Spring Physical Systems - Astrophysics - week 2

EJZ
7 Apr 03

Ch 5: The Interaction of Light & Matter (review of Modern Physics)
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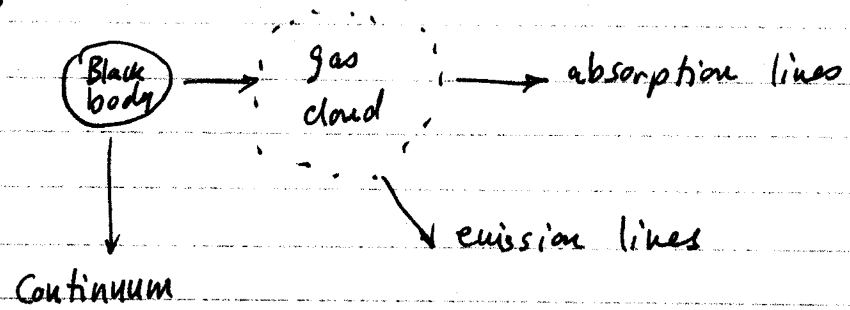


Doppler shift $\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$ (N = # of grating lines)

Diffraction maxima : $d\sin\theta = n\lambda$; Resolution $\Delta\lambda = \frac{\lambda}{nN}$

order number $n = 0, 1, \ldots$

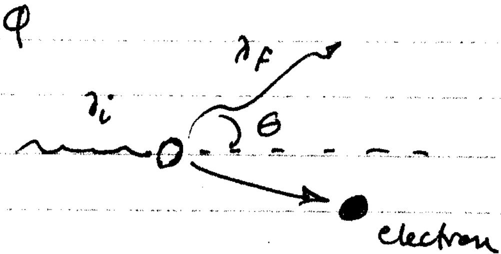
photons: are

$E_{photon} = h\nu = \frac{hc}{\lambda}$ $hc = 1.24\times10^4 \text{ eV}\cdot\text{Å}$

· particles with · momentum

Photoelectric effect $K_{max} = E_{photon} - \varphi$

Compton effect $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$



Bohr model : $r = \frac{\hbar^2 n^2}{\mu e^2} = a_0 n^2$, $E = \frac{-\mu e^4}{2\hbar^2 n^2} = \frac{-13.6\text{eV}}{n^2}$

$L = n\hbar$

reduced mass

$\Delta E = \frac{hc}{\lambda} = R\left(\frac{1}{n^2_{hi}} - \frac{1}{n^2_{lo}}\right)$ $R \approx \frac{1.1\times10^5}{\text{cm}}$

QM: $\Delta x\,\Delta p \geq \frac{\hbar}{2}$ $\Delta E\,\Delta t \geq \frac{\hbar}{2}$ $L = \sqrt{l(l+1)}\,\hbar$

Zeeman effect : $\Delta E = h\delta\nu = \frac{heB}{4\pi\mu c}$
(cgs)

Spring Astrophysics HW

due 14 Apr 03
wk 2

Ch 5 * 4, 9, 14, 17 (15?)

(5.4) The photoelectric effect can be an important heating mechanism for the grains of dust found in interstellar clouds (see Section 12.1). The ejection of an electron leaves the grain with a positive charge, which affects the rates at which other electrons and ions collide with and stick to the grain to produce the heating. This process is particularly effective for ultraviolet photons ($\lambda \approx 1000$ Å) striking the smaller dust grains. If the average energy of the ejected electron is about 5 eV, estimate the work function of a typical dust grain.

(5.9) To demonstrate the relative strength of the electrical and gravitational forces of attraction between the electron and the proton in the Bohr atom, suppose the hydrogen atom were held together *solely* by the force of gravity. Determine the radius of the ground-state orbit (in units of Å and AU) and the energy of the ground state (in eV).

Ap 12

5.14 A white dwarf is a very dense star, with its ions and electrons packed extremely close together. Each electron may be considered to be located within a region of size $\Delta x \approx 1.5 \times 10^{-10}$ cm. Use Heisenberg's uncertainty principle, Eq. (5.18), to estimate the minimum speed of the electron. Do you think that the effects of relativity will be important for these stars?

5.17 The members of a class of stars known as *Ap stars* are distinguished by their strong magnetic fields (usually a few thousand gauss).[19] The star HD215441 has an unusually strong magnetic field of 34,000 G. Find the frequencies and wavelengths of the three components of the H_α spectral line produced by the normal Zeeman effect for this magnetic field.

Ch. 8 : Classification of Stellar Spectra
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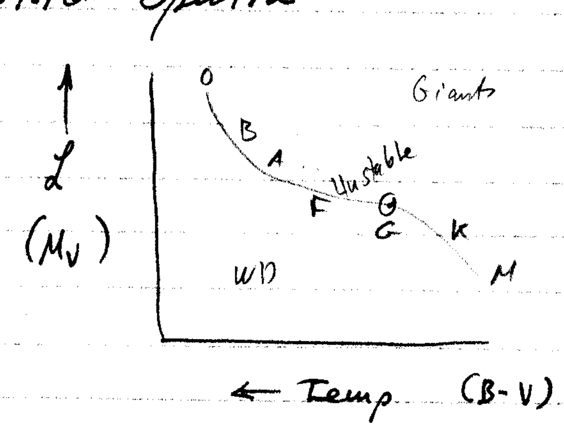
H-R diagram



$k = 8.6174 \times 10^{-5} \frac{eV}{K}$

(8.4) 231 Boltzmann Eqn = ratio of EXCITATION STATES

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$ where $g = 2n^2$ for H.
↑ degeneracy

(8.6) 234 Saha Eqn = ratio of IONIZATION states

$$\frac{N_{i+1}}{N_i} = 2 \frac{Z_{i+1}}{Z_i} \frac{n_Q}{n_e} e^{-\chi_i/kT}$$ where χ_i = ionization energy

$P_e = n_e kT$

$$n_Q = \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}$$

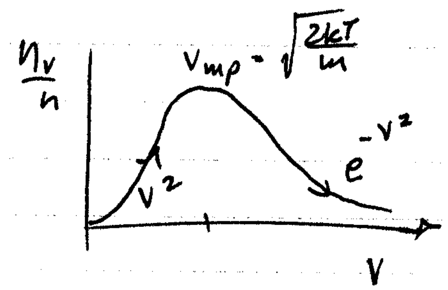
Partition function = number of e^- arrangements for a given energy.

$$Z = g_1 + \sum_{j=2}^{\infty} g_j e^{-(E_j - E_1)/kT}$$: E_j = j^{th} energy level

Boltzmann distribution function

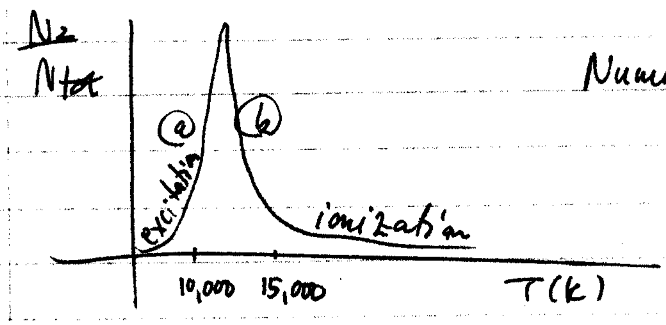
$$n_v \, dv = n \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-mv^2/2kT} dv$$

↑ rise ↑ tail



Boltzmann: Higher T → higher v BUT
Higher T → fewer particles

Saha: ⓐ Higher T → more excited e^- BUT
ⓑ Higher T → more IONIZED atoms (lost their e^-)



Number of H singly ionized

Fig 8.7 / 236

HR / 241 $L = A\sigma T^4 = (4\pi R^2)\sigma T^4$
brighter bigger hotter

244 To find R from HR diagram: spectrum → type, temp
see fig. 8.13 / 245

Stellar ranges $0.1 M_\odot \lesssim M \lesssim 100 M_\odot$

$0.1 R_\odot \lesssim R \lesssim 100 R_\odot$ except giants & dead stars

$\rho_\odot \sim 1\ g/cm^3$ $\rho_{Betelgeuse} \sim 10^{-8} \rho_\odot$

Line broadening · temperatures
· collisions $\propto \rho$

Observe spectrum → read M + measure m → Calculate d
$d = 10^{(m-M+5)/5}$

Spring Astrophysics HW part 2 - wk 2 due 14 Apr 03

Ch. 8 # 1, 4, 12, 13, 15, 16

$T = 300\,K$

8.1 Show that, at room temperature, the thermal energy $kT \approx 1/40$ eV. At what temperature is kT equal to 1 eV? to 13.6 eV?

$kT = 8.6 \times 10^{-5} \frac{eV}{K} (300K) =$ ______

$T_1 = \frac{E}{k} = \frac{1eV}{8.6 \times 10^{-5} eV/K} =$ ______ K, $T_{13.6} =$ ______ K

8.4 Show that the most probable speed of the Maxwell–Boltzmann distribution of molecular speeds (Eq. 8.1) is given by Eq. (8.2).

$\frac{(8.1)}{225}$ $n_v = 4\pi n c v^2 e^{-mv^2/2kT}$ where $c = \left(\frac{m}{2\pi kT}\right)^{3/2}$

$\frac{dn_v}{dv} = 0 =$

$\frac{(8.2)}{229}$ $V_{\text{most probable}} = \sqrt{\frac{2kT}{m}}$

Ap - 17

(8.12) Use the Saha equation to determine the fraction of hydrogen atoms that are ionized, N_{II}/N_{total}, at the center of the Sun. Here the temperature is 15.8 million K and the number density of electrons is about $n_e = 6.4 \times 10^{25}\ \text{cm}^{-3}$. (Use $Z_I = 2$.) Does your result agree with the fact that practically *all* of the Sun's hydrogen is ionized at the Sun's center? What is the reason for any discrepancy?

$T =$

From Example 3 (p.234) $\quad Z_I = 2, \quad Z_{II} = 1, \quad \chi_i = 13.6\,\text{eV}$

Saha eqn (8.6) p.234 $\quad \dfrac{N_{i+1}}{N_i} = 2\,\dfrac{Z_{i+1}}{Z_i}\,\dfrac{n_Q}{n_e}\,e^{-\chi_i/kT}$

(8.6) 234 $\quad n_Q = \left(\dfrac{2\pi m_e kT}{h^2}\right)^{3/2} =$

$$\frac{N_{II}}{N_I} =$$

Then use (p.235) $\quad \dfrac{N_{II}}{N_{total}} = \dfrac{N_{II}}{N_I + N_{II}} = \dfrac{N_{II}/N_I}{1 + N_{II}/N_I} =$

This is less than "practically all" ionized.

Increased density at center of Sun actually increases the amount of ionization.

Apr 18

p. 237

8.13 Use the information in Example 8.4 to calculate the ratio of doubly to singly ionized calcium atoms (Ca III/Ca II) in the Sun's photosphere. The ionization energy of Ca II is $\chi_{II} = 11.9$ eV. Use $Z_{III} = 1$ for the partition function of Ca III. Is your result consistent with the statement in Example 8.4 that, in the solar photosphere, "nearly all of the calcium atoms are available for forming the H and K lines of calcium?"

$Z_{II} = 2.3$

$Z_{III} = 1$

$P_e = 15 \frac{\text{dyne}}{\text{cm}^2}$

$T = 5770\,K$

Saha equ (8.6) 234 $\quad \frac{N_{i+1}}{N_i} = 2 \frac{Z_{i+1}}{Z_i} \frac{n_Q}{n_e} e^{-\chi_i/kT}$

$n_Q = \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} =$

$n_e = \frac{P_e}{kT}$

$\left(\frac{N_{III}}{N_{II}}\right)_{Ca} =$

Compare to $\left(\frac{N_{II}}{N_I}\right)_{Ca} = 903$

Most Ca atoms are singly ionized Ca II : $n=2 \rightarrow n=1$

H lines $\lambda = 3968$ Å

K lines $\lambda = 3933$ Å

p. 239

Ap-19

8.15 Figure 8.13 shows that a white dwarf star typically has a radius that is only 1% of the Sun's. Determine the average density of a 1-$M_\odot$ white dwarf.

density = $\frac{\text{mass}}{\text{volume}}$: $\rho = \frac{M}{\frac{4}{3}\pi R^3}$

8.16 The blue-white star Fomalhaut ("the fish's mouth" in Arabic) is in the southern constellation of Pisces Austrinus. Fomalhaut has an apparent visual magnitude of $V = 1.19$. Use the H–R diagram in Fig. 8.15 to determine the distance to this star.

p. 249

M_v = absolute visual magnitude ~ ______

$m = V$ = apparent visual mag ~ ______

$d \sim 10^{(m-M+5)/5} =$

Ap-20