Physical Systems - week 4 Mon. 27 Jan 03

p. 140 Modern Physics Ch. 5

de Broglie wavelength: for light, $E = pc = hc/\lambda$

for particles, $p = \frac{h}{\lambda} = mv$

$\lambda(v) =$ _____. Write this in terms of $E = \frac{p^2}{2m}$:

↑E Prob 1 / 167 $\lambda(E) =$ _____

Giancoli Ex 38-9 / 971 Calculate the de Broglie wavelength for a $m = 0.20$ kg ball with speed $v = 15$ m/s.

2.2×10^{-34} nm

We showed last week that λ_e leads to Bohr's quantization condition $L = n\hbar$ for the H atom.

1927 DAVISSON-GERMER experiment: Ni crystal diffracted e⁻ EVIDENCE for e⁻ waves

Set up Prob. 14

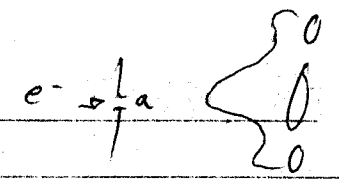
TEM Transmission Electron Microscope :: optical microscope
Illuminate with: e⁻ :: photons
Focus with: magnetic lenses :: glass lenses
Prob. 12 Resolution ~ 10^{-1} nm > λ_e :: $\lambda \sim 500$ nm

SEM SCANNING Electron Microscope: 3D image! (Res ~ 10 nm)
- secondary e⁻ emitted by sample
reveal depth as well as 2D surface features.

$h = 6.63 \times 10^{-34}$ J·s $= 4.14 \times 10^{-15}$ eV·s

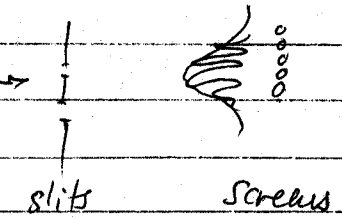
$\Psi(x,t)$

WAVE-PARTICLE-DUALITY: (Δp small) (Δx small)



e^- as waves: Individual electrons fired through a single slit show diffraction!
" " " " " double slit " interference!

e^- as particles: Yet each hits the screen as a particle-like point.



Light as waves: interference & diffraction
Light as particles: photo-electric effect.

Griffiths 110 UNCERTAINTY relations: in QM, we derived $\Delta x \Delta p \geq \frac{\hbar}{2}$ mathematically.

WORKSHEET: derive UNCERTAINTY PRINCIPLE heuristically:

Do Problem 24. Set up Prob. 26 or 28

p. 152 Uncertainty in Bohr atom: $\Delta p > p = \frac{mv}{a_0} = \frac{\hbar}{a_0}$

Prob. 28 NEITHER MOMENTUM NOR ORBIT IS WELL-DEFINED!

COMPLEMENTARITY (of PARTICLE & WAVE nature of QM objects)
the more it resembles a particle (Δx small)
the less it resembles a wave (Δp small)
and vice-versa: $\Delta x \Delta p \neq 0$

We derived Schrödinger's wave equation earlier:

$H\Psi = E\Psi = (T+V)\Psi$

$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$ where $\Psi = \Psi(x,t) = e^{-iwt}\psi(x)$

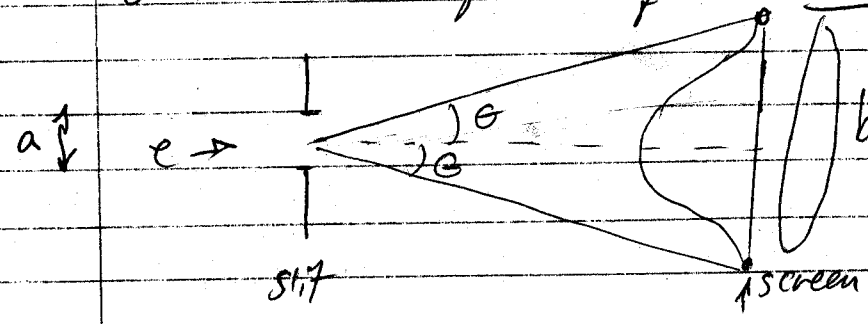
Prob. 38 $E = \hbar w$

Δt: Characteristic time for evolution of wave packet with E: $\Delta E \Delta t \geq \frac{\hbar}{2}$

WORKSHEET - Modern Physics - 27 Jan 03

Heuristic derivation of UNCERTAINTY PRINCIPLE.

Send an e⁻ (or proton...) through a slit of width $a = \Delta y$ = uncertainty in position



diffraction minimum at $a \sin\theta = \lambda$

Width of diffraction pattern $\Delta\theta \geq$ _____

(Hint: for small angles, $\sin\theta \approx \theta$)

momentum = p

p_y = _____ or zero (could be zero)

p_x = _____ (or more: could be as great as $p_x = p$)

Uncertainty in momentum Δp_y = _____

Recall de Broglie relation: $p =$ _____

Substitute this into $\Delta p \geq$ _____

$\frac{2\pi}{\Delta y}$

1. Show that the de Broglie wavelength of an electron of energy E can be expressed as

$$\lambda = \frac{12.26}{\sqrt{E}}$$

where λ is measured in angstroms and E in electron-volts.

4. The electron-microscope picture of Fig. 5.6b was made with an electron beam of energy 40 keV. What is the de Broglie wavelength of such electrons?

12. Since the wavelength of visible light is ≈ 5000 Å, a light microscope cannot resolve any details smaller than this length. What must be the minimum energy of the electrons in an electron microscope if it is to attain a resolution at least as good as a light microscope? (In practice, energies used in electron microscopes are usually more than 10 keV, and their resolution is not limited by the wave properties of the electrons.)

24. Suppose that the x-component of the velocity of an electron has been measured to an accuracy of 10^{-2} m/s. What is the minimum uncertainty in its position along the x-axis? Along the y-axis? Solve the same problem if the particle is a proton.

28. According to de Broglie's naive picture of closed waves traveling around the orbit of an electron in a hydrogen atom, what would be the wavelength of such a wave in the ground state? According to the Uncertainty Principle, what would be the uncertainty in the wavelength? What conclusion can you draw?

38. An electron of 20 keV in the beam of a TV tube moves from the electron gun at the rear of the tube to the picture screen at the front, a distance of 30 cm. At the electron gun, the wave packet has an extent of 10^{-7} m, measured along the direction of the beam; and its uncertainty in momentum is the minimum permitted by the Heisenberg relation. How much does this wavepacket spread while the electron moves to the picture screen?