WORKSHEET - wk 2

Harmonic Oscillator

***Problem 2.13** Using the methods and results of this section,

(a) Normalize ψ_1 (Equation 2.51) by direct integration. Check your answer against the general formula (Equation 2.54). *Note*: In this and most problems involving the harmonic oscillator, it simplifies the notation if you introduce the variable $\xi \equiv \sqrt{m\omega/\hbar}\,x$ and the constant $\alpha \equiv (m\omega/\pi\hbar)^{1/4}$.

(b) Find ψ_2, but don't bother to normalize it.

(c) Sketch ψ_0, ψ_1, and ψ_2.

(d) Check the orthogonality of ψ_0, ψ_1, and ψ_2. *Note*: If you exploit the evenness and oddness of the functions, there is really only one integral left to evaluate explicitly.

$x = \xi\sqrt{\frac{\hbar}{m\omega}}$

$a = \omega\sqrt{2m}$

$\psi_1 = iA_1\omega\sqrt{2m}\; x\, e^{-\frac{m\omega}{2\hbar}x^2}$

$= iA_1 a\; x e$

$= iA_1 a$

$b = a\sqrt{\frac{\hbar}{m\omega}} = \frac{\omega\sqrt{2m\hbar}}{\sqrt{m\omega}} = \sqrt{2\hbar\omega}$

$\psi_1 = iA_1 b\; \xi e^{-\xi^2/2}$

$1 = \int_{-\infty}^{\infty} |\psi_1|^2 dx = \int \psi_1^2 \quad d\xi = \quad \int \xi^2 e^{-2\xi^2/2} d\xi$

$\int_{-\infty}^{\infty} \xi^2 e^{-\xi^2} d\xi = \frac{1}{2}\sqrt{\pi}$

$+1 = \sqrt{\frac{\hbar}{m\omega}}\, A_1^2 (2\hbar\omega)[\quad] \to A_1 =$

ⓑ [2.50] 35 $\quad \psi_n = A_n (a_+)^n e^{-\frac{m\omega}{2\hbar}x^2}$

[2.41] 33 $\quad$ Practice! $\psi_1 = A_1 a_+^1 e^{-\frac{m\omega x^2}{2\hbar}}$, $\quad a_+ \equiv \left[\frac{1}{\sqrt{2m}}\left(-i\hbar\frac{d}{dx} + im\omega x\right)\right]$

$a_+ e^{-\frac{m\omega x^2}{2\hbar}} = \frac{i}{\sqrt{2m}}\left[-\hbar\frac{d}{dx} e^{-\frac{m\omega x^2}{2\hbar}} + m\omega x\, e^{-\frac{m\omega x^2}{2\hbar}}\right]$

$= \frac{i}{\sqrt{2m}} e^{-\frac{m\omega x^2}{2\hbar}}\left[\quad\right]$

$\psi_1 = A_1\left[\quad\right]$ ✓ as above.

ⓑ ..

ⓒ

ⓓ

(b)... $\psi_2 = A_2 (a_+)^2 e^{\frac{-m\omega x^2}{2\hbar}} = A_2 a_+ \psi_1$

$$= \frac{A_2 i}{\sqrt{2m}} \left(-\hbar \overset{①}{\frac{d}{dx}} + \overset{②}{m\omega x} \right) x e^{-m\omega x^2/2\hbar}$$ absorbing constants into A_2

① $\frac{d}{dx} x e^{-m\omega x^2/2\hbar} =$

② $x^2 e^{-m\omega x^2/2\hbar}$

$\psi_2 = \frac{A_2 i}{\sqrt{2m}} \Big[\qquad \Big] e^{-m\omega x^2/2\hbar}$

$=$

$= \frac{A_2 i}{\sqrt{2m}} \left[\hbar (2\xi^2 - 1) \right] e^{\sim}$

(c) Sketch: $\psi_2 \sim e^{-\xi^2/2}(2\xi^2-1)$ $\quad \psi_1 \sim \xi e^{-\xi^2/2}$ $\quad \psi_0 \sim e^{-\xi^2/2}$

(d) Orthogonality: Wavefunctions of opposite parity (or symmetry) are orthogonal by inspection $\int_{-\infty}^{\infty}$ even · odd $dx \equiv 0$

so $\int \psi_1 \psi_2 dx = \int \psi_0 \psi_1 dx = 0$ ∴

$\int \psi_0 \psi_2 dx \approx \int_{-\infty}^{\infty} (2\xi^2 - 1) e^{-\xi^2} dx =$