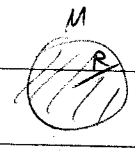
14 Nov 02 – Mechanics Ch 6
Gravitation + central forces.

203 $\vec{F}_g = \frac{G m_1 m_2}{r^2}\,\hat{r}$ between two masses, $G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

$\vec{F} = m\vec{a}$ — m = gravitational mass, m = inertial mass) equivalent!

208 (#6.2 :



$F_{outside}$ = F due ~~to~~ M concentrated at O

$F_{inside} = 0$

209 KEPLER'S LAWS: (2) : $\vec{L} = \vec{r} \times \vec{p}$ = $2m\frac{dA}{dt}$ constant for central force, where $d\bar{A} = \frac{1}{2}|\bar{r}\times d\bar{r}|$ p. 212

EQUAL AREAS in equal times — torque $= \bar{r}\times\bar{F} = \frac{d\bar{L}}{dt}$ (1) $\bar{r} \parallel \bar{F}$

211 DERIVE $L = mr^2\dot{\theta}$ (p. 211) from $\bar{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$ and $\bar{r} = r\hat{e}_r$!

K1 ELLIPTICAL ORBITS: HW: FILL IN DETAILS of § 6.5 pp 212-13

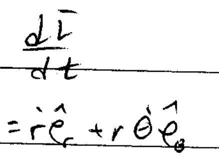
$\vec{F} = m\ddot{\vec{r}}$, define $l = \frac{L}{m} = r^2\dot{\theta}$, trick: $r = \frac{1}{u}$ $\rightarrow$

$$\frac{d^2u}{d\theta^2} + u = -\frac{f}{ml^2u^2}$$

215 HW: FILL IN DETAILS FOR INVERSE SQUARE LAW: $f(r) = -\frac{k}{r^2}$

$\rightarrow \frac{d^2u}{d\theta^2} + u = \frac{k}{ml^2}$ has solution $\frac{1}{u} = r = \frac{\alpha}{1+\epsilon\cos\theta}$, $\alpha = \frac{ml^2}{k}$, $\epsilon = \frac{Aml^2}{k}$

$\epsilon = \frac{Aml^2}{k} = \frac{AL^2}{mk}$
- $= 0$: circle
- < 1 : ellipse
- ≥ 1 : unbound



(K3) $T^2 \propto a^3$: $F = ma$

$$\frac{GmM}{r^2} = m\frac{v^2}{r}$$

$$\frac{GM}{r} = v^2 = \left(\frac{2\pi r}{T}\right)^2$$

$$T^2 = \underline{\qquad} r^3$$

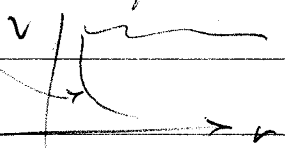
§6.6 p. 21

HW: SHOW this works for elliptical orbits

1846

Examples: Adams + Leverrier predict Neptune based on perts to Uranus

Keplerian velocity distribution



observed in galaxies

→ DARK MATTER

228 POTENTIAL ENERGY: $\vec{F} = -\vec{\nabla} V$

g = field intensity $\quad \frac{\vec{F}}{m} = \vec{g} = -\nabla \Phi \quad \Phi = \frac{V}{m}$ = potential

CENTRAL FIELD: $f(r) = \frac{k}{r^2} \rightarrow V(r) = -\frac{k}{r} = -ku$

232 ENERGY OF ORBIT IN CENTRAL FIELD

(6.9.1) $\quad \vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta \longrightarrow v^2 = \vec{v}\cdot\vec{v} =$ ______

(6.9.2) $\quad E = V(r) + T =$ ______

write in terms of $l = \frac{L}{m} = r^2\dot{\theta}$ to get

(6.11.1a) $\quad E = \frac{m}{2}\dot{r}^2 + \left(\frac{m}{2}\frac{l^2}{r^2} + V(r)\right)$

(b) $\quad E = \frac{m}{2}\dot{r}^2 + U(r) \quad$ where $U(r)$ = EFFECTIVE POTENTIAL.