***Problem 2.14** Using the results of Problem 2.13,

(a) Compute $\langle x\rangle$, $\langle p\rangle$, $\langle x^2\rangle$, and $\langle p^2\rangle$, for the states ψ_0 and ψ_1.

(b) Check the uncertainty principle for these states.

(c) Compute $\langle T\rangle$ and $\langle V\rangle$ for these states (no new integration allowed!). Is their sum what you would expect?

(a) $\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-mx^2\omega/2\hbar}$ $\quad \langle x\rangle_0 = 0$ since ψ_0 is even.

$\langle x^2\rangle_0 = \int_{-\infty}^{\infty} x^2 \psi_0^2\, dx =$

$\langle p\rangle_0 = -i\hbar \int_{-\infty}^{\infty} \psi_0 \frac{\partial \psi_0}{\partial x}\, dx = \int_{-\infty}^{\infty}$ even · odd

$\langle p^2\rangle_0 = -\hbar^2 \int \psi_0 \frac{\partial^2 \psi_0}{\partial x^2}\, dx$

$= m\omega\hbar/2$

(b) $\Delta x \geq \sqrt{\langle x^2\rangle - \langle x\rangle^2} =$, $\Delta p \geq \sqrt{\langle p^2\rangle - \langle p\rangle^2} =$

$\Delta x\, \Delta p \geq$

(a) $\psi_1 = \left[\frac{4}{\pi}\left(\frac{m\omega}{\hbar}\right)^3\right]^{1/4} x\, e^{-mx^2\omega/2\hbar}$ $\quad \langle x\rangle_1 = \int_{-\infty}^{\infty}$ even · odd $=$

$\langle x^2\rangle_1 = \frac{2}{\sqrt{\pi}}\left(\frac{m\omega}{\hbar}\right)^{3/2} \int_{-\infty}^{\infty} x^2 e^{-mx^2\omega/\hbar}\, dx =$

$\langle p\rangle_1 = -i\hbar \int \psi_1 \frac{\partial \psi_1}{\partial x}\, dx = \int$ even · odd $=$

$\langle p^2\rangle_1 = -\hbar^2 \int \psi_1 \frac{\partial^2 \psi_1}{\partial x^2}\, dx =$

(b) $\Delta x\, \Delta p \geq$

2.14 continued $\langle T \rangle = \dfrac{\langle p^2 \rangle}{2m}$, $\langle V \rangle = \frac{1}{2}k\langle x^2 \rangle$

$= \dfrac{m\omega^2}{2}\langle x^2 \rangle$

$\psi_0 \sim e^{-\xi^2/2}$ $\langle p^2 \rangle =$ $\langle x^2 \rangle =$

$\langle T \rangle =$ $\langle V \rangle =$

$\langle H \rangle_0 = \langle T \rangle + \langle V \rangle =$

$\psi_1 \sim \xi e^{-\xi^2/2}$ $\langle p^2 \rangle =$ $\langle x^2 \rangle =$

$\langle T \rangle =$ $\langle V \rangle =$

$\langle H_1 \rangle = \langle T \rangle + \langle V \rangle$

recall $E_n = \hbar\omega(n + \frac{1}{2})$ for Harmonic Oscillator