EM

Ex 5.2 Cycloid motion $\vec{F} = m\vec{a}$ p.206

$\vec{E} = E_z \hat{k}$

$\vec{B} = B_x \hat{i}$

$$q(\vec{E} + \vec{v} \times \vec{B}) = m\ddot{x} + m\ddot{y} + m\ddot{z}$$

$$q(B\dot{z}\hat{j} - B\dot{y}\hat{k} + E\hat{k}) = m\ddot{y}\hat{j} + m\ddot{z}\hat{k}$$

$\hat{j}\,(qB\dot{z} = m\ddot{y})$

$qB\ddot{z} = m\dddot{y}$

$\ddot{z} = \frac{m\dddot{y}}{qB}$

$\hat{k}\,(qE - qB\dot{y} = m\ddot{z})$

$qE - qB\dot{y} = \frac{m^2 \ddddot{y}}{qB}$

Let $\dot{y} = u$, $\ddot{y} = \dot{u}$

$\frac{qB}{m^2}(E - Bu) = \dot{u}$

Let $(E - Bu) = p$, $-B\dot{u} = \dot{p}$

$\frac{q^2B}{m^2} p = -\frac{\ddot{p}}{B}$

$\frac{q^2B^2}{m^2} p = -\ddot{p}$

Guess $p = Ae^{i\omega t}$, $\ddot{p} = -\omega^2 p = -\frac{q^2B^2}{m} p$

$$\omega = \frac{qB}{m}$$

$p = Ae^{i\omega t} = (E - Bu) = (E - B\dot{y})$

$\frac{Ae^{i\omega t} - E}{-B} = \dot{y} = \frac{E - Ae^{i\omega t}}{B} = \frac{E}{B} - Ae^{i\omega t}$

$$y = \int \dot{y}\,dt = \int \left(\frac{E}{B} - Ae^{i\omega t}\right) dt$$

$$y = \frac{E}{B}t - \frac{A}{i\omega} e^{i\omega t} + C_3$$

$$y = \frac{E}{B}t + C_1 e^{i\omega t} + C_3 = \frac{E}{B}t + C_1 \cos\omega t + C_1 \sin\omega t + C_3$$

$\dot{z} = \frac{m}{qB}\ddot{y} = \frac{\ddot{y}}{\omega}$

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$z = \int \frac{dz}{dt}\,dt = \frac{1}{\omega}\int \ddot{y}\,dt = \frac{1}{\omega}\dot{y} = \frac{1}{\omega}\left[\frac{E}{B} - \omega C_1 \sin\omega t + \omega C_2 \cos\omega t + C_3\right]$

$= C_4 - C_1 \sin\omega t + C_2 \cos\omega t$ ✓

$-\frac{d\Phi}{dt}$