th and 7th grade students participated. Out of the 358 students, the study included 68 students with learning disabilities. All students in the study were characterized as having a learning disability or were considered at risk for having algebra difficulties. The researchers grouped the 68 students into pairs based on achievement score, age, pretest score, and class performance. The two groups included a comparison group and a treatment group. In addition, no students from the same classroom were paired together to account for the differences in teaching styles. The same mathematics teacher taught all students in the study and the pair together. If one of the students in the pair moved, missed class or test, then both pairs were lost from the analysis.

To accurately measure the student's ability on the acquisition and maintenance of knowledge, Witzel et al. (2003) created a test of 63 questions that was taken from the classroom curriculum. The students completed the test in the same form as a pretest, posttest and assessment follow-up test. The students completed the posttest on the last day of instruction and the assessment follow-up test was given three weeks later. The researchers using the coin toss method randomly selected the instruction for the classroom teacher to follow. The researchers choose to include the class that followed the explicit algebra curriculum (CRA) and considered the treatment group and the other group was considered the comparison group. Both groups received the same content with both classrooms and the same amount of class time of 50 minutes was given. The main difference between the two groups was that the treatment group used manipulative during instruction.

Witzel et al. (2003) found that there were no differences between the pretest scores ($F(1,33) = 31.98$, $p < 0.01$) between the two groups. Both groups showed significant improvement ($F(2,66) = 13.89$, $p < 0.01$) in single-variable algebraic equations, the students who received the CRA (constructivist) instruction scored higher (56.27%) than the traditional students on both the posttest and the follow-up assessment. To ensure accuracy between the two tests ($\alpha = 0.05/3 = 0.017$) was used to ensure a 95% confidence level. Neither group showed any significant change from the posttest to the follow-up assessment. Witzel et al. concluded that students with learning disabilities have shown to be able to retain mathematical information better and have higher academic achievement over time when they are taught using the CRA curriculum. By having the students grouped into pairs scaffolding of a more capable peer that would enhance the students learning and achievement. In addition, by grouping the students across teachers the researchers were able to account for the differences seen between teaching styles. A weakness of this study is in the small sample size, the research would have been stronger with a larger sample.

In order to develop a stronger understanding of how to predict when a child might have a mathematics learning disability. Mazzocco and Thompson (2005) conducted a four year longitudinal study on kindergarten students. The subjects for this study included 226 children out of which 210 were able to complete the study all four year. In total, at the end of the four-year study, 209 students complete that participation (103 boys and 106 girls). During the course of the study, some students had to repeat a grade, either kindergarten ($n = 2$), first grade (n = 6) or second grade (n = 1). All students in the participation were from one school from one school district.

Each year during the study Witzel et al. (2003) tested each participant two to three times per year. A standardized test measured the students' basic math skills, reading-related and visual-spatial skills. The researchers focused on how the assessment measures in Kindergarten were related to math achievement in second and third grade. The student's math performance skills were assessed using a variety of methods: the KeyMath-Revised achievement tests, the test of Early mathematics Ability-2, and the Woodcock Johnston-Revised Math Calculations subtest. The W-J-R Letter Work Identification subtest was used to analyze the kindergarten reading-related skills. A set of four subtests were used to analyze the students motor-reduced skills. To be able to determine if a student has a math learning disability, the researchers needed a basis for comparison the researchers used a 10th percentile and relied on the TEMA-2 and W-J-R calculations. All of the data collected was analyzed using the statistical software package (STATA 7.0). A univariate analysis was performed on the independent variables. The continuous test scores were analyzed using a t-test.

No statistical differences were found between the Mathematical Learning disability student and the non-mathematical learning disability student in the second and third grade, the researcher excluded these results from the rest of the study and there was no significant difference. Mazzocco and Thompson (2005) found a correlation [X^2 (4, n=202) = 9.85, $p < 0.05$) between the students with the Mathematic Learning Disability and the education of the mother, the higher education the mother received, the less likely the child was to have a Mathematics Learning Disability (< 3 vs. ~ 15%). The analysis of the spatial test scores indicate that students that are unable to develop numeric skill at a young age are more likely to have lower math achievement at a later age. No definite answer was able to be produced to determine if a child with a mathematics learning disability might also have a reading disability. The researchers conclude that being able to predict then a student might have a mathematics learning disability as a main factor in the child receiving intervention sooner and building their math skill. While the researchers had a large and random sample size, their analysis of the results of the test scores was weak.

In a comparative study between mathematics learning disabled (MLD), low achieving (LA) and typically achieving (TA) students Geary, Hoard, Byrd-Craven, Nugent, and Numtee sought to find the cognitive ability of MLD students in comparison to LA and TA students. In an analysis of test score of 278 students from 12 different elementary schools, 15 were found to have a mathematics learning disability (n=15). A comparison group a sample of LA (n=44) and TA (n=46) was used for comparison. The Mean age for the children in the study was 6 years. The ethnic background of the students in the study consisted of a majority (67%) white populations with lower sample of African Americans and Asians. No significant difference [X^2 (2) = 2.00, p > .50) between the minority students in the TA and LA groups. However, there were more African American students and fewer Asian students in the MLD group.

Students abilities were measured on intelligence, number sets, counting knowledge, addition strategy, working memory, phonological loop, visuopatial sketch pad, and speed of processing. The students were tested on their abilities twice, in the spring of their Kindergarten year and during the fall of their first grade year. During the spring assessments, students were tested on achievement and intelligence and on speed of processing and mathematical tasks during the fall. Each of the testing sessions lasted for approximately 40 minutes. During the spring testing session the following assessment were used: Word Reading and Numerical Operations tests, Progressive Matrices in Kindergarten and Vocabulary and Matrix Reasoning in the First Grade. In addition to the fall and spring assessments, an additional test was given to the students over the summer between Kindergarten and First Grade. The WMTB-C test was given to the students from a van and each student was given approximately 60 minutes to complete the exam.

An overall $\propto$ of .01 was used to analyze the results of the testing. The researchers found there to be a statistical difference [F (1,102) = 25.21] between the student IQ testing in the first testing session, but not for the second. [F (1, 102) = 1.05; ds = -1.1 and -0.8. The two achievement tests showed a significant difference [Fs (1,101) > 25.00] with the MLD group have a much lower (ds < -1.3) score than the LA (d=-0.7) and the TA group (d = - 0.8). In addition, the MLD (ds = - 2.5) students showed a large achievement gap in relation to numerical operations compared to the LA (ds = -1.0 and -0.9) group and the TA (ds = =1.5, an -1.5) group during both years. The students were analyzed on their ability to correctly [Fs (1,100) > 12.00] identify number sets and the amount of times they missed [F (1,100) = 8.75]. It was found that the TA students were more accurate, and performed faster than the LD students and the LD students performed better than the MLD students. The results showed that the MLD students scored lower than the LD and TA students in all areas of cognition, memory and processing. This study provided detail analysis of any areas of the students mathematical abilities, and was able to provide a detailed analysis. The research would have been stronger if there were more than two test times and expanded over a longer period of time. In addition, the sample size for the MLD students was considerably smaller an the LD and TA students that could have skewed the results.

Groebecker (1999), Witzel et al. (2003) and Mazzocco and Thompson (2005) investigated the mathematics achievement in relation to students with learning disabilities. Groebecker focused the study on two students (one male and one female) with learning disabilities and found that the female student would benefit from constructivist teachings while the male student would benefit from a more traditional approach to mathematics. Witzel (2003) compared a new algebra curriculum that focuses on the traditional approach to constructivist mathematics and found that the students with learning disabilities had a higher achievement when engaged in the constructivist curriculum. Mazzocco and Thompson (2005) sought to understand how to predict when a child might have a learning disability in mathematics. To investigate this question they conducted a four year study on kindergarten students. The researchers found a correlation between the students with a mathematics learning disability and the education of the mother, the higher the education of the mother, the less likely it was for the student to have a learning disability. The research showed the students with a mathematics learning disability scored lower in the areas of cognition, memory and processing. The researchers believed that being able to predict when a student might have a learning disability will enable the student to receive help sooner.

Section Six: Mathematics assessments' around the world

The studies in this section analyze the effect that constructivist and traditional mathematics teachings has on students in other countries, Greece, Korea and the Netherlands. Barakats and Malone (2005) explored the effect that teacher belief systems in effects the mathematical achievement of secondary mathematics students in Greece. In addition, Kim (2005) observed what effect constructivist teaching methods has had on the mathematical achievement of students in Korea. Kroesbergen (2004) continues with evaluating the effect of constructivist teaching for low income students in the Netherlands.

In a quasi-logical study in Greece, Barkatas and Malone (2005) sought to fine the beliefs of secondary mathematics teachers in relation to the learning and teaching of mathematics and to observe if there were any inconsistencies between teacher belief and practice. The study was broken in to two portions, a collection of surveys and one case study. The case study included a random selection of 600 7th to 12th grade mathematics teachers. Out of the 600 chosen, 465 completed the survey, (276 males, 145 females and 44 with no gender specified). Out of the 465 teachers that completed the survey, 24 were secondary school principals, 10 were regional mathematics consultants. The case study included one female teacher in her late 40's who has been teaching in Greece for 22 years. She has taught students from 10th to 12th grade and from a range of socio economic status. During the time of the study she was teaching mathematics for the 10th and 11th grade.

Between 1999 and 2000, 600 surveys were sent out with a return rate of 75% (n=465). The survey was a 34 item questionnaire that covered beliefs about mathematics, beliefs about mathematics learning and beliefs about mathematics teachings. The responses to the survey were analyzed using the SPSSwin Principal Components Analysis, Cluster Analysis, Multiple Discriminant Analysis, and Trends Analysis. The survey results were analyzed with a Cronbachs alpha ($p < .01$) to ensure that the items comprising the results produces a reliable scale. The case study was a series of questions about her beliefs about mathematics.

In analysis of the survey results, Barkatas and Malone (2005) discovered that 15.3% of the teachers were found to have a socio-constructivist orientation of mathematics. 23.2% of the teachers were found to have a dynamic problem-driven orientation to mathematics. 30.8% of the teacher have a static-transmission orientation to mathematics. 26.2% of the teachers had a mechanistic-transmission orientation to mathematics. 4.5% of the teachers were found to have a cooperative orientations to mathematics. The researchers concluded that you can not separate how teachers feels about mathematics and how mathematics is taught. The teacher's beliefs about mathematics could not be separated from their beliefs about teaching and learning mathematics. The findings suggest that the broad social and cultural climate of the classroom may impact teachers' espoused beliefs on mathematics, mathematics learning, teaching and assessment. The case study research reviled that her school experience and her teaching experience were the main influences on her beliefs about teaching, learning and assessment. The case study held a socio-constructivist nature in the teaching of mathematics. The study concluded that teachers can not be expected to teach in the social constructivist way unless they already have those views.

While the study provided a strong base with their larger sample size, because the survey was mail in and optional to respond is a weakness. In addition, the case study portion was weak and did not include any information as to how the case study was conducted, who conducted it, how they developed the questions or how it was recorded. The results could have been stronger with more complete information on the participants and a larger case study.

In a study on student achievement in mathematics, Kim (2005) sought to find what effect constructivist teaching would have on the academic achievement of student in Korea. The study included a total of 76, 6th grade students. The group of students was divided into two groups, an experimental group and a control group. The Experimental group consisted of 21 males and 17 females. The control group consisted of 22 males and 16 females.

The academic achievement of students was observed for forty hours over a nine week period. The students were tested on the mathematical skills in counting, calculating area, volume, ratio and proportions. Both groups were given a pretest then received instruction in the constructivist or traditional teachings and then given a post test. The test results were analyzed using a Cronbachs alpha (range .74 to .81). The rest-retest correlation coefficient ranges from .85 to .93.

Kim (2005) concluded that in overall academic achievement, there was a significant differance ($F=89.11$, $p < .001$) in academic achievement with the students who learned mathematics constructively out performed the traditional instruction group. No significant difference ($F=103.56$, $p<.001$) between the two groups in relation to their self-concept in relation to mathematics. In relation to learning strategies, no significant difference ($F=7.21$, $p < .01$) was found. In addition, the researcher concluded that students preferred to constructivist mathematics teaching.

In an effort to discover the effect that constructivist teachings has had on low-income students in the Netherlands. Kroesbergen (2004) wanted to know if low-achieving mathematics students would benefit more from constructivist teaching which requires the learner to be an active participant in their environment, or from traditional mathematics instruction which involves direct instruction. To investigate this information, Kroesbergen (2004) conducted a study of 265 students between the ages of eight and eleven. Thirteen elementary schools and eleven special education schools participated in the research. The students that were selected to participate in the study were selected based on low math performance skills with a grade below 25%. In addition, for the students to participate in the study, they had to be able to count and/or add to 100 and have no mastery skills in multiplication. Once the students were selected then they were assigned to one out of two groups, either experimental or control. The study was comprised of 153 boys and 112 girls with a mean age of 9.7. While the students ranged in grade level from second and third grades and special education schools, they all were at a similar level with mathematics performance. Out of the 265 students that participated in the study, 22% of the student were of an ethnic minority and were equally distributed across all three test groups. In addition, the students from the special education schools were older than the general elementary school students, (t (263) = 12.796, $p < .001$) and generally had a lower IQ level, (t (263) = 9.7222, $p < .001$). In addition, the boy and girls in the study were not equally distributed across the groups, (x^2 (1) = 34.245, $p < .001$) with more girls in the regular elementary school and more boys in the special education school.

For the basis of the research, students were divided into three groups: Experimental - Constructivist Instruction group (CI) (N=85), Explicit Instruction group (EI) (N=83) and a control group (N=97) that had no change in instruction. To observe any changes each group was given a pre-test, a post-test and a follow-up test three months after completion of the study. Each test included problems that tested the student's mathematical abilities in multiplication, automaticity, problem solving and strategy. In addition, students were received a motivational questionnaire before and after testing. They used the MMQ 8-11 test to measure students motivation regarding mathematics.

The students in the experimental group received a total of 30 lessons in the area of multiplication, a duration of 30 minutes each. The students took the test outside of their classroom for special instruction while the teacher continued with the rest of the class as normal. The special instruction continued for 4 to 5 months. In the times that the students were not involved in special instruction they continued on their regular studies with the rest of the class. In addition, extra time during the instructional practices is made for the students to be able to actively contribute to their learning. The students in the explicit instruction group, the students follow the teacher's direct instruction, and leave little room in the instructional period for student contribution to the learning. Students, who were in the control group, followed the regular instruction all 5 days of the week with no special specific instruction in the area of multiplication. In addition, the majorities of the students in the control group had learning difficulties and were able to receive more instruction and guidance from their teachers. To observe any changes in the student's mathematical achievement, Kroesbergen (2004) administered a total of four multiplication tests, two of the test included problems that involved problem solving, and two of the tests included problems that involved automaticity that is given to the students orally. For the multiplication tests, students are given 20 minutes to complete regular multiplication problems and 20 minutes to complete the problem solving tests. In addition, there were some variances as to how the instruction was taught in the schools. While most schools followed the Realistic Mathematics Education (RME) some teachers did not follow this teaching method and would switch to a more traditional, direct instruction approach to teaching.

The results from the pre and posttests were combined and the paired-sample t tests showed significant improvement on all four tests. The multivariate multilevel analyses of the multiplication tests were measured using four outcome measures ($p=0$). The univariate test results showed that students that had a higher pretest score had a higher posttest score as well. Furthermore, the students that scored higher on the pre-test showed more improvement than those students that scored lower on the pre-test. On the problem solving section of the test the post test results showed that students receiving explicit instruction scored better then the students who received the constructivist instruction ($p < .05$). In addition, the special education student scored lower than the general education students. In regards to the problem solving test, it was found that the boys outperformed the girls.

Follow up tests were given to the students three months after the original tests and after their summer break. Out of the 265 original students, only 250 students were able to be tests in the post test. In general, the students performed at the same level as the previous test, and scoring lower in the areas of automaticity and problem solving. The multivariate analysis showed a significant variance ($p= .011$). Differences were found between the control group and the explicit instruction in that the explicit group used a more diverse means of strategy to solve the problems. There was no significant difference found between the experimental group and the other groups ($p > .05$).

Overall, Kroesbergen (2004) found that explicit instruction proved to be the most effective way to increase student's mathematical achievement in regards to solving multiplication problems. The researcher also found that while both instructional methods could be used to improve automaticity and problem solving skills that low-achieving students benefited from explicit instruction more than the constructivist approach to teaching. The strengths of this study were illustrated in that the ethnic students were separated equally into the three groups and students were taken from several different schools and age levels. However, the weakness of this study was in the grouping of the special education students together and by the uneven distribution in the educational system. In addition, with the experimental group being the only students taken out of the room and taught in a different setting would have different test results. Also, by conducting some of the test orally, it would not be conducive to all learners.

The section investigated the mathematical achievement of students around the world and focused on Greece, Korea and the Netherlands. In the Greece study, Barkatas and Malone (2005) on a study of teacher beliefs, the researchers concluded that teachers would not be willing to teach in a constructivist setting unless they already possessed those beliefs. In Korea, Kim (2005) investigated the results of constructivist teaching on students academic achievement and found no significant differences in the students academic achievement but found that students preferred to be taught mathematics using constructivist methods. Kroesbergen (2004) investigated the results of low-achieving mathematics students in the Netherlands to determine if the students would benefit from constructivist teachings. The researchers found that in regards to solving multiplication problems, students responded best to explicit instruction. Overall, the students had higher achievement rates when given explicit, or traditional teaching methods.

Summary

This chapter examined a range of research regarding constructivist and traditional mathematics instruction, student assessment, mathematics and students with learning disabilities and mathematical practices around the world. Much of the research is in favor of constructivist mathematical teachings in the United States and for students with learning disabilities. As the research showed, students who were taught with the constructivist method often had higher achievement levels in mathematics. The mathematics teachings around the world varied with some countries preferring traditional methods over a more interactive curriculum. Chapter four will provide the conclusion for the entire paper.

CHAPTER FOUR: CONCLUSION

Introduction

In Chapter One, a summary was given of the rationale for the study of student achievement in relation to their mathematical achievement and the controversy that surrounds the changes in mathematics instruction between constructivist and traditional teachings. Chapter Two discussed the history of mathematics starting with the ancient world and moving up to the mathematical debates in America starting with the new math movement and stopping with the NCTM standards. Chapter Three provided a review of the current research literature on the effects of student achievement in relation to traditional or constructivist teaching, explores the Everyday Mathematics curriculum, mathematical achievement in relation to students with learning disabilities and mathematics around the world This chapter will summarize those findings, investigate classroom applications and discuss implications for further research.

Summary of Findings

This section will summarize the main findings found in Chapter three. It will begin with summarizing the results of student assessment that shows support for constructivist instruction. Then, it will move into the analysis of the Everyday Mathematics curriculum, and provide the results of research that support traditional mathematics. The paper will them move into comparative results of both constructivist and traditional mathematics, and further investigate which learning method appears to provide better achievement for students with learning disabilities. Finally, the section will summarize mathematics across the world.

Student Achievement In Relation to Instruction

This section researched student academic achievement in mathematics in relation to the type of instruction that the student received. Gales and Yan (2001) found that the beliefs of the teacher had a direct impact on the achievement of the student and that the students had better achievement when they were able to relate mathematics to real world situations. The researchers did not present any specific information and failed to take into account social factors, psychological factors, and organization factors into student achievement. Anderson (1997) concluded that children, when given the opportunity, are able to learn mathematics in the constructivist methods when they are able to learn the mathematics in context to experiences. Morrone et al. (2004) surveys college mathematics students and found that 90% of the students preferred constructivist methods of teaching but did no compare the results to students who received traditional mathematics instruction. Bay et al. (1999) also reviewed student responses to the type of instruction that they received, like Morrone, the students preferred the constructivist methods of teaching, in addition, Bay used two different curriculums for comparison with the students and the majority of the students preferred the constructivist curriculum. Both Clark (1997) and Simon and Schifter (1993) found that student and teachers beliefs and attitude toward mathematics has an effect on students mathematical achievement. In addition, the teacher must believe in the constructivist method of teaching as well as believe that it is important for it to be effective for the students. Alsup (2004) found that constructivist method of teaching mathematics greatly reduced a student math anxiety in the classroom.

Investigating The Everyday Mathematics curriculum

This section investigated students' mathematical achievement in relation to specific mathematics curriculum. The academic achievement of students who used the Everyday Mathematics curriculum (EM), Math Trailblazers, Investigation in Numbers, Data and Space and one other mathematics curricula, title not listed. Two different researchers investigated the Everyday Mathematics curricula and both studies concluded that students achieved greater academic achievement when instructed using the constructivist teaching methods in the EM curriculum. (Carroll 1997; Fuson, Carroll and Drueck 2000; Sood & Jitendra 2007). While the EM curriculum showed to achieve higher student results, Carroll who developed the study is also one of the main developers for the EM Curriculum. Some variances were found in relation to student achievement. Carroll (1997) reported that students achieved higher results using the constructivist curriculum, but failed to compare results of students using traditional mathematics curriculum and did not report any significant data to support the findings. Sood and Jiterndra (2007) recorded the results that traditional textbooks (Math Trailblazers, Investigation in Numbers and the Data in Space curriculum provided students with more practice problems and feedback that the researchers believed to be essential in student scaffolding. In addition the EM and the SF curriculum that provided more constructivist methods of teaching related the text to real world application and one lesson builds off of the previous lesson. Huntley et al (2000) completed research on specific instructional methods to determine which way students had the greatest achievement. Their research found that students had a higher achievement rate when they are able to relate their mathematics to real world connections but failed to support either constructivist or traditional teaching methods.

Traditional mathematics

The main focus of the researchers of these studies aimed to prove that traditional mathematics instructions yielded better results. The results of the findings had surprising results with only one study that proved that traditional mathematics instruction improves student achievement. Flores and Kaylor (2007) found that at risk students perform better, have higher student achievement and are more attentive in class when given direct instruction with traditional teaching methods. Alsup and Springer (2003) presented strong results with comparison of student achievement and results over a two year period comparing both tradition and constructivist mathematics text books. The researchers sought to prove traditional teachings achieved higher SAT scores, however they were unable to provide any significant differences between the two curriculums. Carpenter (1998) found that real world connections had a positive impact on student learning.

Constructivism and Traditional mathematical approaches

In one case study conducted by Jennings and Pawat (1997) the researchers recorded and observed both constructivist and traditional mathematics teachings. The study showed that the teachers believed that traditional mathematics teachings resulted in better student achievement. The participants in the case study attempted constructivist teaching methods but showed resistance to change and did not believe it to be more effective. Hollar and Norwood (1999) sought to find if student achieved higher results in algebra using a graphing calculator. They found that students in the experimental group that used the calculators in math had a higher positive attitude towards mathematics and their abilities. However, no difference in their academic achievement was found. Arra and Bahr (2005) found no differences between which instructional method, constructivist or traditional, that students preferred. In researching two algebra classes in a regular college algebra class and a algebra for science class Elliot et al. (2001) found that the college algebra students achieved higher results in solving exponential equations and algebra for science students had higher achievement in critical thinking. No information on how the students received instruction was given. Chung (2004) believed that students had better achievement when instructed using constructivist methods, however, their results proved to show no significant changes in student achievement. Chung believed that this was a result of teacher error in not teaching constructivist mathematics effectively.

Mathematics in relation to learning disabilities and student achievemen

Groebecker (1999), Witzel et al. (2003) and Mazzocco and Thompson (2005) investigated the mathematics achievement in relation to students with learning disabilities. Groebecker focused the study on two students (one male and one female) with learning disabilities and found that the female student would benefit from constructivist teachings while the male student would benefit from a more traditional approach to mathematics. Witzel (2003) compared a new algebra curriculum that focuses on the traditional approach to constructivist mathematics and found that the students with learning disabilities had a higher achievement when engaged in the constructivist curriculum. Mazzocco and Thompson (2005) sought to understand how to predict when a child might have a learning disability in mathematics. To investigate this question they conducted a four year study on kindergarten students. The researchers found a correlation between the students with a mathematics learning disability and the education of the mother, the higher the education of the mother, the less likely it was for the student to have a learning disability. The research showed the students with a mathematics learning disability scored lower in the areas of cognition, memory and processing. The researchers believed that being able to predict when a student might have a learning disability will enable the student to receive help sooner.

Mathematics Assessments' Around the World

The studies in this section analyzed the effect that constructivist and traditional mathematics teachings had on students in other countries, Greece, Korea and the Netherlands. Barakats and Malone (2005) explored the effect that teacher belief systems in effects the mathematical achievement of secondary mathematics students in Greece. In addition, Kim (2005) observed what effect constructivist teaching methods has had on the mathematical achievement of students in Korea. Kroesbergen (2004) continues with evaluating the effect of constructivist teaching for low achieving students in the Netherlands. The section investigated the mathematical achievement of students around the world and focused on Greece, Korea and the Netherlands. In the Greece study, Barkatas and Malone (2005) on a study of teacher beliefs, the researchers concluded that teachers would not be willing to teach in a constructivist setting unless they already possessed those beliefs. In Korea, Kim (2005) investigated the results of constructivist teaching on students academic achievement and found no significant differences in the students academic achievement but found that students preferred to be taught mathematics using constructivist methods. Kroesbergen (2004) investigated the results of low-achieving mathematics students in the Netherlands to determine if the students would benefit from constructivist teachings. The researchers found that in regards to solving multiplication problems, students responded best to explicit instruction. Overall, the students had higher achievement rates when given explicit, or traditional teaching methods.

Classroom Implications

This section explores what the research findings suggest teachers should be doing in the classroom. As section one of Chapter three suggests, student would benefit from constructivist mathematics teachings. Students being taught using constructivist mathematics methods would become active learners in their own environment, develop cognitive thinking and be able to relate mathematics on real world applications. One constructivist mathematics curriculum is the Everyday Mathematics Curriculum as outlined in section two.

The majority of the research presented that students achieved higher mathematical skills and higher attitudes about mathematics when the Everyday Mathematics curricula was used in the classroom. Several studies from section three indicated that student were able to achieve better skills and achievement when taught with the traditional, route memorization of mathematics. One study found that at risk students had higher achievement rates and were more attentive in class when given direct instruction. While the study stated that students scores had improved significantly from previous instruction, it failed to note what the previous instruction was, and or any relevant test results. In section four, while the research explored both traditional and constructivist instruction, it was unable to find any significant differences between the types of instruction and student achievement. The research in section five and section six presented mixed results in relation to student with learning disabilities and which instructional methods would best benefit the student. Overall, the majority of the research seemed to support constructivist mathematics teaching in the classroom. However, much of the research was one sided and did not provide an equal comparison between constructivist and traditional teaching methods. In addition, in several of the studies on new mathematics curriculum, the authors of the articles developed the curriculum that was being evaluated.

Suggestions for Further Research

This section examines how the research that has already been completed to provide a stronger research base. The majority of the research does not mention the socio-economic status of the students. If the SES is stated, all the students in the study come from the same SES and no cross-comparison. In addition, a trend was seen in that the sample size of many of the research was relatively small. In addition, very few of the research stated how or why the students were chosen and or broken down into groups. There appeared to be a large amount of research support constructivist mathematical teachings in recent years, but there is lacking evidence that support traditional teaching methods. The researcher would provide stronger evidence if studies contained both constructivist and traditional teaching methods for comparison. In addition, when a new curriculum is being researched, the results would have a stronger impact if the research was completed by someone other than the developer.

Conclusion

Chapter one provided a basis for the research focusing on comparing constructivist teaching methods and traditional teaching methods. In addition, the achievement of student in mathematics in relation to instructional methods is important area that will be further explored in chapter three. Chapter two explored the general history of mathematics in the ancient world and Greece continuing with the math wars in the United States and the changes in mathematic teaching over the last 50 years. Chapter Three presented a critical review of the literature to include student achievement in mathematics in relation to instruction, examines the everyday mathematics curriculum, students with learning disabilities in mathematics and mathematics instruction around the world. This chapter discussed the conclusions that can be drawn from the research, suggestions for teacher practices, curriculum use and additional research that would be useful.

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