Introduction

Spiral phyllotaxis is a characteristic arrangement of leaves, petals, scales or seeds that is seen in a variety of plants including daisies, sunflowers, pine cones and cauliflower. The cause of this beautiful natural pattern has only recently been explained fully. The explanation involves an interesting mix of mathematics, biology and physics. At the growth tip of a plant (called the **meristem**) small protusions called **primordia** emerge at regular intervals and move away radially from the center. Eventually these primordia go on to develop into various features of the plant such as its petals or seeds. In this workshop you will investigate how the primordia arrange themselves into a the characteristic spiral pattern and why the number of spirals is so often a Fibonacci number.

1. Before getting started with phyllotaxis it is worth getting familiar with plotting functions on polar graph paper. Each member of the group should choose a different one of the following polar functions and make a table of values for the radius r and the angle θ. Start with $\theta = 0^o$ and go up to at least 720^o in increments of 30^o. Plot the points on the polar graph paper provided and connect the points in order.

   (a) $r = 10 + \theta/30$

   (b) $r = 2^{\theta/180}$

   (c) $r = \phi^{\theta/90}$ where ϕ is the golden ratio.

   (d) $r = 720/\theta$ and $\theta \neq 0$

   (e) $r = 10 + 10\cos\theta$. This curve is called a cardioid. What does it remind you of?

   (f) $r = 10 + 10\cos 3\theta$. This curve is called a trifolium. What does it remind you of?

   Draw a sketch of each function plotted by your group members in your workbook and describe the differences and similarities between them with appropriate mathematical language.

2. One model of growth for the primordia is to assume that each primordium emerges from the meristem at a fixed angle relative to the previous primordium. The angle separating neighbouring primordia is called the **divergence angle**. In this activity we will try out various divergence angles to see which gives the most efficient packing. We will do this by plotting points (or preferably coloured dots) representing primordia on polar graph paper. Plot the first dot at $\theta = 0$ and $r = 10$. For each subsequent point increase θ by the chosen divergence angle and increase r by one unit. **Do not connect the dots.** The spiral that is formed by connecting the dots together in the order they are plotted (i.e. in the order that the primordia grow) is called the **generating spiral** However, we are not interested in the generating spiral, but rather in the spirals or lines that emerge as our eyes make connections between nearest neighbour dots. These spirals are called **parastichies**.

   (a) Each member of the group should choose a different one of the following divergence angles 30^o, 45^o, 90^o, 120^o, 135^o and 150^o. Plot enough points so that the parastichies become evident (you will need as many as 36 in some cases) and in pencil connect the dots along each parastichy.

   (b) Now each member of the group should repeat the exercise above but choose an angle that is 5^o larger than the one they chose at first.

(c) Finally everyone should repeat the exercise using a divergence angle of 137.5^o (it may be helpful to measure out this angle once on a triangular wedge to use as a template). You will likely need 36 or more primordia to see the parastichies.

3. Draw a rough sketch of each pattern of parastichies created by the members of the group in your workbook. Compare your patterns and observe the different features of these patterns. Discuss the following questions in your group and answer them in your workbook.

   (a) Which divergence angles result in parastichies that are straight lines? How many are there in each case. Explain, mathematically, why they line up in a straight line. Explain how the divergence angle relates to the number of straight lines that emerge.

   (b) When the divergence angles are changed by 5^o describe the changes in the parastichies pattern you observe. Is the shape of the parastichies the same? In which cases do the number of parastichies stay the same and in which cases do the number of parastichies change? Explain mathematically.

   (c) In the case where the divergence angle is 137.5 you should be able to see two different ways to connect the dots to form parastichies. How many spirals do you get by each method of connecting the dots? Are these numbers familiar?

   (d) Of all the patterns which results in the arrangement that is most uniformly packed?

4. You will probably find that the divergence angle of 137.5^o will lead to the most efficient packing of primordia. The number of spirals in each way of forming the parastichies should be neighbouring Fibonacci numbers. There is a mathematical reason for this

   (a) For any given divergence angle α there is a complementary angle $360^o - \alpha$ which would generate an identical pattern. For example a divergence angle of 270^o generates the same pattern as a divergence angle of 90^o. Why?

   (b) For the divergence angle of 137.5^o find the complementary angle. Find the ratio of the complementary angle to the divergence angle in this case. What is this number? What is its connection to Fibonnaci numbers?

5. Finally create a drawing on a blank piece of paper of the pattern that emerged for the divergence angle that is most efficient for packing. Replace the dots in your diagram with some representation of a plant structure of your choice (eg seeds, scale, petals, or florets). Consider altering the size and shape of the structures as you move radially outward to make your pattern more natural.

6. Follow up: While mathematicians have proven that the golden angle 137.5^o is the divergence angle that leads to closest packing of primordia, it is remains doubtful that plants are privy to this proof! How do they know? Some may say it is the hand of God, others would invoke the power of evolution. In fact such appeals to authority are not needed – the explanation lies in simple dynamics. When a new primordium emerges it is repelled by all existing primordia and so moves to a region with the most open space. Existing primordia will also drift a bit to make room for it. When only two primordia exist then they will be separated by 180^o, but a soon as another is added this angle decreases. As, more and more primordia are added the divergence angle converges closer and closer to 137.5^o. This is why the number of parastichies for large flowers like sunflowers are large Fibonacci numbers – since these ratios give a close approximation to the golden ratio ϕ.