Physical Systems - winter qtr - 13 Jan 03
EJZ

Answers to QM HW

Ch 1 # 1, 3, 6, 7, 12, 14 ; Ch 2 # 2, 6, 13, 14, 17, 19, 38

Ch 1 #1 $459.57 = \langle j^2 \rangle$, $\langle j \rangle^2 = 441$, $\sigma = 4.309$

#3 $\rho(\theta) = \begin{cases} \frac{1}{\pi} & \text{for } 0 \le \theta \le \pi \\ 0 & \text{otherwise} \end{cases}$ $\langle \theta \rangle = \frac{\pi}{2}$, $\langle \theta^2 \rangle = \frac{\pi^2}{3}$, $\sigma = \frac{\pi}{2\sqrt{3}}$

$\langle \sin\theta \rangle = \frac{2}{\pi}$, $\langle \cos\theta \rangle = 0$, $\langle \cos^2\theta \rangle = \frac{1}{2}$

#6 $A = \sqrt{\frac{\lambda}{\pi}}$, $\langle x \rangle = a$, $\langle x^2 \rangle = a^2 + \frac{1}{2\sqrt{\lambda}}$, $\sigma = (4\lambda)^{-1/4}$

#7 $A = \sqrt{\frac{3}{b}}$ $P = \frac{a}{b}$ $\langle x \rangle = \frac{2a+b}{4}$

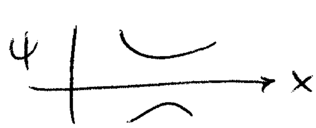
#12 Hint: $\int_{-\infty}^{\infty} \left[\psi^* \frac{\partial^3 \psi}{\partial x^3} - \frac{\partial^2 \psi^*}{\partial x^2} \frac{\partial \psi}{\partial x}\right] dx = 0$, integrating by parts twice

and $\left[V \psi^* \frac{\partial \psi}{\partial x} - \psi^* \frac{\partial}{\partial x}(V\psi)\right] = -|\psi|^2 \frac{\partial V}{\partial x}$

#14 $A = \left(\frac{2am}{\pi\hbar}\right)^{1/4}$, $V(x) = 2ma^2 x$, $\langle x \rangle = 0$, $\langle x^2 \rangle = \frac{\hbar}{4am}$

$\langle p \rangle = 0$ $\langle p^2 \rangle = am\hbar$ $\sigma_x = \sqrt{\frac{\hbar}{4am}}$ $\sigma_p = \sqrt{am\hbar}$

Ch 2 #2 Hint:



but $\lim_{x \to \pm\infty} \psi = 0$.

#6 $A = \sqrt{\frac{1}{2}}$ $\psi = \frac{1}{\sqrt{a}} e^{-i\omega t}\left[\sin\left(\frac{\pi}{a}x\right) + \sin\left(\frac{2\pi x}{a}\right) e^{-3i\omega t}\right]$

$\langle x \rangle = \frac{a}{2}\left[1 - \frac{32}{9\pi^2}\cos(3\omega t)\right]$ $\langle p \rangle = \frac{8\hbar}{3a}\sin(3\omega t)$

$\langle H \rangle = \frac{5\pi^2\hbar^2}{4ma^2} = \frac{1}{2}(E_1 + E_2)$

2.13 $A_1 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{\hbar\omega}}$ $\psi_2 = A_2\hbar\omega\left(1 - \frac{2m\omega}{\hbar}x^2\right)e^{-m\omega x^2/2\hbar}$

2.14 $\langle x\rangle = 0$ $\langle p\rangle = 0$

	$\langle x^2\rangle$	$\langle p^2\rangle$	$\sigma_x\sigma_p$	$\langle T\rangle$	$\langle V\rangle$
$n=0$:	$\frac{\hbar}{2m\omega}$	$\frac{m\hbar\omega}{2}$	$\frac{\hbar}{2}$	$\frac{\hbar\omega}{4}$	$\frac{\hbar\omega}{4}$
$n=1$:	$\frac{3\hbar}{2m\omega}$	$\frac{3m\hbar\omega}{2}$	$\frac{3\hbar}{2}$	$\frac{3\hbar\omega}{4}$	$\frac{3\hbar\omega}{4}$

2.17 $A = \sqrt{\frac{1}{2}}$, $\psi = \frac{1}{\sqrt{2}}e^{-i\omega t/2}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}e^{-\xi^2/2}\left(1+\sqrt{2}\,\xi\cos\omega t\right)$

$\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}\cos\omega t$, $\langle p\rangle = m\sqrt{\frac{\hbar}{2m\omega}}\left(-\omega\sin\omega t\right)$

$\psi = \frac{1}{\sqrt{2}}e^{-i\omega t/2}\left(\psi_0 + \psi_1 e^{-i\omega t}\right)$

2.19 $C = A+B$, $D = i(A-B)$, $F = 2\sqrt{|A|^2+|B|^2}$ $G = 2|A|$

$\tan 2\alpha = \frac{Im(A/B)}{Re(A/B)}$ $\tan(2\beta+\pi) = \frac{Im(A/B)}{Re(A/B)}$