spring Physical Systems - Astrophysics - wk 6

Ch 25 - Structure of the Universe

5 May 03
E12

25.1 The extragalactic distance scale
1097

1761 Kepler: Venus parallax → $R_{solar\ system}$

1862 Bessel: 61 Cygni parallax → $d_{61Cyg} = 6.5 \times 10^5$ AU parallax → kpc

moving cluster method (22.3)

main-sequence fitting + spectroscopic parallax ~~clusters~~ → 7kpc

p. 1098 WILSON-BAPPU effect I_x: narrow emission line ~ M_V

Cepheids: Period-luminosity relation $M_{\langle V \rangle} = -2.80 \log_{10} \Pi^d - 1.43$

1913 Hertzsprung: Sun's motion → secular parallax → $d_{Cepheids}$ (0-pt)

Cepheids PLC relation $M_{\langle V \rangle} = -3.53 \log_{10} \Pi^d - 2.13 + 2.13 (B-V)$

p. 1099-1100 caution (needs extinction corrections - good to 20 Mpc) i dosome in IR

Caution - (RR Lyrae + W Virginis) = Pop II ≠ Pop I Cepheids

PRIMARY measure aut SN expansion $\omega = \frac{\Delta\theta}{\Delta t}$ + $V_\theta = \omega d$ = V_{ejecta} from Doppler shift → 1000 Mpc!

or find $L = 4\pi R^2 \sigma T_e^4$ $\frac{L}{L_\odot} \to M - M_\odot$

T_e from spectra

$R = v_{ej} t$, t = age of SN

Novae (accretion onto WD) decline rapidly: $M_V^{max} = -9.96 - 2.31 \log_{10} \frac{dm}{dt}$
in the MW

$d_{\text{virgo cluster}} = 14.9 \pm 1.2$ Mpc

SECONDARY MEASUREMENTS of galaxies at great distances: use brightest objects in a galaxy 1103

Brightest red supergiants $M_V = -8.0$ → 7 Mpc

Globular cluster (of stars): luminosity turnover magnitude $M_0 \sim -6.5$ → 50 Mpc

Planetary nebula $M_{5007\,max} = -4.48$ → 30 Mpc

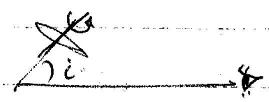
p.1106 Surface brightness is SMOOTHER for more distant galaxies! → 50-200 Mpc

TULLY FISHER relation

V_{rot} ← galactic mass → M_V
observe, infer

$$M_H = -10.0 \log_{10}\left(\frac{2v_r}{\sin i}\right) + 3.61$$

↑
IR: 1.66 μm
less reddening (extinction)



→ 100 Mpc
high accuracy

p.1108 Faber-Jackson relation for elliptical galaxies
σ = velocity dispersion ~ angular size ~ $\frac{1}{\text{distance}}$

Measurements to galaxy clusters: using brightest galaxy - caution: evolving!

25.2 Expansion of Universe. REDSHIFT $v = H_0 d$

Wendy Freeman: $H_0 = 100\, h \frac{km}{s \cdot Mpc}$ $h = .73 \pm .01$ $\frac{1}{h} = 1.4$

$H_0 = 3.24 \times 10^{-18}\, h \frac{1}{s} = 4.44 \times 10^{-18} \frac{1}{s}$

Doppler shift: $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$ (4.38) $z =$

p.1114 $z < 0.4$: $d = \frac{cz}{H_0}$ / $z \leq 2$: $d \approx \frac{c}{H_0} \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$

Expansion of universe = Hubble flow → cosmological redshift
= motion of galaxy WITH space

~~pe~~
peculiar velocity = motion THROUGH space → true Doppler shift

p.1118 Age of universe $t_H \equiv \frac{1}{H_0} = \frac{9.78 \times 10^9}{h}$ yr = 13.4 Byr for $h = .73$

25.3 Clusters of galaxies

VT = VIRIAL THM; recall that that for orbits in a $F \propto \frac{1}{r^2}$ field,
$E = K + U = K - 2K = -K = \frac{U}{2}$
$U = -2K$ p.138

One consequence of VT is that (p.365) gravitational collapse convert half the potential energy into heat.

p.330 Total U_{grav} for star with constant density is $U_g \sim -\frac{3}{5}\frac{GM^2}{R}$

Therefore VT yields total mechanical energy $E \sim -\frac{3}{10}\frac{GM^2}{R}$

p.1008 Dispersion in radial velocity $\sigma_r^2 = \langle v_r^2 \rangle$ of galaxies in cluster is related to mass of cluster!

$-3m\sigma_r^2 \approx U_g \rightarrow \sigma_r^2 = \frac{GM_{virial}}{5R}$ (23.8) 1008

(dark matter)

25.3 Clusters of Galaxies

1119

d from MW		N	SIZE (Mpc)	σ ($\frac{km}{s}$)	$M_\odot$	$M_\odot/L_\odot$
	GROUPS:	<50	$1.4\ \frac{1}{h}$	150	$2\times10^{13} h^{-1}$	260 h
	poor clusters	50		800	$10^{15} h^{-1}$	400 h
	rich clusters	10^3	$6\ h^{-1}$	>1000		
	Local Group	30	1			
	(Andromeda) M31					
			d = .77 Mpc			
	(Milky Way) MW					50-80
					bulge + disk:	3
16 Mpc	Virgo Cluster	250 +2000	3	$660\frac{km}{s}$	10^{15}	
	giant Elliptical in HSE M87		0.3		10^{10} gas, $3\times10^{1?}$ tot	750
90	Coma Cluster	10^5!	6 Mpc	$977\frac{km}{s}$	3.3×10^{15} tot, 3×10^{14} gas	
>2000	Serpens	(9 Gyr)			3×10^{15}	660
	LOCAL SUPERCLUSTER		20 Mpc			

(Andromeda → Milky Way) d = .77 Mpc, approaching at $v = 119\frac{km}{s}$, will collide in $t \sim 6.3$ Gyr

$M_{tot} > 3\times10^{12} M_\odot$

Magellanic stream flows into MW with $v = 220\frac{km}{s}$

$v \sim 600\frac{km}{s}$ Large-scale streaming carries us all toward Great Attractor past Centaurus

Large-scale structure: Galaxies cluster like soap bubbles around great voids ~100 Mpc across

"peculiar velocities" (local differences from Hubble flow) ~ 600 km/s

Galaxies could not cross voids in t_H ∴

LARGE SCALE STRUCTURE to PRIMORDIAL (3K fluctuations COBE & MAP)

25.4 / 1141 Mystery of GAMMA RAY BURSTS

once a day - from all directions - Energies 1 keV - 100 MeV
Δt ms - hrs
rise times > 10^{-4} s

most have no clear optical counterpart
three known sources: SN remnants

MAGNETAR = Strongly magnetic NS (for repeaters) (R ≈ 30 km)
$B \sim 10^{12}$ G!

Collisions between NS (for one-timers)

25.1 By equating the period–luminosity relation, Eq. (14.2), and the period–luminosity–color relation, Eq. (25.1), estimate the range of the $B-V$ color index for Cepheid variable stars.

(25.1)

(14.2) 544 $M_{\langle V \rangle} = -2.80 \log_{10} \Pi^d - 1.43 = -3.53 \log_{10} \Pi^d - 2.13 + 2.13 (B-V)$

$-2.80\, P - 1.43 = -3.53\, P - 2.13 + 2.13\,(B-V)$

$\frac{1}{2.13}\left[(3.53 - 2.80)\, P + (2.13 - 1.43)\right] = (B-V)$

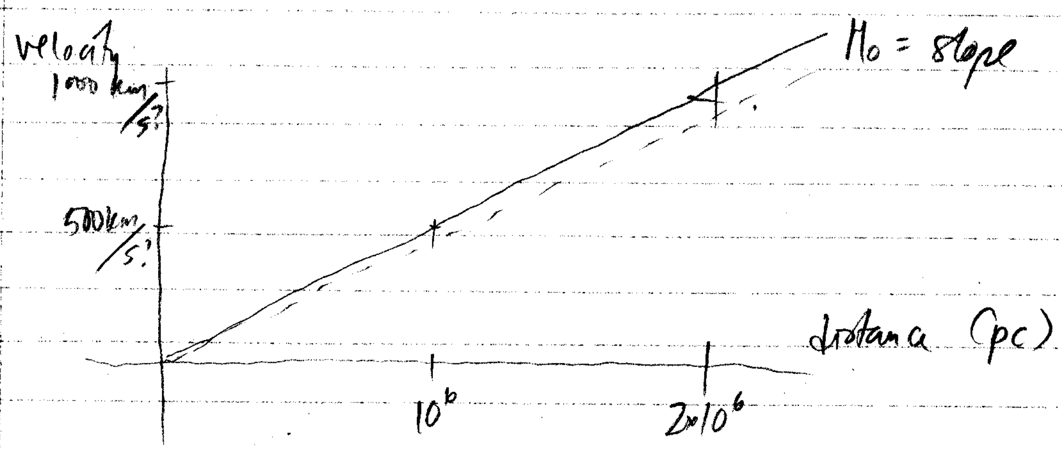
$\_\_\_\_ \, P + \_\_\_\_ = (B-V)$

p. 542 SMC Cepheids have $\Pi \sim 1\text{–}50\,d$ or $\log_{10} \Pi^d \sim 0 - 2.5$ p. 543

$(B-V)_{max} =$ $(B-V)_{min} =$

Compare to p. 108: $(B-V) \sim 0.4 - 1.1$

25.6 Use the solid line in Hubble's velocity–distance diagram, Fig. 25.6, to determine his value of H_0. Why was his result so different from today's range of values?



$\text{slope} = \frac{\Delta v}{\Delta d} =$

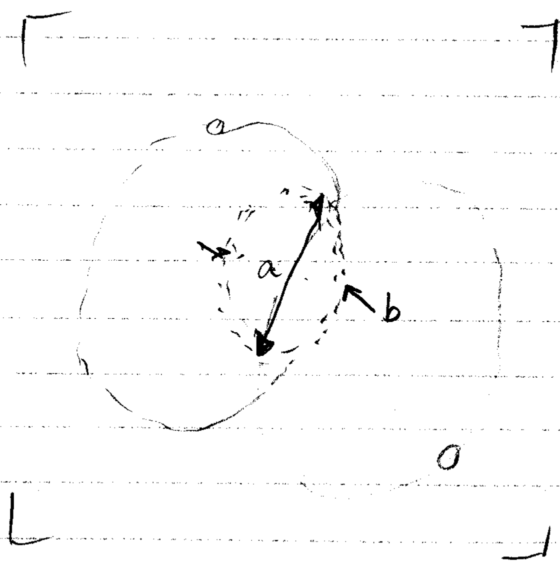
Wendy Freeman: $H_0 = 73 \pm 1 \ \frac{km}{s \cdot Mpc}$

25.2 The central light ring produced by SN1987A, shown in Fig. 13.22, gave astronomers a unique chance to determine its distance (and therefore the distance to the Large Magellanic Cloud). The supernova heated the ring of gas, causing it to glow. The ring, which has an angular diameter of 1.66″ (long axis), is presumed to be circular but tilted from the perspective of Earth. Light was received from the near side of the ring 340 days before light arrived from its far side.

(a) Carefully measure the photograph, and determine the angle (θ) between the plane of the ring and the plane of the sky.

(b) Use the time delay to determine the diameter of the ring in parsecs, and compare your result with the caption to Fig. 13.22. p. 523

(c) Use trigonometry to find the distance to SN 1987A.



If ring is circular, then b looks smaller than a because it is tilted by an angle θ

$b = a\cos\theta$

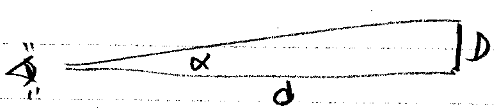
(If $\theta = 0$, $a = b$)

(a) $\theta =$ ______

(b) Since ring is tipped, the projection of its diameter D facing Earth is $x = D\sin\theta$.

$$\text{speed} = c = \frac{x}{\Delta t} = \frac{D\sin\theta}{\Delta t} \rightarrow D = \_\_\_\_\_\_$$

(c)



$$\alpha\,(\text{rad}) = \frac{D}{d} = 1.66'' \left| \frac{\text{rad}}{''} \right| = \_\_\_\_\_\_ \text{ rad}$$

Distance to SN 1987A is $d =$ ______

(25.7) Use the relative motion of the Andromeda and Milky Way galaxies to estimate their total mass for the case $h = 0.\cancel{5}$. What is the corresponding mass-to-light ratio of these galaxies? 0.73

following p. 1121

(2.34) Energy conservation

(25.14) $v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) = \frac{2GM}{r} + \left(\frac{2\pi GM}{P}\right)^{2/3}$

approach speed = 119 km/s; r = separation = 770 kpc; a = orbit's semi major axis

K3: $P^2 = \frac{4\pi^2}{GM} a^3$

$P = t_H + t_c = 13.4\,\text{byr} + \frac{r}{v}$

t_H = Hubble time ≈ age of universe $= 13.4\,\text{byr} \left| \frac{\pi \times 10^7\,\text{s}}{\text{yr}} \right| =$ ______ sec

We are told that the distance between Andromeda & MW is $r = 770$ kpc and their orbit speed is $v = 119$ km/s

$\frac{r}{v} =$ ______

Period of mutual orbit $P =$ ______

Solve the cubic for $M_{tot} =$ ______ $M_\odot$

We are told that $L_{MW} = 2.3 \times 10^{10} L_\odot$ and $L_A \approx 2 L_{MW}$ so together the Milky way + Andromeda have total luminosity $L_{tot} =$ ______

$\frac{M}{L} =$ ______ $=$ ______ $\frac{M_\odot}{L_\odot}$ $48 \frac{M_\odot}{L_\odot}$

∝ ratio of dark to luminous matter.

25.11 Suppose that in M87, $\rho_{\text{dark matter}} = a r^x$ and $\rho_{\text{interstellar gas}} = b r^x$

(Same r-dependence.) Show that the gas is isothermal.

$\frac{(25.15)}{1124}$ Interior Mass (inside r) $M_r = -\frac{kTr}{\mu m_H G}\left(\frac{\partial \ln \rho}{\partial \ln r} + \frac{\partial \ln T}{\partial \ln r}\right)$

Hint: assume $T = c_T r^\alpha$ and show $\alpha = 0$
Use flat rotation curves and use

$\frac{(22.36)}{958}$ $M_r = \frac{v^2 r}{G} = C_M r$ $\frac{(22.37)}{959}$ $\rho(r) = \frac{V^2}{4\pi G r^2} = \frac{C_\rho}{r^2}$

assuming v = constant $\left(C_M = \frac{V^2}{G}\right)$

$\left(C_\rho = \frac{V^2}{4\pi G}\right)$

$+\alpha)$

$M_r = C_M r = -\frac{k(\quad\quad)r}{\mu m_H G}\left[\frac{\partial \ln(\quad\quad)}{\partial \ln r} + \frac{\partial \ln(\quad\quad)}{\partial \ln r}\right]$

Match powers of r to find α

$T = C_T r^\alpha =$ ______

25.12 Like the Coma cluster, the Virgo cluster contains a large amount of hot (70 million K) intracluster gas that emits x-rays.

$= T = 7 \times 10^7$ K

(a) If the x-ray luminosity of the Virgo cluster's intracluster gas is $L_x =$ about 1.5×10^{43} ergs s^{-1}, use Eq. (25.18) to find the electron number density and the mass of the gas. Assume that the Virgo cluster is a sphere of radius 1.5 Mpc $= R$ that is filled with completely ionized hydrogen.

(25.18) 1129 $L_x = \frac{4}{3}\pi R^3 \mathcal{L}_{vol}$ where the luminosity density (25.17) 1128

$\mathcal{L}_{vol} = 1.42 \times 10^{-27} n_e^2 T^{1/2} \frac{erg}{s \cdot cm^2}$

Once you find n_e, Then the mass of Hydrogen is

$M_{gas} = \frac{4}{3}\pi R^3 m_H n_e$ where $m_H = m_{proton} =$ ______

(b) Use $L_V = 1.2 \times 10^{12}$ $L_\odot$ for the visual luminosity of the Virgo cluster to estimate the cluster's luminous mass. How does this compare with your answer to part (a) for the mass of the intracluster gas?

Table 22.1 p. 918

$\frac{M}{L} \simeq 3 \frac{M_\odot}{L_\odot}$

for Milky way

Amount of luminous matter in Virgo Cluster

$M_{lum} = \frac{M}{L} \cdot L_V =$

(c) Assuming that the gas has no energy source and that it is simply losing energy via thermal bremsstrahlung, use Eq. (10.22) for the average kinetic energy per gas particle (protons and electrons) to estimate how long it will take for the gas to lose all of its energy. (Assume that the x-ray luminosity remains constant throughout your calculation.) How does your answer compare with the Hubble time, t_H, for $h = 0.75$?

weighted average p. 326

$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$

average KE per particle.

Energy density = $\frac{\text{\# particles}}{\text{vol}} \times \frac{\text{average energy}}{\text{particle}}$

$u = 2 n_e \times \frac{3}{2} kT$

$\mathcal{L}_{vol} = \frac{\text{energy}}{\text{time} \cdot \text{vol}} = \frac{\text{energy density}}{\text{time}} = \frac{u}{t}$

$t =$

Hubble time (25.13) 1118 $t_H = \frac{1}{H_0} = \frac{3.09 \times 10^{17}}{h}$ sec $= \frac{9.78 \times 10^{9}}{h}$ yr

25.16 For the galaxy NGC 5585, the quantity $2v_r/\sin i = 218\ \text{km s}^{-1}$, and its apparent H magnitude is $H = 9.55$ (already corrected for extinction). Use the Tully–Fisher method to determine the distance to this galaxy.

(25.5) / 1106 Absolute magnitude at the IR H wavelength (1.66 μm) is

$$M_H = -10.0 \log_{10}\left(\frac{2v_r}{\sin i}\right) + 3.61$$

$$=$$

Then the distance to this galaxy can be found from

(3.5) / 6e

$$d = 10^{(m-5+5)/5}\ \text{pc}$$

25.17 The brightest galaxy in the cluster A1060 has an apparent visual magnitude of $V = 10.99$. Estimate the distance to the cluster. Use the uncertainty in the average absolute magnitude of the brightest galaxy to determine how far off your answer could be.

We were told that the absolute visual magnitude for the brightest galaxy in any cluster is about $M_V = -22.83 \pm 0.61$

Use (3.5) to find $d =$

If it's brighter, $M_V = -22.83 - 0.61 =$ ______
then $d_{min} =$

If it's dimmer, $M_V = -22.83 + 0.61 =$ ______
then $d_{max} =$

N

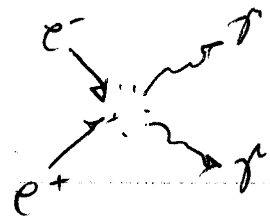
25.19 It is estimated that there are approximately 100,000 neutron stars in the Milky Way Galaxy. Show that if the observed gamma-ray bursts are associated with neutron stars in our Galaxy, then each source *must* repeat. If you make the extreme assumption that each neutron star produces bursts, what would be the average time between bursts?

p.1141: these bursts happen about once a day (every 25 hours) $= t_0$

If every NS in the MW bursts once, then bursting lasts for $t = N t_0 =$

If every NS bursts repeatedly, this is the average time interval between each star's γ-ray burst.

25.20 Consider an electron and positron that annihilate each other at the surface of a neutron star ($M = 1.4\ M_\odot$, $R = 10$ km), producing two gamma-ray photons of the same energy.



(a) Show that each gamma ray has an energy of at least 511 keV.

(b) As a 511 keV photon leaves the surface of the neutron star, its energy is reduced by the gravitational redshift. Find the energy of the gamma ray when it arrives at Earth.

(a) $E_e = m_e c^2 = (0.911 \times 10^{-27}\ g)(\qquad)^2 \left|\dfrac{kg}{10^3 g}\right| \dfrac{eV}{1.602\times10^{-19} J}$

$E_\gamma =$ ______________ eV

(b) $E_\gamma = \dfrac{hc}{\lambda} = hf$ Gravitational redshift

p.645: $\dfrac{\Delta f}{f_0} = \dfrac{-v}{c} = -\dfrac{gh}{c^2}$ (h = distance traveled upward from the mass with surface gravity g)

$\displaystyle\int_{f_0}^{f} \frac{df}{f} \simeq -\int_{r_0}^{\infty} \frac{GM}{r^2 c^2}\,dr$ (Show that this yields:) p.646

(16.7) 646 $\dfrac{f_\infty}{f_0} = \left(1 - \dfrac{2GM}{r_0 c^2}\right)^{1/2}$ where f_∞ is the shifted received frequency far away.