ORBITS REVEAL CENTRAL MASS

workshop summary - Astronomy + Cosmology — May 03, E1Z

p 1-3 **WEIGHING JUPITER** with orbits of moons:

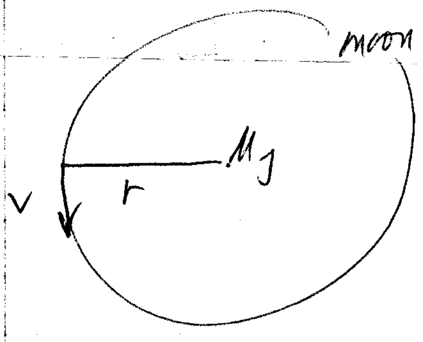


Fig.1a

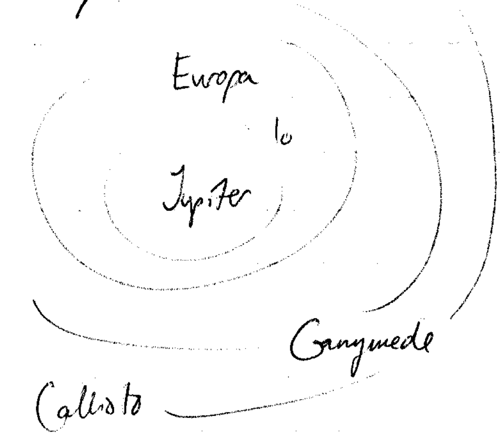


Fig.1b

r = moon's orbit radius around Jupiter
v = orbital speed (assuming circular orbits) $\quad v = \frac{2\pi r}{T}$
T = orbit period = time to go around once

(Universe 6 p. A-2)

MOON	r(km)	r(m)	T(days)	T(sec)*	$v\,(\frac{m}{s})$
Io	4.2×10^5	4.22×10^8	1.77	1.53×10^5	1.73×10^4
EUROPA	6.7×10^5	6.71×10^8	3.55	3.07×10^5	1.37×10^4
GANYMEDE	1.1×10^6	1.07×10^9	7.16	6.19×10^5	1.09×10^4
CALLISTO	1.9×10^6	1.88×10^9	16.69	1.44×10^6	8.20×10^3

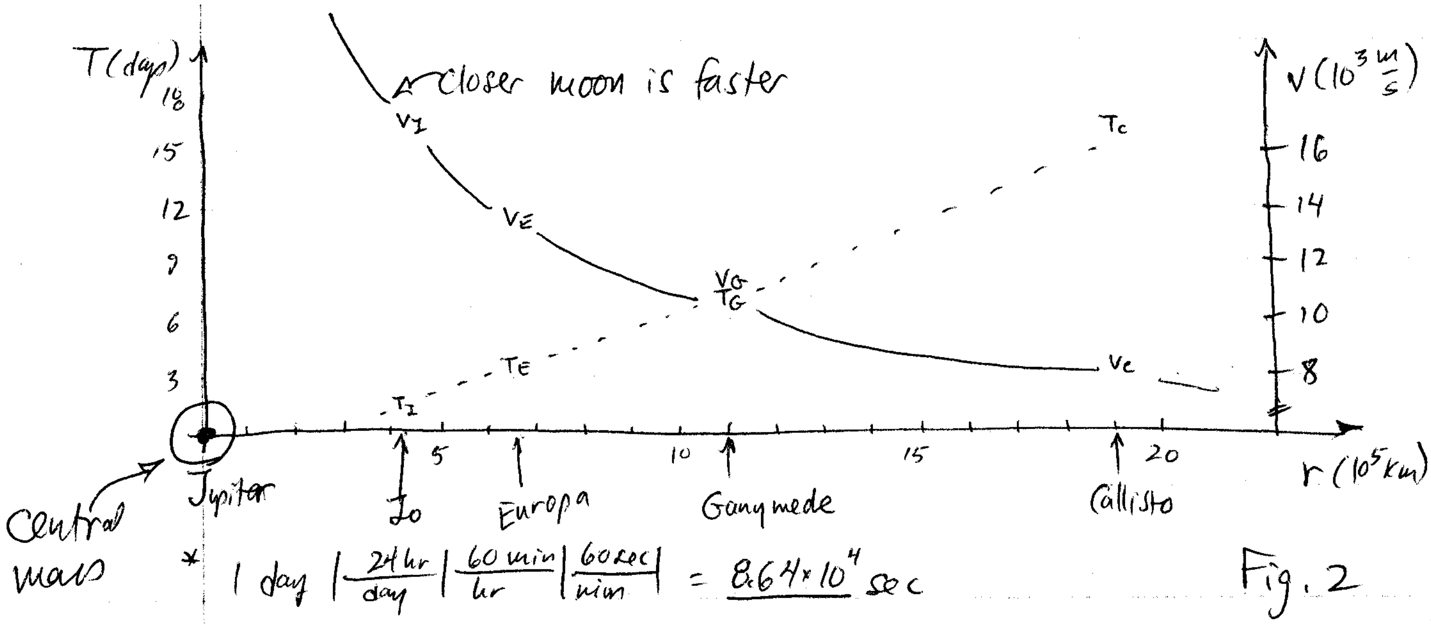


Fig. 2

* $1\text{ day}\left|\frac{24\text{ hr}}{\text{day}}\right|\frac{60\text{ min}}{\text{hr}}\left|\frac{60\text{ sec}}{\text{min}}\right| = \underline{8.64\times10^4}\text{ sec}$

We can calculate the mass of Jupiter from Kepler's 3d Law:

DERIVING KEPLER'S 3d law from NEWTON'S 2d law:

Newton's 2d:

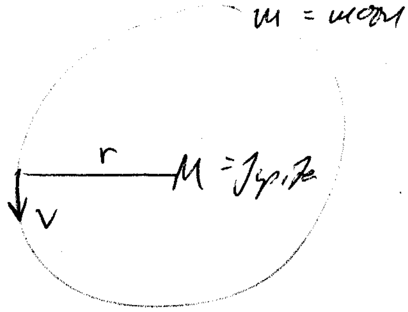
$$F = ma$$

acceleration in a circular orbit

$$\frac{GmM}{r^2} = m\frac{v^2}{r}$$

Gravity keeps moon in orbit around Jupiter

$$\frac{GM}{r} = v^2 \quad (1)$$



Recall the definition of speed = $\frac{\text{distance}}{\text{time}}$

orbital speed $v = \frac{\text{circumference}}{\text{period}} = \frac{2\pi r}{T} \quad (2)$

Combine (1) and (2) to eliminate v^2:

$$v^2 = v^2$$

$$\frac{GM}{r} = \left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r^2}{T^2}$$

$$\boxed{M = \frac{4\pi^2 r^3}{GT^2}}$$

This is Kepler's 3d law.

Apply this to Jupiter. Pick any moon, say Europa

$r_E = 6.71\times10^8$ m, $T_E = 3.07\times10^5$ sec

$$M_{Jupiter} = \frac{4\pi^2(6.71\times10^8\,m)^3}{\left(6.67\times10^{-11}\,\frac{m^3}{kg\,s^2}\right)(3.07\times10^5\,s)^2} = 1.90\times10^{27}\,kg \checkmark$$

G = universal gravitational constant

This works for each moon. Try another!

§4. "WEIGHING" the Galaxy with orbits of Stars

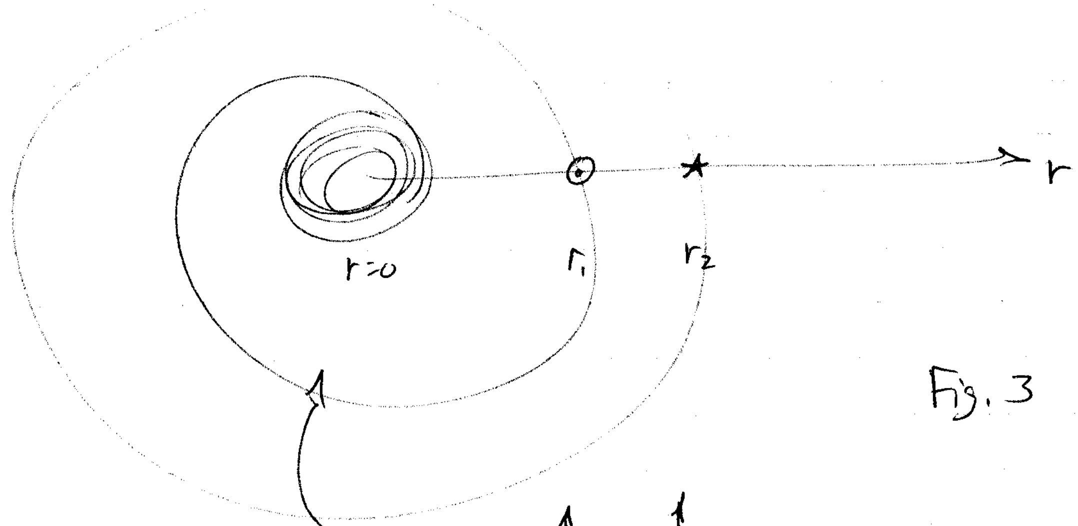


Fig. 3

Most of the Galaxy's light comes from the center

Star at $r_2 = 15$ kpc has orbital speed $V_2 = 250 \frac{km}{s}$

Sun at $r_1 = 9.1$ kpc has $V_1 = 250 \frac{km}{s}$

Notice that near and far stars have about the same orbital speed about the Galactic center!

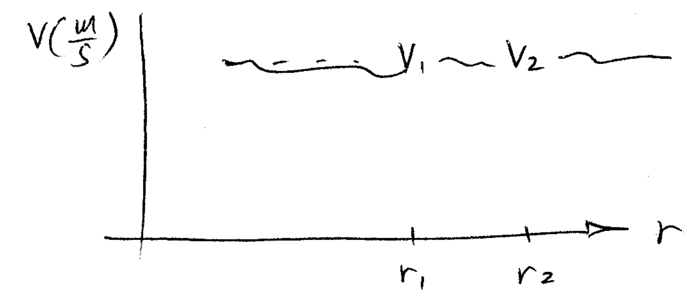


Fig. 4

This FLAT ROTATION (velocity) CURVE is <u>NOT</u> Keplerian.

This suggests that the Galaxy's mass is NOT central.

Compare to Fig. 2 for Jupiter.

To calculate the Galactic mass inside each star's orbit, first find their orbital periods T:

$$V = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{V}$$

① SUN: $T_1 = \frac{2\pi r_1}{V_1} = \frac{2\pi \times 9.1 \text{ kpc}}{250 \text{ km/s}} = \frac{2\pi \times 9.1 \text{ pc}}{250 \frac{m}{s}} \Big| \frac{\_\_\_\_\_\_ m}{pc}$

$T_1 = \_\_\_\_\_\_ s \Big| \frac{\_\_\_\_\_\_ yr}{s} \Big| = \_\_\_\_\_\_ yr$

(CONVERSION RATIOS)

Where we used light year = ly = one year × speed of light

year = $365 \text{ days} \Big| \frac{24 h}{day} \Big| \frac{3600 s}{hr} \Big| = \_\_\_\_\_\_ s$

ly = $s \times 3 \times 10^8 \frac{m}{s} = \_\_\_\_\_\_ m$

And parsec = pc = 3.26 ly = $\_\_\_\_\_\_$ m

to find the star's orbit radius in meters:

$r_1 = 9.1 \text{ kpc} = 9.1 \times 10^3 \text{ pc} \Big| \frac{3.26 \text{ ly}}{pc} \Big| \frac{9.46 \times 10^{15} m}{ly} \Big| = \_\_\_\_\_\_ m$

② STAR: $r_2 = 15 \times 10^3 \text{ pc} \Big| \frac{3.09 \times 10^{16} m}{pc} \Big| = \_\_\_\_\_\_ m$

$T_2 = \frac{2\pi r_2}{V_2} = \frac{2\pi (\_\_\_\_\_\_ m)}{260 \times 10^3 \text{ m/s}} = \_\_\_\_\_\_ s \Big| \frac{yr}{3.156 \times 10^7 s}$

$T_2 = \_\_\_\_\_\_ yr$

Finally, we can apply Kepler's 3d law to each star to find the Galactic mass that keeps it in orbit:

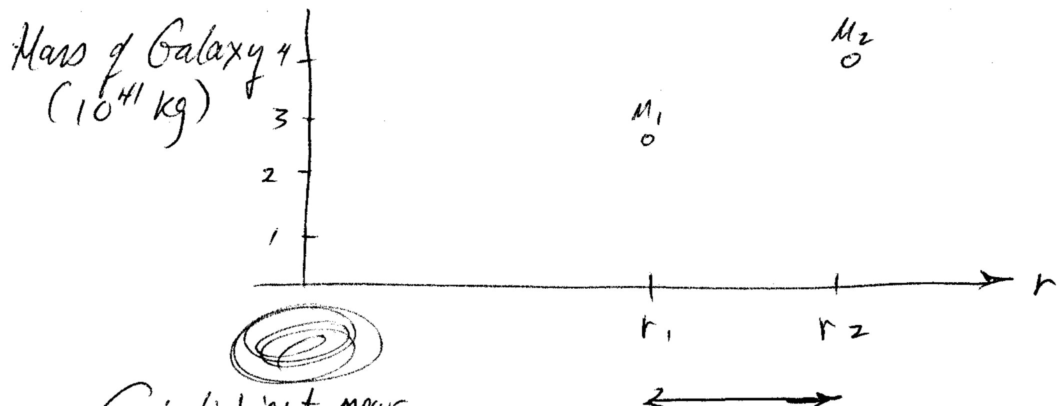
SUN ① $M_1 = \dfrac{4\pi^2 r_1^3}{G T_1^2} = \dfrac{4\pi^2 (2.8\times10^{20}\,m)^3}{6.67\times10^{-11}\,\frac{m^3}{kg\cdot s^2}\,(7.1\times10^{15}\,s)^2} =$ ______ kg

M_1 is the Galactic mass inside r_1, our Sun's orbit radius.

Star ② $M_2 = \dfrac{4\pi^2 r_2^3}{G T_2^2} = \dfrac{4\pi^2 (4.6\times10^{20}\,m)^3}{6.67\times10^{-11}\,\frac{m^3}{kg\cdot s^2}\,(1.2\times10^{16}\,s)^2} =$ ______ kg

M_2 is the Galactic mass inside r_2, the star's orbit radius.

Q: WHAT PERCENTAGE OF THE MASS OF THE MILKY WAY GALAXY LIES BETWEEN $r_1 = 9.1$ kpc and $r_2 = 15$ kpc, that is between the orbits of our Sun and the star we have analyzed?



Galactic light near $r=0$

$\Delta M = M_2 - M_1 =$ ______ $\times10^{41}$ kg

$\dfrac{\Delta M}{M_2} =$ ______ $=$ ______ % of the Galaxy's mass lies out in the edge where there is little light: THIS MASS IS DARK!