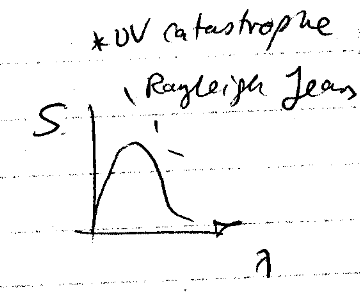
Modern Physics - Wk 2 - Ch 3 - Thurs 16 Jan 03

QUANTA OF ENERGY




Blackbody = perfect absorber = perfect emitter

Stefan Boltzmann: Intensity $\propto$ Temperature$^4$

$S = \sigma T^4$, $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$

Wien's Law: $\lambda_{max} T = 3 \times 10^{-3}\ m \cdot K$

Ex1/80: $S_{\odot\, at\, \oplus} = 10^3 \frac{W}{m^2} = \sigma T^4 \rightarrow T_{\oplus} = \left(\frac{S}{\sigma}\right)^{\frac{1}{4}} = 364K = 91°C$

Earth is cooler because some sunlight is reflected, & Earth radiates

DO Prob 4/102

Rayleigh's explanation for blackbody $\rightarrow$ UV catastrophe

EQUIPARTITION: ENERGY EQUALLY DISTRIBUTED

$S_\nu = \frac{\text{intensity}}{\text{frequency}} = \frac{c}{4} u_\nu = \frac{c}{4} \frac{\text{energy}}{\text{volume}} = kT \cdot n$

harmonic oscillators on every point of cavity wall

Tipler 78/105

where $n = \frac{\#\text{ of modes of oscillation}}{\text{volume}} = \frac{8\pi}{\lambda^4} \longrightarrow S_\nu = \frac{8\pi k T}{\lambda^4}$

Fine at long wavelengths, but $S \rightarrow \infty$ for short $\lambda$.

Planck's explanation for blackbody $\longrightarrow$ QUANTIZATION

Guess $S_\nu = \frac{2\pi h}{c^2} \frac{\nu^3}{e^{h\nu/kT} - 1}$

where $\nu$ = frequency
$h$ = phenomenological constant $= 6.63 \times 10^{-34}\ J \cdot s$

Do #10

Assume harmonic oscillators have a DISTRIBUTION of energies $f(E)$
Assume these energies are discrete: $E = nh\nu$
Calculate average oscillator energy $\times \frac{\#\text{ oscillators}}{\text{volume}} \rightarrow$ TA DA
ENERGY NOT EQUALLY DISTRIBUTED.

# Planck's blackbody - worksheet

$\frac{\text{Energy}}{\text{frequency interval}} = \text{Average } \frac{\text{energy} \times \text{number of modes}}{df} = \frac{\text{energy}}{\text{volume}} \cdot \text{volume}$

(0) $$\frac{E \cdot dn}{df} = u \cdot V$$

(1) we showed before that Intensity of radiation $S = \frac{uc}{4} \to u =$ ______

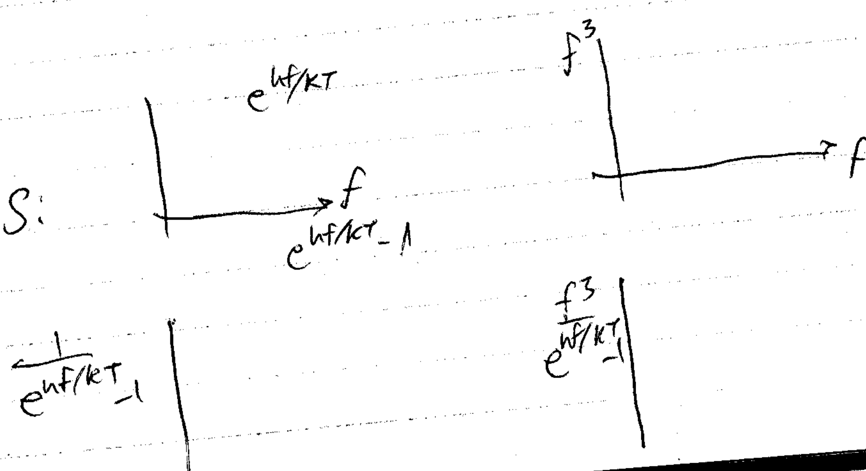
(2) Number of vibration modes per freq interval $\frac{dn}{df} = V \frac{8\pi f^2}{c^3}$

Assume energy is QUANTIZED! $E = nhf$ ($n = 0, 1, 2 \ldots$) (§8.5)

(3) Average oscillator energy (p. 84) $\langle E \rangle = \frac{hf}{e^{hf/kT} - 1}$ DERIVE attached p. 84

Sub (1),(2),(3) into (0) to get $S$:

Sketch shape of $S$:



$$E = nh\nu \qquad n = 0, 1, 2, 3, \ldots \tag{14}$$

The integer $n$ is called the **quantum number** of the oscillator.

With this quantization condition, Planck calculated the average energy of th oscillator as follows. From statistical mechanics, it is known that, at thermal equi librium, the probability that an oscillator (or any other system) be found in a stat of energy $E$ is proportional to the exponential factor $e^{-E/kT}$. This exponential facto is called the **Boltzmann factor** (we will discuss the Boltzmann factor in mor detail in Chapter 8). For an oscillator of energy $E = nh\nu$, the Boltzmann factor i $e^{-nh\nu/kT}$, and therefore the average energy of the oscillator is

$$\begin{aligned}\bar{E} &= \sum_n nh\nu \times (\text{probability for } nh\nu) \\ &= \sum_n nh\nu \times \frac{e^{-nh\nu/kT}}{\sum_{n'} e^{-n'h\nu/kT}}\end{aligned} \tag{15}$$

where the summation in the denominator is the constant of proportionality tha converts the Boltzmann factor into a probability, normalized to 1. For convenienc we write $x = e^{-h\nu/kT}$, so

$$\bar{E} = h\nu \frac{\sum_n nx^n}{\sum_{n'} x^{n'}} \tag{16}$$

To evaluate this, we note that the infinite $\sum x^n$ is simply a geometric series. As i well known, such a geometric series has the value

$$\sum x^n = \frac{1}{1-x} \qquad \text{for } |x| < 1 \tag{17}$$

If we differentiate this equation with respect to $x$, we obtain

$$\sum nx^{n-1} = \frac{1}{(1-x)^2}$$

and if we multiply by $x$, we obtain

$$\sum nx^n = \frac{x}{(1-x)^2} \tag{18}$$

Substituting (17) and (18), respectively, into the denominator and the numerator Eq. (16), we find

$$\bar{E} = h\nu \frac{x/(1-x)^2}{1/(1-x)} = h\nu \frac{x}{1-x} = h\nu \frac{1}{1/x - 1} = \frac{h\nu}{e^{h\nu/kT} - 1} \tag{19}$$

## 3.3 PHOTOELECTRIC EFFECT : ML by Andy

$\frac{3.3}{86}$

Einstein (1905) used Planck's idea (1900) that $E = h\nu = \hbar\omega$

QUANTA of LIGHT = PHOTONS → Bose-Einstein distribution

$f =$

1887 Hertz observed that light can knock electrons off metal

$$\text{Energy of photon} = \begin{pmatrix}\text{kinetic}\\ \text{energy of electron}\end{pmatrix} + \begin{pmatrix}\text{work done}\\ \text{on metal}\end{pmatrix}$$

Do #24 $$h\nu = K + \phi$$

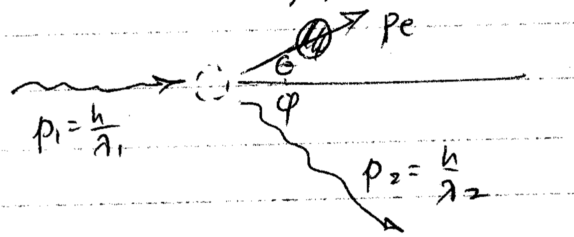
## 3.4 COMPTON EFFECT : collisions change energy of scattered ~~photons~~ X-rays

Classical scattering of light

||| → e ← ||| →

incoming light of $E = \hbar\omega = \frac{hc}{\lambda}$ oscillates e, which radiates light of same $\omega$, $\lambda$

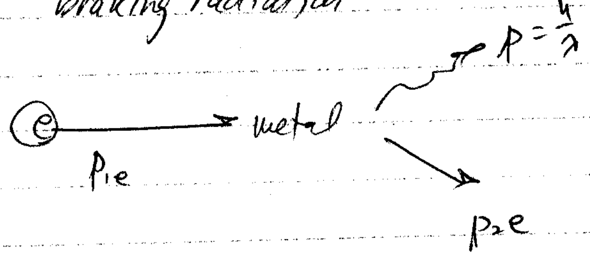
QM scattering of light : $\frac{T87}{116}$




Do #33

REVERSE process = BREMSSTRAHLUNG = "braking radiation"

electrons collide → decelerate
→ radiate

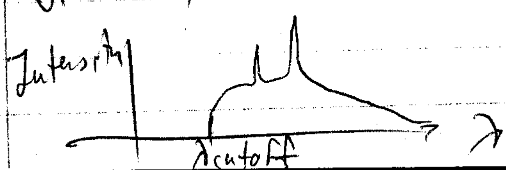



Characteristic x-rays

If e loses ALL its energy, $K = \frac{hc}{\lambda_{cutoff}} = eV_0$

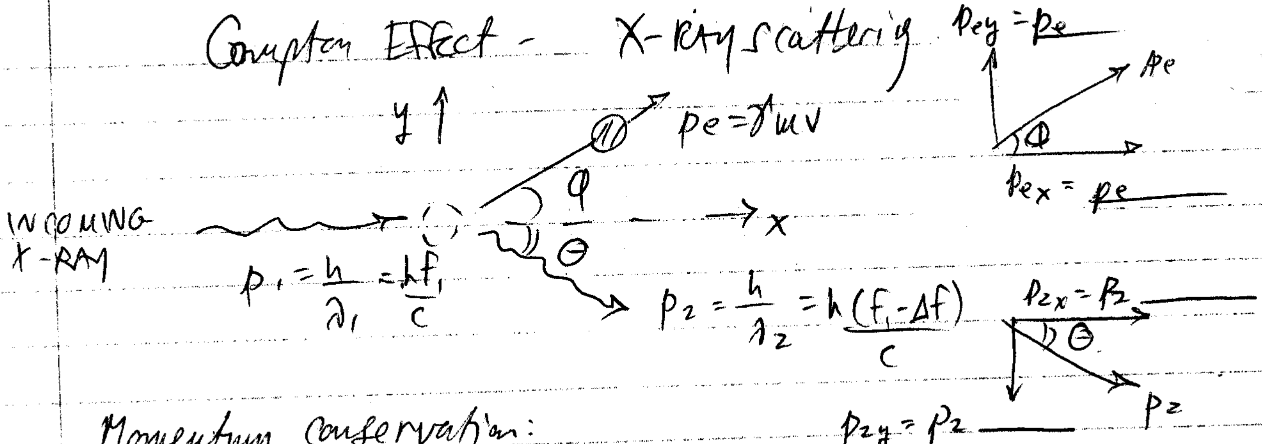
$V_0$ ↑ potential which accelerated e

Do #44



Worksheet

Compton Effect - X-ray scattering




$p_e = \gamma m v$

$p_1 = \frac{h}{\lambda_1} = \frac{hf_1}{c}$

$p_2 = \frac{h}{\lambda_2} = \frac{h(f_1 - \Delta f)}{c}$

$p_{ey} = p_e$ ______

$p_{ex} = p_e$ ______

$p_{2x} = p_2$ ______

$p_{2y} = p_2$ ______

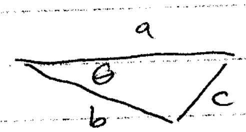
Momentum conservation:

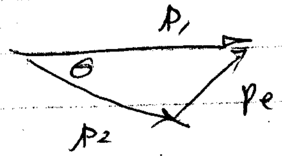
$\Sigma p_x(\text{before}) = \Sigma p_x(\text{after})$ $\qquad$ $\Sigma p_y(\text{before}) = \Sigma p_y(\text{after})$

$p_1 = p_{ex} + p_{2x}$ $\qquad$ $0 = p_{ey} - p_{2y}$

We want $\lambda_2$ in terms of $\lambda_1$, $m$, and $\theta$. TRIG TRICK:

Law of cosines




$c^2 = a^2 + b^2 - 2ab\cos\theta$

$p_e^2 =$

Find $p_e^2$ in terms of $\left(\frac{hf_1}{c}\right)^2 \{\text{function of } \cos\theta, \left(1 - \frac{\Delta f}{f_1}\right)\}$

**33.** A photon of initial energy $2.4 \times 10^3$ eV is deflected by 120° in a collision with a free electron, initially stationary. What energy does the electron acquire in this collision?

Giancoli 955, Tipler 115

Compton Continued...

$$\frac{p_e^2}{2} = \left(\frac{hf_1}{c}\right)^2\left(1-\frac{\Delta f}{f_1}\right)(1-\cos\theta)$$

Energy lost by X-ray photon = Energy gained by electron

SUB IN

Collect $\Delta f$ terms

$$= \frac{h}{mc}(1-\cos\theta)$$

$$\Delta\lambda = (\lambda+\Delta\lambda) - \lambda = \left(\frac{c}{f-\Delta f}\right) - \frac{c}{f} =$$

Eliminate $f$, $\Delta f$ terms: $\Delta\lambda =$

Calculate Compton factor $\frac{h}{m_e c} =$

Practice: Problem