A SECOND ORDER REACTION

$$a\,A + b\,B \longrightarrow \text{Products}$$

α = extent of reaction

	A	B	P
$t=0$	A_0	B_0	0
# of moles reacted	$-a\alpha$	$-b\alpha$	α
t	$A_0 - a\alpha$	$B_0 - b\alpha$	α
$\therefore$ Conc. at time t	$[A]$	$[B]$	$[P]$

$$\text{Rate} = \frac{d[P]}{dt} = k[A][B] = k[A_0 - a\alpha][B_0 - b\alpha]$$

$$\therefore \frac{d\alpha}{dt} = k[A_0 - a\alpha][B_0 - b\alpha]$$

$$\int \frac{d\alpha}{[A_0 - a\alpha][B_0 - b\alpha]} = \int k\,dt \qquad \text{——} (1)$$

$$\frac{1}{[A_0 - a\alpha][B_0 - b\alpha]} = \frac{P}{[A_0 - a\alpha]} + \frac{Q}{[B_0 - b\alpha]}$$

$$\therefore 1 = P[B_0 - b\alpha] + Q[A_0 - a\alpha]$$

$$\text{Let } \alpha = \frac{B_0}{b} \Rightarrow 1 = Q\left[A_0 - \frac{a B_0}{b}\right) \Rightarrow Q = \frac{1}{\left(A_0 - \frac{a B_0}{b}\right)}$$

$$\text{Let } \alpha = \frac{A_0}{a} \Rightarrow 1 = P\left[B_0 - \frac{b A_0}{a}\right] \Rightarrow P = \frac{1}{\left(B_0 - \frac{b A_0}{a}\right)}$$

$$\therefore \frac{1}{[A_0 - a\alpha][B_0 - b\alpha]} = \frac{1}{\left(B_0 - \frac{b A_0}{a}\right)(A_0 - a\alpha)} + \frac{1}{\left(A_0 - \frac{a B_0}{b}\right)(B_0 - b\alpha)} \qquad (2)$$

① and ② ⇒

$$\int_0^{\alpha} \frac{d\alpha}{\left(B_0 - \frac{bA_0}{a}\right)(A_0 - a\alpha)} + \int_0^{\alpha} \frac{d\alpha}{\left(A_0 - \frac{aB_0}{b}\right)(B_0 - b\alpha)} = \int_0^{t} k\,dt$$

$$\frac{-(1/a)}{\left(B_0 - \frac{bA_0}{a}\right)} \left[\ln (A_0 - a\alpha)\right]_0^{\alpha} + \frac{-(1/b)}{\left(A_0 - \frac{aB_0}{b}\right)} \left[\ln (B_0 - b\alpha)\right]_0^{\alpha} = kt$$

$$\frac{-(1/a)}{\left(\frac{aB_0 - bA_0}{a}\right)} \ln\left(\frac{A_0 - a\alpha}{A_0}\right) - \frac{(1/b)}{\frac{(bA_0 - aB_0)}{b}} \ln\left(\frac{B_0 - b\alpha}{B_0}\right) = kt$$

$$\left(\frac{1}{bA_0 - aB_0}\right) \ln\left(\frac{A_0 - a\alpha}{A_0}\right) - \left(\frac{1}{bA_0 - aB_0}\right) \ln\left(\frac{B_0 - b\alpha}{B_0}\right) = kt$$

$$\left(\frac{1}{bA_0 - aB_0}\right) \left[\ln\left(\frac{A_0 - a\alpha}{A_0}\right) - \ln\left(\frac{B_0 - b\alpha}{B_0}\right)\right] = kt$$

$$\frac{1}{bA_0 - aB_0} \left[\ln\left(\frac{A_0 - a\alpha}{A_0}\right)\left(\frac{B_0}{B_0 - b\alpha}\right)\right] = kt$$

$$\Rightarrow \frac{1}{bA_0 - aB_0} \ln \frac{[A]B_0}{A_0[B]} = kt$$

This is eqⁿ 18.27 in your text book.