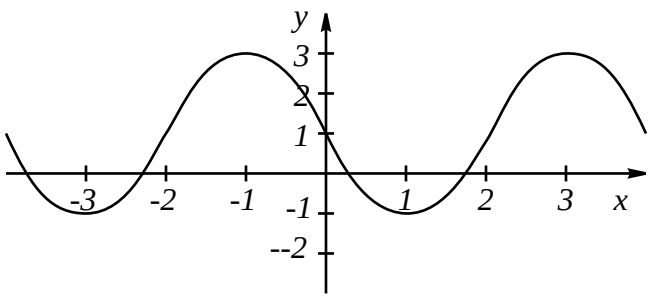
1. The equation of the graph on the right can be expressed in the form

$$y = a\sin(bx + d) + c\ .$$

State, with explanation, the values of a, b, c and d.



The graph is a sine graph which with amplitude 2 and is reflected in the midline so $a = -2$.

The midline is at $y = 1$ (ie the graph has been shifted up one unit) so $c = 1$.

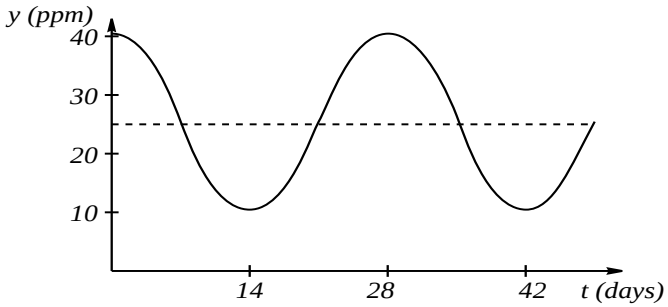
The graph is not shifted horizontally so $d = 0$.

The period is 4 so $b = 2\pi/period = 2\pi/4 = \pi/2$. Thus

$$y = -2\sin(\pi t/2) + 1$$

2. The concentration of a hormone in the body is observed to oscillate between a high of 40 ppm and a low of 10 ppm over a 28 day cycle

   (a) Draw a graph of the concentration of this hormone as a function of time in days assuming the concentration is high at the start of the cycle and that the graph has a sinusoidal shape.



   (b) Model the graph using a suitable sinusoidal function.
   This is a cosine function The amplitude is 15 ppm, so $a = -15$. The midline is at $y = 25$ ppm so $c = 25$. The period is 28 days so $b = 2\pi/28 = \pi/14$. Since there is no horizontal shift $d = 0$ so an appropriate model would be

$$y = 15\cos\left(\frac{\pi t}{14}\right) + 25$$

   where y is hormone concentration in ppm and t is time in days.

3. The number of hours of daylight in Olympia during different times of the year is modeled by the function

$$y = 12 - 3\cos\left(\frac{\pi}{6}t + \frac{\pi}{18}\right)$$

where y is hours of of daylight and t is the number of months since January 1st.

(a) What are the amplitude, period and midline of this function?

$a = -3 \Rightarrow$ Amplitude is 3 hours. $b = \pi/6 \Rightarrow$ period $= 2\pi/b = 2\pi/(pi/6) = 12$ months. $c = 12$ so the midline is $y = 12$.

(b) Draw a sketch of this function

(c) By considering your answers in (a) how many hours are there in the longest day and in the shortest day of the year?

The longest day will be 12+3=15 hours. The shortest day will be 12-3=9 hours.

(d) According to the model at what time of the year does the longest day of the year occur?

The longest day is 15 hours so we solve the equation $15 = 12 - 3\cos\left(\frac{\pi t}{6} + \frac{\pi}{18}\right)$ for t.

So $3 = -3\cos\left(\frac{\pi t}{6} + \frac{\pi}{18}\right) \Rightarrow -1 = \cos\left(\frac{\pi t}{6} + \frac{\pi}{18}\right) \Rightarrow \frac{\pi t}{6} + \frac{\pi}{18} = \cos^{-1}(-1) = \pi$

$\Rightarrow \frac{\pi t}{6} = \frac{17\pi}{18} \Rightarrow t = \frac{17}{3} = 5\frac{2}{3}$ months since January 1st.

So the model predicts correctly that the longest day of the year is June 20th.