Mechanics week 3 - Ch 3 - oscillations

Hw p 128 #1 5 7 8 9 10 21 22 C3.1

3.1 A guitar string vibrates harmonically with a frequency of 512 Hz (one octave above middle C on the musical scale). If the amplitude of oscillation of the centerpoint of the string is 0.002 m (2 mm), what are the maximum speed and the maximum acceleration at that point?

$f = 512 \frac{1}{s}$

$A = 2 \times 10^{-3}\ m$

Harmonic oscillation: $x = A \sin(\omega_0 t + \phi_0)$, $v = \frac{dx}{dt}$

amplitude

phase given by $x(0)$

angular frequency $\omega_0 = 2\pi f = \frac{2\pi}{\text{period}}$

$\omega_0 = 2\pi \cdot 512 \frac{1}{s} = \_\_\_ \frac{rad}{sec}$

Speed $v = \frac{dx}{dt} =$

$v_{max} =$

acceleration $a =$

$a_{max} =$

3.5 A particle undergoing simple harmonic motion has a velocity $\dot{x}_1$ when the displacement is x_1 and a velocity $\dot{x}_2$ when the displacement is x_2. Find the angular frequency and the amplitude of the motion in terms of the given quantities.

$x(t) = A\cos(\omega t + \varphi)$ $\quad$ $v(t) = A\omega \sin(\omega t + \varphi)$

$x_1 = A\cos(\omega t_1 + \varphi)$ $\quad$ $v_1 = A\omega \sin(\omega t_1 + \varphi)$

$x_2 = A\cos(\omega t_2 + \varphi)$ $\quad$ $v_2 = A\omega \sin(\omega t_2 + \varphi)$

How to solve these for A and ω? Easier with ENERGY: $T = \frac{1}{2}mv^2$, $V = \frac{1}{2}kx^2$ can be used for any simple harmonic oscillator

Recall relation between ω, k, m:

$$F = -kx = ma = m\ddot{x} = -m\omega^2 x$$

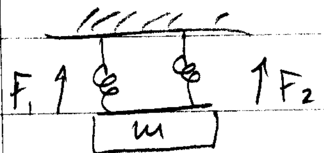
$$k = m\omega^2 \rightarrow \omega = \sqrt{\frac{k}{m}}$$

Total energy $= T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

3.7 Two springs having stiffness k_1 and k_2, respectively, are used in a vertical position to support a single object of mass m. Show that the angular frequency of oscillation is $[(k_1 + k_2)/m]^{1/2}$ if the springs are tied in parallel, and $[k_1 k_2/(k_1 + k_2)m]^{1/2}$ if the springs are tied in series.

PARALLEL



$$\Sigma F = ma$$

$$F_1 + F_2 = m\ddot{x}$$

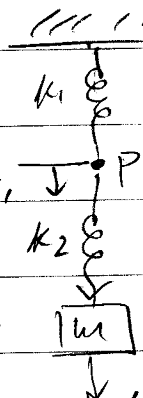
If $x = A\cos\omega t$
then $\ddot{x} = -A\omega^2\cos\omega t = -\omega^2 x$

$$-k_1x - k_2x = -m\omega^2 x$$

$$k_1 + k_2 = m\omega^2$$

$$\omega_{par} = \sqrt{\frac{k_1+k_2}{m}}$$

SERIES



displacement of spring joint $= x_1$

$\uparrow k_1x_1$ = upward force on 2 due to 1

$\downarrow k_{eq}x$ = downward force on spring 2

$F = k_{eq}x$

x = displacement of mass

Consider forces where springs join at P

Spring 2 stretches a distance $(x - x_1)$

$\uparrow k_1x_1$
$\downarrow k_2(x-x_1)$

Equal & opposite forces: $k_1x_1 = k_2(x - x_1)$

$$x_1(k_1+k_2) = k_2 x$$

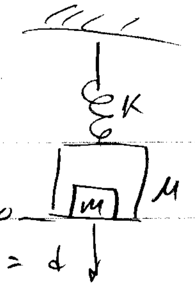
$$x_1 = \frac{k_2}{k_1+k_2}x$$

Forces on Spring 2:

$$k_{eq}x = k_1x_1 = k_1\left(\frac{k_2}{k_1+k_2}\right)x$$

$$k_{eq} = \frac{k_1k_2}{k_1+k_2} \rightarrow \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$ (like resistors in parallel)

3.8 A spring of stiffness k supports a box of mass M in which is placed a block of mass m. If the system is pulled downward a distance d from the equilibrium position and then released, find the force of reaction between the block and the bottom of the box as a function of time. For what value of d will the block just begin to leave the bottom of the box at the top of the vertical oscillations? Neglect any air resistance.



$$\sum F_{M+m} = m_{tot}\, a$$

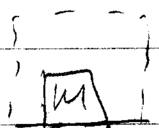
$$-kx = (M+m)\ddot{x}$$

Solve for $x(t)$ = position of bottom of either block:

① $x(t) =$

$$\omega = \sqrt{\frac{k}{m+M}}$$

Now consider the forces on inner mass alone:



mg ↓ ↑ F_r = force due to bottom of big box = reaction force

$$\sum F_m = ma$$

$$mg - F_r = m\ddot{x}$$

Sub in ①:

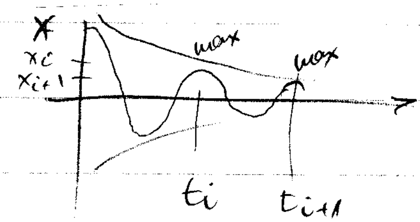
Solve for

② $F_r = mg + m\omega^2 d\cos\omega t$

When the mass m just loses contact with box M at $x = -$

$F_r = 0$; solve for d:

3.9 Show that the ratio of two successive maxima in the displacement of a damped harmonic oscillator is constant. (*Note:* The maxima do not occur at the points of contact of the displacement curve with the curve $Ae^{-\gamma t}$.)



Damped harmonic oscillator:

(3.4.15) p.87 $\quad x = Ae^{-\gamma t}\cos(\omega_d t + \varphi)$

$$\frac{dx}{dt} =$$

Max at $\frac{dx}{dt} = 0 =$

which boils down to

$$\tan(\omega_d t + \varphi) = -\frac{\gamma}{\omega_d}$$

So relative maxima happen once per period, where $T = \frac{2\pi}{\omega_d}$

i-th max : $x_i = Ae^{-\gamma t_i} A\cos(\omega_d t_i + \varphi)$

(i+1)th max : $x_{i+1} = Ae^{-\gamma t_{i+1}} A\cos(\omega_d t_{i+1} + \varphi)$ (where

$= Ae^{-\gamma(t_i + T)} A\cos(\omega_d t_i + \varphi)$ $\quad t_{i+1} = t_i + T$

$$\frac{x_{i+1}}{x_i} = \underline{\qquad\qquad}$$