HELP SHEET on $\frac{2.28}{87}$ Griffiths (20oct02 - E/Z)

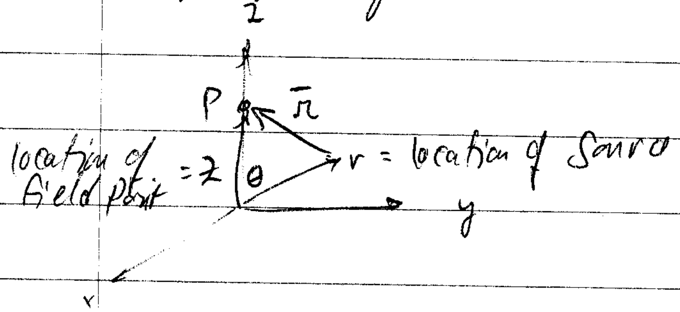
$V(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{\mathscr{r}}\, d\tau'$ $\frac{(2.29)}{p.84}$

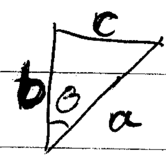
spherical element of volume $d\tau$ = ______________________ p.40

charge density ρ = constant

Consider a point P on the z-axis. Use spherical coordinates. (p.40)

By symmetry, V(r) on the z-axis is V(r) everywhere.





Law of cosines:

$c^2 = a^2 + b^2 - 2ab\cos\theta$

$\mathscr{r}^2$ = ______________________

For simplicity of notation, drop the ' on r'. Then

$V = \frac{\rho}{4\pi\epsilon_0}\int$ ______________________

Do the ϕ integral first : $\int dr\, d\theta\, d\phi = \int_{r=0}^{R}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} dr\, d\theta\, d\phi$

$\int_0^{2\pi} d\phi$ = __________

Do the θ integral next, for a fixed r:

$$I^{(\theta)} = \int_0^{\pi} \frac{\sin\theta}{\sqrt{r^2+z^2-2rz\cos\theta}}\, d\theta$$

Change variables: let $\alpha = r^2+z^2-2rz\cos\theta$

$$d\alpha = -2rz(-\sin\theta)\, d\theta$$

Sub into I to get simple integral in terms of α:

$I(\alpha) =$

Solve the integral:

Sub back in $\alpha = r^2+z^2-2rz\cos\theta$:

Evaluate limits:

Show that

$$I(\theta) = \frac{1}{rz}\left[\sqrt{r^2+z^2+2rz} - \sqrt{r^2+z^2-2rz}\right] = \frac{1}{rz}\left(r+z-|r-z|\right) =$$

$I(\theta) = \frac{2}{z}$ if $r<z$, $I(\theta) = \frac{2}{r}$ if $r>z$.

Finally, show that $V = \frac{\rho}{2\epsilon_0}\int_0^R r^2 I\, dr$ and finish up:

$$= \frac{\rho}{2\epsilon_0}\int_0^z r^2\left(\frac{2}{z}\right)dr + \frac{\rho}{2\epsilon_0}\int_z^R r^2\left(\frac{2}{r}\right)dr$$

sources inside point — sources outside point