CM.

Ch 6 HW #2, 3, 5, 6, 7, 9, ~~23~~ 26 (a) (not C6.2)

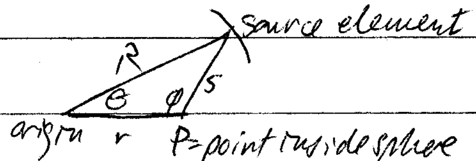
p 248 Also derive results in §6.6 (b) pp 215-216, and (c) 6.137

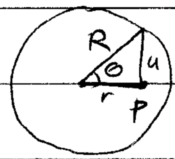
**6.2** Show that the gravitational force on a test particle inside a thin uniform spherical shell is zero at P

**(a)** By finding the force directly

**(b)** By showing that the gravitational potential is constant

See §6.2 p. 208




$r$ = position of point where we want to know the field

$R$ = radius of sphere

$s$ = distance from source (mass element) and pt. of interest

(6.22) Force on P due to entire shell $\Delta F = \frac{Gm 2\pi\rho R^2 \sin\theta \cos\varphi \, \Delta\theta}{s^2}$

(6.23) $$F = Gm 2\pi\rho R^2 \int_0^{\pi} \frac{\sin\theta \cos\varphi \, d\theta}{s^2}$$

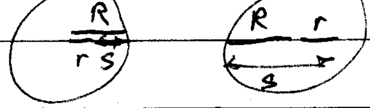
(6.24) Law of Cosines: $r^2 + R^2 - 2rR\cos\theta = s^2$

$\theta$ ← angle opposite side $s$

(6.25) Differentiate ($r$ & $R$ are constant) $s\,ds =$

(6.26) $\cos\varphi = \frac{s^2 + r^2 - R^2}{2rs}$

Substitute everything into (6.23). HINT: limits of $ds$ integral are $\int_{R-r}^{R+r} \ldots ds$



**6.3** Assuming Earth to be a uniform solid sphere, show that if a straight hole were drilled from pole to pole, a particle dropped into the hole would execute simple harmonic motion. Show also that the period of this oscillation depends only on the density of Earth and is independent of the size. What is the period in hours? ($R_{earth} = 6.4 \times 10^6$ m.)

Assume uniform: $\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi r^3} \rightarrow m(r) =$ ________

Find $\vec{F}(r) =$ ________

$\vec{F}(r) = -k r \hat{e}_r = m\ddot{r}\,\hat{r}$ where $k =$ ________

This has SHO solutions $r(t) =$ ________ (guess)

sub in $\ddot{r}$ to find $\omega = \frac{2\pi}{T}$

Solve for $T$, in general and at surface of Earth

**6.5** Assuming a circular orbit, show that Kepler's third law follows directly from Newton's second law and his law of gravity: $GMm/r^2 = mv^2/r$.

$F = ma \qquad a = \frac{v^2}{r}$ , Use $v = \frac{2\pi r}{T}$

@

Also FILL IN DETAILS of §6.6, p221 to show you get the Same expression for ELLIPTICAL orbits.

**6.6** **(a)** Show that the radius for a circular orbit of a synchronous (24-h) Earth satellite is about 7 Earth radii.

**(b)** The distance to the Moon is about 60 Earth radii. From this calculate the length of the month (period of the Moon's orbital revolution).

Use Ex. 6.5.3

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**6.7** Show that the orbital period for an Earth satellite in a circular orbit just above Earth's surface is the same as the period of oscillation of the particle dropped into a hole drilled through Earth (see Problem 6.3).

(b)

Also FILL IN DETAILS for the derivation of INVERSE SQUARE LAW, pp 215 - 216.

**6.9** If the solar system were embedded in a uniform dust cloud of density $\rho$, show that the law of force on a planet a distance $r$ from the center of the Sun would be given by

$$F(r) = -\frac{GMm}{r^2} - \left(\frac{4}{3}\right)\pi\rho m G r$$

Find the mass of dust $M_d(r)$ inside an orbit radius $r$.

Then find the force due to the dust: $F_d = -\frac{G M_d m}{r^2}$

$F_{total} = F_{due\ to\ Sun} + F_{due\ to\ dust}$

**6.26** **(a)** Show that a circular orbit of radius $r$ is stable in Problem 6.22 if $r$ is less than $b^{-1}$.

**(b)** Show that circular orbits are unstable in an inverse-cube force field.

a

$$f(r) = -k\frac{e^{-br}}{r^2}$$

a FIRST, show that the stability condition for a circular orbit is (6.13.7) p. 245.

Then apply that condition to the force in a and

b $f = \dfrac{-k}{r^3}$.