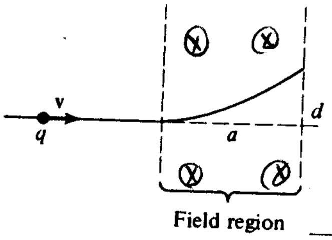
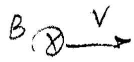
Problem 5.1 A particle of charge q enters a region of uniform magnetic field $\mathbf{B}$ (pointing *into* the page). The field deflects the particle a distance d above the original line of flight, as shown in Fig. 5.8. Is the charge positive, or negative? In terms of a, d, B, and q, find the momentum of the particle.

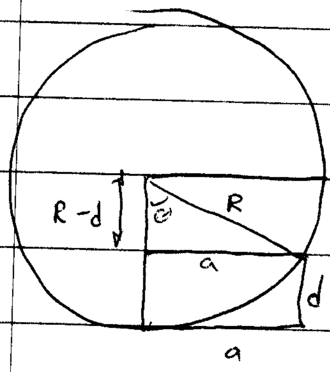


$\uparrow \bar{F} = q\bar{v} \times \bar{B}$



First find p in terms of radius of curvature R and B & q.

Then use geometry to find R in terms of a & d.



Finally, sub in $R(a,d)$ into $p(R,B,q)$ to get $p(a,d,B,q)$

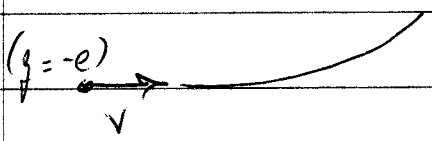
Problem 5.3 In 1897 J. J. Thomson "discovered" the electron by measuring the charge-to-mass ratio of "cathode rays" (actually, streams of electrons, with charge q and mass m) as follows:

(a) First he passed the beam through uniform crossed electric and magnetic fields **E** and **B** (mutually perpendicular, and both of them perpendicular to the beam), and adjusted the electric field until he got zero deflection. What, then, was the speed of the particles (in terms of E and B)?

(b) Then he turned off the electric field, and measured the radius of curvature, R, of the beam, as deflected by the magnetic field alone. In terms of E, B, and R, what is the charge-to-mass ratio (q/m) of the particles?

(a) As in #1 above, with $\bar{E}=0$, $\bar{B}$ deflects the beam:

DRAW:

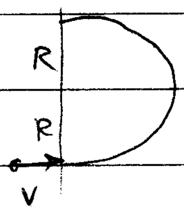


$\uparrow \; \bar{F}_B = q\bar{v} \times \bar{B}$ Is B in or out of page?

$\downarrow \; \bar{F}_E = q\bar{E}$ What direction is $\vec{E}$?

Find $|\vec{v}| = v =$ speed required to yield $\Sigma\vec{F} = \vec{F}_B - \vec{F}_E = 0$

(b)



When $\bar{E}=0$, it is easy to measure R due to bending $\bar{B}$. Use your v above to find $\frac{q}{m}$

Problem 5.4 Suppose the magnetic field in some region has the form

$$\mathbf{B} = kz\hat{\mathbf{i}} \quad (k \text{ is some constant})$$ greater B at higher z, as in Fig(b)

Find the force on a square loop of side s, lying in the yz plane, centered at the origin, which carries a current I, flowing counter clockwise, looking down the x-axis or clockwise, looking into origin

Fig. a

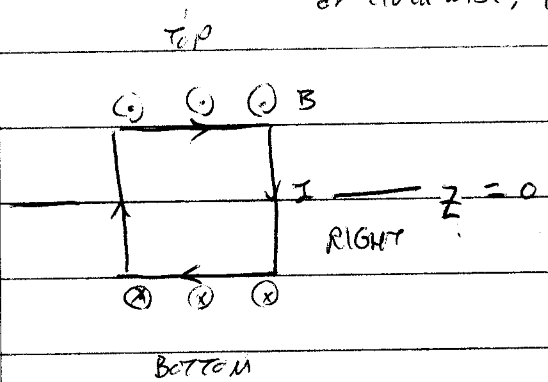
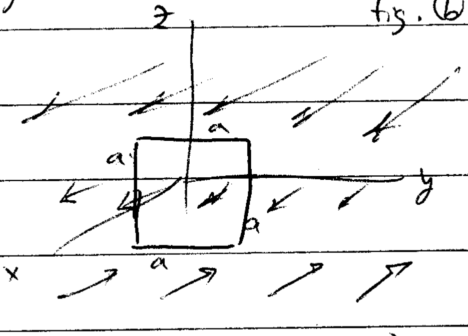


Fig. (b)



(B<0 when z<0)

Draw the force vectors $\vec{F} = \vec{I}\ell \times \vec{B}$ at every point on loop in Fig. 1a. Find the overall direction of net force

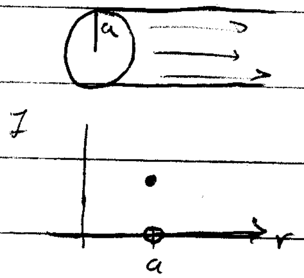
Then calculate the value of the total force on each side of length a, using $\vec{F} = I \oint d\vec{l} \times \vec{B}$.

Problem 5.5 A current I flows down a wire of radius a.

(a) If it is uniformly distributed over the surface, what is the surface current density K?

(b) If it is distributed in such a way that the volume current density is inversely proportional to the distance from the axis, what is J?

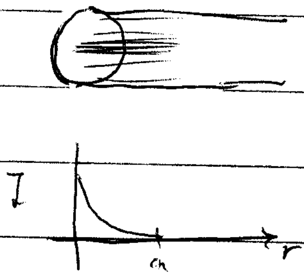
(a)



Surface current density

$$K = \frac{\text{current}}{\text{circumference}} =$$

(b)



volume current density $J = \frac{k}{r}$

$J = \frac{\text{current}}{\text{area}}$ so current $I = \int J(r)\, da$

where $da = r\,dr\,d\theta$

First evaluate $I = \int_0^a \frac{k}{r}\, da =$

Then solve for constant $k =$

Substitute in k to find $J = \frac{k}{r} =$

(c) what if $J = kr$? find $I(r)$ (You'll use this in #5.13)

Problem 5.12 Suppose you have two infinite straight line charges λ, a distance d apart, moving along at a constant speed v (Fig. 5.26). How great would v have to be in order for the magnetic attraction to balance the electrical repulsion? Work out the actual number... Is this a reasonable sort of speed?[8]

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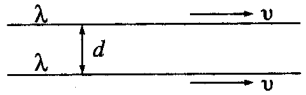
[8]If you've studied special relativity, you may be tempted to look for complexities in this problem that are not really there—λ and v are both measured *in the laboratory frame*, and this is *ordinary electrostatics* (see footnote 4).



Consider the electrostatic repulsion of λ_1 by the field E_2 of λ_2:

(1) $F = q_1 E_2 \rightarrow \frac{F}{l} = \lambda_1 E_2$ where $\oint E_2 \cdot da = \frac{q}{\epsilon_0}$. find the field $E_2(d)$ a distance d away from the source λ_2.

(2) $E_2(d) =$

Substitute (2) into (1) to find the mutual electrostatic repulsion of the two lines of charge: $\frac{F_E}{l} =$

Now consider the magnetostatic attraction of I_1 by the field B_2: current $\frac{dq}{dt} = \frac{dq}{dx}\frac{dx}{dt} = \lambda v$. First find the field due to I_2:

(3) $\oint B_2 \cdot dl = \mu_0 I_2 = \ldots$ $B_2(d) =$ ______

Then find the force between the two currents, in terms of v, λ, and d:

(4) $\frac{|F_M|}{l} = |\bar{I}_1 \times \bar{B}_2| = I_1(\qquad) =$

Finally, consider what is required for these two forces to balance:

(2) $$\frac{F_E}{l} = \frac{F_M}{l}$$ (4)

At what speed v does this relationship hold?

(electrostatic)
permittivity of free space $\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

(magnetostatic) permeability of free space $\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$