Mechanics Ch 4 - 7 Nov 02 - Zita @ fac meeting

Hw # 1, 2, 3, 4, 6, 17, 18

4.1 Find the force for each of the following potential energy functions: $\vec{F} = -\vec{\nabla} U$

(a) $U = cxyz + C$

(b) $U = \alpha x^2 + \beta y^2 + \gamma z^2 + C$

(c) $U = ce^{-(\alpha x + \beta y + \gamma z)}$

(d) $U = cr^n$ in spherical coordinates

(d) $\vec{F} = -\hat{e}_r \frac{\partial U}{\partial r} - \hat{e}_\theta \frac{1}{r}\frac{\partial U}{\partial \theta} - \hat{e}_\varphi \frac{1}{r\sin\theta}\frac{\partial U}{\partial \phi} = -\hat{e}_r\, cnr^{n-1}$

4.2 By finding the curl, determine which of the following forces are conservative: If $\vec{\nabla}\times\vec{F} = 0$

(a) $\mathbf{F} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$

(b) $\mathbf{F} = \mathbf{i}y - \mathbf{j}x + \mathbf{k}z^2$

(c) $\mathbf{F} = \mathbf{i}y + \mathbf{j}x + \mathbf{k}z^3$

(d) $\mathbf{F} = -kr^{-n}\mathbf{e}_r$ in spherical coordinates $\nabla\times F = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\varphi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\varphi \\ F_r & rF_\theta & r\sin\theta\, F_\varphi \end{vmatrix}$

4.4 A particle of mass m moving in three dimensions under the potential energy function $U(x, y, z) = \alpha x + \beta y^2 + \gamma z^3$ has speed v_0 when it passes through the origin.

(a) What will its speed be if and when it passes through the point (1, 1, 1)?

(b) If the point (1, 1, 1) is a turning point in the motion ($v = 0$), what is v_0?

(c) What are the component differential equations of motion of the particle?

(*Note:* It is *not* necessary to solve the differential equations of motion in this problem.)

$E = \text{constant} = U(x,y,z) + \frac{1}{2} m v^2$. At the origin,

$E_0(x=0, y=0, z=0) = \underline{\qquad\qquad} + \frac{1}{2} m v_0^2$

(a) $E_1(1, 1, 1) = \underline{\qquad\qquad} + \frac{1}{2} m v_1^2$. Let $E_1 = E_0$, solve for v:

(b) If $v_1 = 0$ then solve $E_0 = E_1$ for v_0:

(c) $\vec{F} = m\vec{a} = m\ddot{\vec{x}} = -\frac{\partial U}{\partial x}\hat{x}_i$ is equivalent to

$m\ddot{x} = -\frac{\partial U}{\partial x}$ $\qquad m\ddot{y} = -\frac{\partial U}{\partial y}$ $\qquad m\ddot{z} = -\frac{\partial U}{\partial z}$

4.6 Show that the variation of gravity with height can be accounted for approximately by (a) the following potential energy function:

$$V = mgz\left(1 - \frac{z}{r_e}\right)$$

(b) in which r_e is the radius of the Earth. Find the force given by the above potential function. From this find the component differential equations of motion of a projectile un- (c) der such a force. If the vertical component of the initial velocity is v_{0z}, how high does (d) the projectile go? (Compare with Example 2.3.2.)

From Ex. 2.3.2 (and we did this in class week 2!)

53 $\quad F(z) = -mg\dfrac{r_e^2}{(r_e+z)^2} \rightarrow V(z) = -mg\dfrac{r_e^2}{r_e+z}$

~~App D~~ A.9 For small x, $((1+x)^n = 1 + nx + \frac{1}{2}n(n-1)x^2 + \ldots$ small terms

Let $\left(x = \frac{z}{r_e}\right)$: $\left(1+\frac{z}{r_e}\right)^{-1} =$

Write $V(z)$ in terms of x, then apply that expansion.

(b) $\vec{F} = -\vec{\nabla} V = -\hat{k}\dfrac{\partial}{\partial z} V(z) =$

4.6 (a) Diffeq is $\vec{F} = m\vec{a}$:

$F_x = m\ddot{x}$ $F_y = m\ddot{y}$ $F_z = m\ddot{z}$

(d) Solve the diffeq as you usually do for space-dependent forces:

$$F_z(z) = m\ddot{z} = m\frac{d^2z}{dt^2} = m\frac{dz}{dt}\frac{d\dot{z}}{dz} = m\dot{z}\frac{d\dot{z}}{dz}$$

$$\int_{v_{0z}}^{0} \dot{z}\,d\dot{z} = \int_0^h \frac{F(z)}{m}\,dz$$ Integrate, evaluate, solve for h.

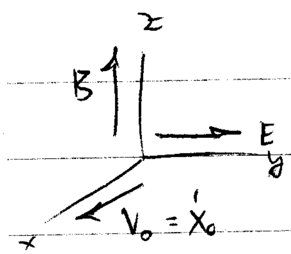
4.17 An electron moves in a force field due to a uniform electric field **E** and a uniform magnetic field **B** that is at right angles to **E**. Let $\mathbf{E} = \mathbf{j}E$ and $\mathbf{B} = \mathbf{k}B$. Take the initial position of the electron at the origin with initial velocity $\mathbf{v}_0 = \mathbf{i}v_0$ in the x direction. Find the resulting motion of the particle. Show that the path of motion is a cycloid:

$$x = a \sin \omega t + bt \qquad b = \dot{x}_0 - \omega a$$

$$y = a(1 - \cos \omega t) \qquad a = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega \dot{x}_0\right)$$

$$z = 0$$

Cycloidal motion of electrons is used in an electronic tube called a magnetron to produce the microwaves in a microwave oven.



This is very similar to what we're doing in E&M Ch 5!

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = m\vec{a}$$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dot{X}_0 + \dot{X} & \dot{y} & \dot{z} \\ 0 & 0 & B \end{vmatrix} =$$

$F_x = \_\_\_\_\_\_ = m\ddot{x} \qquad F_y = \_\_\_\_\_\_ = m\ddot{y} \qquad F_z = \_\_\_\_\_\_ = m\ddot{z}$

- Integrate or differentiate to uncouple equations and solve for $(\dot{x} - \dot{x}_0) = \frac{qB}{m} y$ and $\dot{y} = -A\omega \sin(\omega t + \Theta_0)$
- Then apply initial conditions $\dot{y}(0) = 0$ to find Θ_0
  $y(0) = 0$ to find A
- Integrate $\dot{x}$ and $\dot{y}$ to get $x(t)$ and $y(t)$

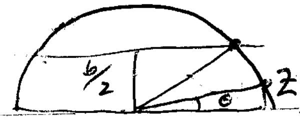
TRICK: Show that $\ddot{y} + \omega^2 y = -\frac{eE}{m} + \omega \dot{x}_0$ has solution

$$y = \frac{1}{\omega^2}\left(-\frac{eE}{m} + \omega \dot{x}_0\right) + A\cos(\omega t + \Theta_0)$$

to get $\dot{y} = 0 \qquad -A\omega \sin(\omega t + \Theta_0)$

Sub $y = a(1 - \cos \omega t)$ into $\dot{x}$ equation.

$h_i = b/2$



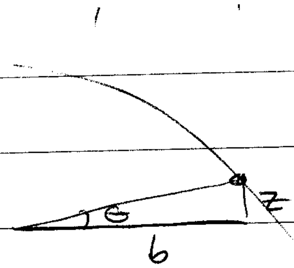
4.18 A particle is placed on a smooth sphere of radius b at a distance $b/2$ above the central plane. As the particle slides down the side of the sphere, at what point will it leave?

$E_i = E_f$ when it falls off at same height z

$\frac{1}{2} m h_i^2 = \frac{1}{2} m v^2 + \frac{1}{2} m h_f^2$ $\qquad h_f = z$

Solve for v above; $\quad v^2 =$

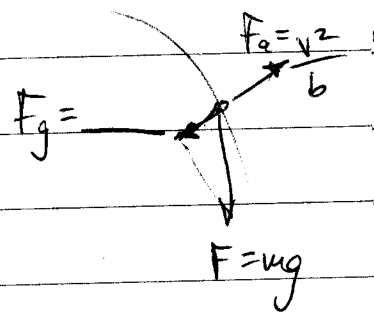
Now from $\quad F = -\frac{mv^2}{r} = \frac{-mv^2}{b}$



$\frac{z}{b} =$ ________

Ball falls off when

$F_g = F_a$



Find z where the ball falls off.