PS- Spring week 7 12 May 2003 E 12

Ch 27 - Cosmology = origin, structure, evolution of universe

1222 Olbers' paradox resolved: - Stars have finite lifetime
- universe is expanding

27.1 NEWTONIAN COSMOLOGY : Energy conservation → geometry of Univ.

$$T + U = E = -\frac{1}{2} m k c^2 \varpi^2 \rightarrow v^2 - \frac{8}{3}\pi G \rho r^2 = -kc^2 \varpi^2$$

"comoving coord"
$\varpi = r(t_0)$ = present radius of mass shell in dusty U.

k = geometric constant = 0 : flat or < 0 : open

(27.63) 1255 k from E of universe or CURVATURE $K(t) = k/R^2(t)$

coordinate distance of shell $r(t) = R(t)\varpi$ $R = \frac{1}{1+z}$ (27.31)

dimensionless scale factor.
Ex: $R = 0.4$ means Univ was 40% of current size

Hubble "constant" can change in time: $H(t) = \frac{1}{R(t)} \frac{dR}{dt}$

(27.8) 1227 RESULT: $\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G \rho\right] R^2 = -kc^2 = H_0^2 - \frac{8}{3}\pi G \rho_0$

(27.65) 1256 EQUIV: FRIEDMANN EQN = solution to GR equations (0 = current value)
$t = t_0$, $R(t_0) = 1$
$r(t_0) = \varpi$

corollary: $\frac{d^2R}{dt^2} = -\frac{4}{3}\pi G \rho R$ (BIRKHOFF'S Thm)

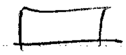
acceleration depends on density of matter within shell.

(27.13) 1228 JF CRITICAL DENSITY $\rho_c(t) = \frac{3H^2(t)}{8\pi G}$ → flat universe ($k = 0$)

(Universe be) $\rho_{matter} \sim 0.2 - 0.4\rho_c$, $\rho_{rad} \approx \frac{1}{6000}\rho_{matter} = \frac{aT^4}{c^2}$ (27.51)

(27.20) 1229 DENSITY PARAMETER $\Omega(t) \equiv \frac{\rho(t)}{\rho_c(t)}$ → $\underline{H_0^2(\Omega_0 - 1) = kc^2}$

(27.40) 1235 $= 1 + \frac{kc^2}{(dR/dt)^2}$ we currently believe expansion is ACCELERATING

	k	Ω	EXPANSION	q		proper distance d	HORIZON
OPEN	$k<0$	$\Omega<1$	$\begin{pmatrix} R \propto \cosh x - 1 \\ t \propto \sinh x - x \end{pmatrix}$	<0.5		$d < d_{coord}$	less
FLAT	$k=0$	$\Omega=1$	$R \propto t^{2/3}$	0.5		$d_{coord} = R(t)\varpi$	$d_h = 3ct$
CLOSED	$k>0$	$\Omega>1$	$\begin{pmatrix} R \propto 1-\cos x \\ t \propto x - \sin x \end{pmatrix}$	>0.5		$d > d_{coord}$	greater
					p.1253	p.1259	p.1261

(27.32) 1231 LOOKBACK TIME: $\frac{t(z)}{t_H} = \frac{2}{3}\frac{1}{(1+z)^{3/2}}$ for $k=0$

(SEE EX.27.1 p.1235)

HUBBLE TIME $t_H \equiv \frac{1}{H_0}$ (25.13) p.1118

1235 1236 INFLATION: greater expansion ($H > H_0$) at high z
early universe was flat

DECELERATION PARAMETER $q(t) \equiv -\frac{R\,\frac{d^2R}{dt^2}}{(dR/dt)^2} = \frac{1}{2}\Omega(t)$

(See Perlmutter et. al. article for current estimates!)

Ch. 27.2: COSMIC BACKGROUND RADIATION (CBR)

1948 – predicted 5K by Alpher + Herman

1238 – $\alpha\beta\gamma$: BB → cosmic abundances, account for He (stellar abundances: B^2FH 1957, He created but then burned)

$T_0 = R\,T(R) \simeq 3.68$ K — This disproved SS Theory (Hoyle)

1964 – Peebles calculated $T_0 \sim 10$ K (1948 work unknown!)

1965 – Penzias & Wilson detected 3 K radiation @ 4.08 GHz

1242 1991 – COBE measured 2.73 K (many f: excellent Blackbody fit)

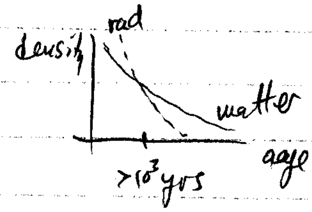
1244 3K radiation doppler shifted by solar & galactic motions

AFTER THE BIG BANG:

1245 RADIATION ERA lasted until $\rho(R) = \rho_{rad}(R)$: $t \sim$ few 1000 yrs

See Ex 27.2 for t, R when the nuclei formed ($T \sim 10^9$K, $t \sim 230$s, $R \sim 10^{-9}$, $z \sim 10^8$)

1247 MATTER ERA



DECOUPLING ~ RECOMBINATION : formation of neutral atoms
radiation streams freely

1248 $t \sim 300{,}000$ yrs, $T \sim 3000$K, $z \sim 1100$

UNIVERSE TRANSPARENT

Ch 27.3 RELATIVISTIC COSMOLOGY

p. 1250

(16.18) 657 Schwarzschild metric describes spatial curvature OUTSIDE MATTER

651 ds = interval between two events in spacetime

$ds^2 > 0$: TIME-LIKE : light can travel between events: can be causally connected

$ds^2 = 0$; LIGHT-LIKE :

$ds^2 < 0$; SPACELIKE - light does not have time to travel between events
no causal connection

PROPER DISTANCE $dd = \sqrt{-(ds)^2}$

(27.64) 1255 ROBERTSON-WALKER metric describes ISOTROPIC, HOMOGENEOUS UNIV.
same in all directions; smooth

(27.66) 1257 FRIEDMANN EQN w/ Λ: $\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right]R^2 = -kc^2$

COSMOLOGICAL CONSTANT Λ PREVENTS COLLAPSE of EINSTEIN'S GR EQNS.

COORDINATE DISTANCE $d_{coord} = \varpi R$ from expansion of universe

p.1250 PROPER DISTANCE = integral of $\sqrt{-(ds)^2}$ from spacetime metric

p.1261 HORIZON DISTANCE $d_h = R(t) \int_0^t \frac{c\,dt'}{R(t')}$ to furthest observable pt (size of largest causally-connected region)

see my p.2 OPEN UNIVERSE: $d_{coord} > d,$ $d_{horizon} < 3ct = d_h$

FLAT UNIVERSE : $d_{h,0} = \frac{2c}{H_0} = \frac{6000}{h}$ Mpc = SIZE OF OBSERVABE UNIV.

p.1262 (See Ex 27.3)

Ch. 27.4 OBSERVATIONAL COSMOLOGY

Measure REDSHIFT + RADIANT FLUX → find q_0
z F

(27.86) 1265 COSMOLOGICAL REDSHIFT $\frac{R(\text{now})}{R(\text{when photon was emitted})} = \frac{\lambda_0}{\lambda_e} = 1+z = \frac{1}{R}$

(27.90) 1266 Proper distance to object $= d(t_0) = R(t_0)\varpi = \frac{2c}{H_0}\left(1 - \frac{1}{\sqrt{1+z}}\right)$ $(\Lambda=0)$

(27.91) 1268 Text derives $\varpi = \frac{c}{H_0(1+z)q_0^2}\left[q_0 z - (1-q_0)(\sqrt{1+2q_0 z} - 1)\right]$ (FOR ANY k)

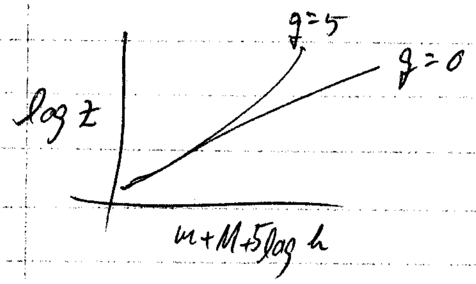
(27.95) 1269 LUMINOSITY DISTANCE $d_L = \left(\frac{L}{4\pi F}\right)^{1/2} = \varpi(1+z)$ (27.96)

$$= \frac{cz}{H_0}\left[1 + \frac{1}{2}\left(1 - \frac{\Omega_0}{2}\right)z + \dots\right]$$

(27.100) 1270 $$= \frac{cz}{H_0}\left[1 + \frac{1}{2}(1 - q_0)z + \dots\right]$$

$\boxed{\text{How to find } q_0?}$ (and Ω_0)

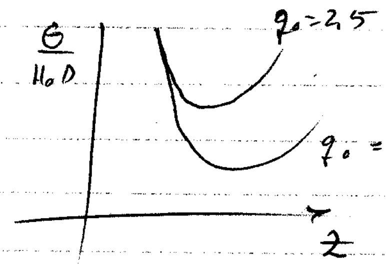
(27.101) 1271-1273 (1) REDSHIFT - MAGNITUDE relation $m - M + 5\log h = a + 5 \log z + \dots$



insensitive to calibrations!

SHAPE of curve yields q_0

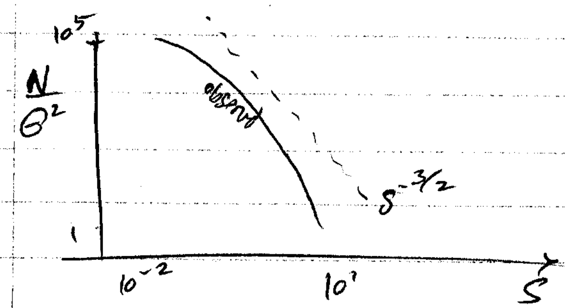
1274-75 (2) ANGULAR DIAM OF GALAXIES: (27.102)



MIN of curve $\longrightarrow q_0$

1275-77 (3) NUMBER OF GALAXIES brighter than $S = F_{min}$:

$$N(F) = n \, \tfrac{4}{3}\pi r^3(F) = \tfrac{4}{3}\pi n \left(\frac{L}{4\pi F}\right)^{3/2}$$



Complications: evolution of distant galaxies : $L(t)$
z, dust reddening, gravitational lensing
fuzzy boundaries
uncertain upper limit to gal. brightness

CONCLUSION: universe is NEARLY flat and EXPANDING
Ω mostly due to DARK ENERGY of unknown form
$H_0 \sim 75$

Ch 27 COSMOLOGY HW #5, 9, 10, 11, 15, 17, 27

and derive 27.45

ⓐ

($k>0$)

27.9 Find the lifetime of a closed universe in terms of density parameter Ω_0 (expressed as a multiple of the Hubble time $t_H = \frac{1}{H_0}$

(27.24) 1229 $R = \frac{1}{2} \frac{\Omega_0}{\Omega_0 - 1} [1 - \cos(x)]$. Trick: set $R=0$ and solve for x

This yields lifetime of closed universe : $\cos x =$ ______

$x =$ ______

$\sin x =$ ______

Sub into

(27.26) 1230 $t = \frac{1}{2H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} (x - \sin x)$

ⓑ Suppose $k>0$. From the facts that Sun is $t_\odot \sim 4.5 \times 10^9$ years old and we live in expanding universe, what limit could you put on the present value of Ω_0? Consider $h = 0.5$ and $h = 1$.

Assume $t_{universe} > 2t_\odot$ and define $\alpha = \frac{2t_\odot}{\pi t_H}$. Sub into result of ⓐ and solve cubic

$$\Omega_0^3 - [3 + \tfrac{1}{\alpha^2}]\Omega_0^2 + 3\Omega_0 - 1 < 0$$

h=1

h=0.5