Part I

1. A particle executes simple harmonic motion. When the velocity of the particle is a maximum which one of the following gives the correct values of potential energy and acceleration of the particle.

   (a) potential energy is maximum and acceleration is maximum.

   (b) potential energy is maximum and acceleration is zero.

   (c) potential energy is minimum and acceleration is maximum.

   (d) potential energy is minimum and acceleration is zero.

   Answer (d). When velocity is maximum displacement is zero so potential energy and acceleration are both zero.

2. A mass vibrates on the end of the spring. The mass is replaced with another mass and the frequency of oscillation doubles. The mass was changed by a factor of

   (a) 1/4 (b) 1/2 (c) 2 (d) 4

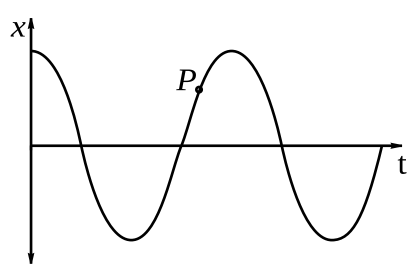
   Answer (a). Since the frequency has increased the mass must have decreased. frequency is inversely proportional to the square root of mass, so to double frequency the mass must change by a factor of 1/4.

3. A mass vibrates on the end of the spring. The mass is replaced with another mass and the frequency of oscillation doubles. The maximum acceleration of the mass

   (a) remains the same. (b) is halved. (c) is doubled. (d) is quadrupled.

   Answer (d). Acceleration is proportional to frequency squared. If frequency is doubled than acceleration is quadrupled.

4. A particle oscillates on the end of a spring and its position as a function of time is shown below.



   At the moment when the mass is at the point P it has

   (a) positive velocity and positive acceleration

   (b) positive velocity and negative acceleration

   (c) negative velocity and negative acceleration

   (d) negative velocity and positive acceleration

   Answer (b). The slope is positive so velocity is positive. Since the slope is getting smaller with time the acceleration is negative.

Part II

1. A clock maker wants to design a grandfather clock which keeps time from a 1.0 kg mass which vibrate on the end of a spring.

   (a) What should the spring constant be if the mass is designed to oscillate with a period of 1 second?

   $T = 2\pi\sqrt{m/k} \;\Rightarrow\; 1.0 = 2\pi\sqrt{1/k} \;\Rightarrow\; \sqrt{k} = 2\pi \;\Rightarrow\; k = 4\pi^2 = 39.5$ N/m.

   (b) After constructing the clock she notices that on a particularly hot day the clock does not keep the correct time. Explain what might be happening to cause this?

   As the spring heats up it lengthens and so the spring constant decreases. So we would expect the period to increase.

   (c) After careful observation she determines that the clock is losing 1 second every minute. What is the actual period and what is the actual spring constant.

   The clock ticks 59 times in 60 seconds so the frequency is 59/60 ticks per second and the period is 60/59=1.02 seconds.

   Solving the equation $T = 2\pi\sqrt{m/k}$ for the spring constant with this period gives so $1.02 = 2\pi\sqrt{1/k} \;\Rightarrow\; \sqrt{k} = (2\pi)/1.02 \;\Rightarrow\; k = 38.2$ N/m.

   (d) To compensate for this problem she decides to replace the 1.0 kg mass on the end of the spring with a different one. What should the new mass be?

   With a spring constant $k = 38.2$ N/m and a desired period of $T = 1.0$ seconds we solve the equation $T = 2\pi\sqrt{m/k}$ for m now.

   $T^2 = 4\pi^2 m/k \;\Rightarrow\; m = T^2 k/(4\pi^2) = 1.0^2(38.2)/(4\pi^2) = 0.967$ kg.