

X _____

I affirm that I have worked this exam independently, without texts, outside help, integral tables, calculator, solutions, or software. (Please sign legibly.)

Read this cover page completely before you start.

*Read through the whole exam before you start, and do the easiest problems first.
Don't stay on any one problem for more than half an hour.
Move on and come back to a problem later if necessary.*

*You may use your HOMEWORK ONLY for reference on this exam.
If you feel you need more information, don't hesitate to ask Zita.
There are no trick questions.*

*This is designed as a one-hour exam. You have two hours to do it.
Solutions will be made available after all exams are turned in.*

*Please show your work, and explain your methods concisely, for full credit.
Do the problems in the simplest meaningful way you know.
Leave expressions in their simplest exact form (e.g. $2\pi/\sqrt{3}$).
Always include units where appropriate, label axes, and so forth.*

*Please indicate your answer clearly, e.g. by boxing it.
If you need more space for your work, feel free to attach additional sheets or use the back.
Direct me to look on back if necessary, or staple any extra sheet(s) in order neatly.*

A meaningful method and demonstrated understanding of concepts is more important than memorized equations or a numerical answer.

***** Leave the space below for Zita to make notes*****

Qualitative questions about conductors fields potentials Gauss law

Div Grad Curl Conservative Forces

Gauss Law: general H atom sketching

q(r)

E(r)

V(r)

A. Qualitative questions about ELECTROSTATICS:

1. Materials which conduct are good conductors because they:
 - (a) are metal
 - (b) have unattached protons
 - (c) are not insulators
 - (d) have many free electrons

2. The electrostatic field of a charged particle
 - (a) increases directly with distance
 - (b) increases inversely with the distance
 - (c) increases directly with the square of the distance
 - (d) decreases directly with the square of the distance

3. Electric field lines can cross only if both positive and negative charges are involved.
TRUE or FALSE?
Why?

4. What can you say about each of these? Constant? Zero? Variable?

 E field inside a hollow metal object

 Potential V inside a hollow metal object

 E field along the surface of a conductor

 Potential V along the surface of a conductor

 Perpendicular E field across a sheet of charge

 Perpendicular V across a sheet of charge

5. Gauss' law can be applied using any surface you like.
TRUE or FALSE?
Why?

6. Outside a spherically symmetric charge distribution of net charge Q, Gauss' law can be used to show that the electric field at a given distance
 - (a) must be directed outward
 - (b) must be directed inward
 - (c) must be zero
 - (d) acts as if it originated in a point charge Q at the center of the distribution

7. An advantage in evaluating surface integrals in Gauss' law for symmetric charge distributions is that
 - (a) the electric field is always a constant on any surface
 - (b) the electric field is of constant magnitude on certain surfaces
 - (c) the charge is always on the surface
 - (d) the flux is outward

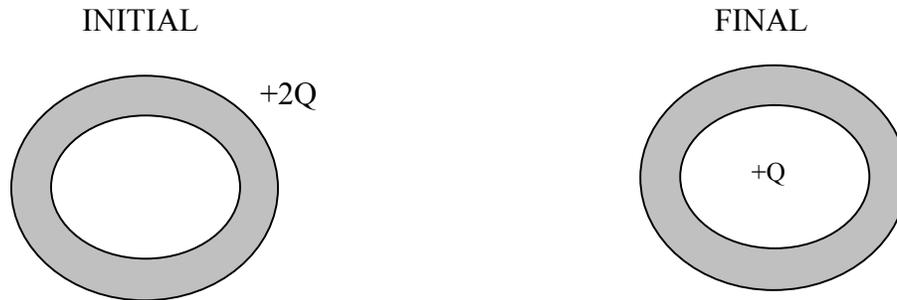
B. Basic relations and operations

1. Find the **divergence, gradient, and curl** of $\mathbf{E} = (2xy - z^3) \mathbf{i} + x^2 \mathbf{j} - (3xz^2 + 1) \mathbf{k}$.

2. Is \mathbf{E} (above) conservative? How can you tell?

3. Consider a hollow conductor with a net charge of $+2Q$ on it (initially).
A point charge of $+1Q$ is then placed inside the conductor (without touching).
Draw and describe the final charge distribution on the conductor.
(No calculations necessary!)

- (a) on the inner surface, the total charge is _____
- (b) inside the material of the conductor, the total charge is _____
- (c) on the outer surface, the total charge is _____



4. What **charge** would a raindrop have to carry to be suspended against gravity in the Earth's field of 100 V/m ? Assume the raindrop has a radius of 1 mm . Water has density 10^3 kg/m^3 . Show your work.

C. Gauss' Law

(a) For a spherically symmetric charge distribution of volume charge density $\rho(r)$ and total radius a , **show how to find the charge $q(r)$** enclosed by a radius ($r < a$), in general. Specify the limits of all integrals and evaluate them as far as possible, without knowing the functional form of the radial dependence of $\rho(r)$. Hint: $d\tau = r^2 dr \sin \theta d\theta d\phi$, and $\rho(r)$ is independent of θ and ϕ .

(b) Now apply your general rule above to the special case of the electron cloud of a neutral Hydrogen atom, roughly modeled by $\rho(r) = \rho_0 a/r$ (where a is the Bohr radius and ρ_0 is a constant such that the total amount of charge is $-e$, the charge of one electron). **Find and plot $q(r)$.**

(c) What is the total charge Q enclosed in the sphere of radius a ?

(d) **Show** how to find the electric field for an arbitrary spherically symmetric charge distribution of radius a , if you know the charge $q(r)$ enclosed by a given radius r . Draw a diagram and specify your Gaussian surface. **Explain** each step concisely.

(e) Apply Gauss' law to the electron cloud of the Hydrogen atom to find its **electric field**.

(f) **Plot** the electric field vs. r and **describe** how you would find the potential everywhere.
(optional: find the potential if you have extra time).

How do you feel about this exam, now that you have taken it?

What question(s) do you wish had been on this exam?