Help Sheet for Classical Mech HW #2 - EZ

10 Oct 02

2.14 A particle of mass m is released from rest a distance b from a fixed origin of force that attracts the particle according to the inverse square law:

$$F(x) = -kx^{-2} = ma = m\frac{dv}{dt} = mv\frac{dv}{dx}$$

Show that the time required for the particle to reach the origin is

$$t = \pi\left(\frac{mb^3}{8k}\right)^{1/2}$$

Separable integral: $\int_0^v v\,dv = \int_0^x -\frac{k}{m}\frac{dx}{x^2}$

You should find $v = A\sqrt{\frac{b-x}{x}}$ where $A =$ ________

To get $x(t)$, we must integrate $v = \frac{dx}{dt} = A\sqrt{\frac{b-x}{x}}$

Another separable integral: $\int \sqrt{\frac{x}{b-x}}\,dx = \int A\,dt$

Trick: to integrate $\frac{x}{b-x} = \frac{x/b}{b/b - x/b} = \frac{x/b}{1-x/b}$, let $\frac{x}{b} = \sin^2\theta$

Then $dx =$ ________

Show that $\int \sqrt{\frac{x}{b-x}}\,dx = 2b \int \sin^2\theta\, d\theta$ and

look this up in Schaum $\frac{14.21}{p.58}$ $\int \sin^2 u \, du = \frac{1}{2}(u - \sin u \cos u)$

Now consider the limits.

$x = b$; $\sin^2\theta =$ ____ : $\theta =$ ____

$x = 0$: $\sin^2\theta =$ ____ : $\theta =$ ____

Show that $\int_{x=b}^{0} \sqrt{\frac{x}{b-x}}\,dx = b(\theta - \sin\theta\cos\theta)\Big|_{\theta=-\pi/2}^{\theta=0}$

and finish the problem!