Chapter 3

Probability

This chapter introduces the general concepts of probability which form the basis for many of the other ideas covered in this book. Try to keep the big picture in mind – given some initial probabilities, we can use some simple rules to determine combined probabilities. Since the chapter uses dice as its main example, you might want to have some dice on hand. There are also some links to flipping coins and rolling dice on the Web. You may find the dice simulation especially helpful.

3.1 Vocabulary

Make sure you understand each of the following terms well enough to use them appropriately when discussing situations which involve probability:

- random experiment
- elementary outcome
- sample space
- probability
- event
- logical and
- logical or
- logical not
- conditional probability
- independence

3.2 Symbols

It is important to be able to immediately recognize some mathematical symbols and even be able to translate them into words. Here are some important symbols from this chapter:

- $P(X)$ – the probability that X occurs or just the probability of X
- $P(A \text{ or } B)$ – the probability that either A occurs or B occurs (which includes the possibility that both occur)
- $P(A \text{ and } B)$ – the probability that both A and B occur
- $P(\text{ not } A)$ – the probability that A does not occur
- $P(A|B)$ – translated as 'probability of A given B' – the probability that A occurs under the condition that B is known to occur.

3.3 Equations, specific rules, etc.

This chapter has several formulas you should be able to use.

- Probabilities are always numbers from 0 to 1: $0 \leq P(X) \leq 1$.
- The sum of the probabilities of all possible outcomes is 1.

$$\sum_i P(X_i) = 1$$

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- $P(\text{ not } A) = 1 - P(A)$ This should not need explanation. Think of $P(\text{ not } A)$ as the probability that something besides A happens.
- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ This makes sense when you realize that $P(A \text{ and } B)$ gets counted both in $P(A)$ and in $P(B)$, so it must be subtracted out once to compensate for being counted twice.
- If A and B are mutually exclusive then $P(A \text{ and } B) = 0$, so $P(A \text{ or } B) = P(A) + P(B)$.
- $P(A \text{ and } B) = P(A|B)P(B) = P(B|A)P(A)$ This formula is not at all obvious, so it is best to memorize it. You can figure out why it works, but it is not clear at first glance.
- If A and B are independent then $P(A|B) = P(A)$, so $P(A \text{ and } B) = P(A)P(B)$.

3.4 Specific notes

- pp. 27-34: Aside from some history, the point of these pages is to get across a rather simple idea – you can assign a numerical probability to each of the possible outcomes in a situation.

- p. 35: This page is very important. It is difficult, if not impossible, to determine the actual probabilities of most events. One must assume certain probabilities and then work from there. These assumptions can be based on rules (classical probability, sometimes called theoretical probability), repeated measurements (relative frequency) or one's gut (personal probability). One of the goals of science is to conduct investigations which move more knowledge from the realm of personal probability to the realm of relative frequency or even to theoretical probability.

- pp. 36-44: If you find these pages confusing, just focus on the summary at the top of page 44. Work through these formulas. If you understand them, you are fine. Try the exercises below to make sure you understand them.

- pp.46-51: Although Bayes theorem leads to some very important results concerning conditional probabilities, it is not as important as the rest of chapter 3 in terms of forming a foundation for the rest of the book. So if Bayes theorem confuses you, don't fret. Notice that it has not been included in either the vocabulary or the equations of this study guide.

- p. 48: Think carefully about what each part of the box represents here. Think of a pregnancy test. (A and B) means you are pregnant and the test says you're pregnant. (not A and not B) means you are not pregnant and the test says you're not pregnant. These two are the desired results of a good test. (A and not B) means that you are pregnant but the test says you're not! The test has failed, and the failure is called a false negative. (not A and B) means that you are not pregnant even though the test says you are. This is another failure of the test, and it is called a false positive. When it comes to tests, it is often important to consider false negatives as well as false positives. Sometimes one is more important than the other; but in general, both are undesirable.

3.5 Exercises

Exercise 3.1: You are given a normal deck of 52 cards.

(a) What is the probability of randomly drawing an ace from the deck?

(b) What is the probability of randomly drawing a diamond?

(c) What is the probability of randomly drawing an ace of diamonds? Does this make sense both from the rules of probability and from what you know about the deck?

(d) What is the probability of randomly drawing an ace or a diamond? Does this make sense both from the rules of probability and from what you know about the deck?

(e) If you draw a card and someone tells you it is a diamond, what is the probability it is an ace? Does this make sense both from the rules of probability and from what you know about the deck?

Exercise 3.2: A room filled with 100 people has 40 who are registered Republicans, 40 who are registered Democrats, and 10 who are Independents. 70 of the people are female, and half of the females are registered Democrats.

(a) With regard to political affiliation, what are the elementary outcomes of selecting people from the room; and what are the probabilities associated with each of these outcomes?

(b) Ignoring the gender of the people, define a reasonable sample space for the task of selecting two people from the room.

For the following questions, you should be able to get the answers by using the probability rules or by using common sense. Do it both ways!

(c) If you randomly select one person from the room, what is the probability that the person is not a Democrat?

(d) If you randomly select one person from the room, what is the probability that the person is a female Democrat?

(e) If you randomly select one person from the room, what is the probability that the person is either female or a Democrat?

(f) If you randomly select one of the males in the room, what is the probability that he is a Democrat?

(g) If you randomly select two people from the room, what is the probability that both people are registered Democrats?

(h) If you randomly select two people from the room, what is the probability that at least one of them is a Democrat?