Chapter 7

Confidence Intervals

As the text says, this chapter is basically the reverse of chapter 6. In chapter 6, you had known distributions and you found that samples taken from those distributions followed some nice patterns. So in this chapter, you will start with samples (which is where you will generally start in real investigations) and make claims about unknown characteristics of the population from which the samples were taken. Unfortunately, the text makes things look a little more complicated than they need to be for our purposes. In reality, this chapter is pretty simple and makes a lot of sense.

This is one chapter where it might be best to focus on the material presented here in the study guide and just skim the chapter in the book.

7.1 Vocabulary

There is no new vocabulary in this chapter, although there is more focus on standard error, the t-distribution, and the 95% confidence interval to which you have been introduced in previous chapters.

7.2 Equations, specific rules, etc.

The formulas in this chapter are the same as in the previous chapter except that now we admit that we have no knowledge of the population and we are thus limited to the parameters we can get from the sample. We use these sample parameters to quantify our claims about the full population.

- To estimate the probability of success in the full population, you can report a range around the probability of success $\hat{p}$ found in the sample. The size of this range depends on how confident you want to be. Usually, a 95% confidence interval is sufficient, but you should keep in mind that other intervals can be reported.

A 95% confidence interval is created simply by using the 'two standard deviations' rule except that you use the standard error instead of the standard deviation. The standard error is defined as

$$SE_{\hat{p}} = \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$$

where n is the sample size. (Notice the similarity to and the difference from the forumula for $\sigma_{\hat{P}}$ in the previous chapter.)

So... you can claim with 95% confidence that the probability of success in the full population is in the interval $\hat{p} \pm 2SE_{\hat{p}}$.

Notice that the size of the standard error (and therefore the size of the confidence interval) depends on the sample size. Larger sample size reduces the standard error, providing a smaller confidence interval.

- To estimate the mean of some full population, you can report a range around the mean $\bar{x}$ of the sample. Again, the size of the range depends on the desired confidence level. For a 95% confidence interval you again use twice the standard error where the standard error is defined as

  $$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$$

  where s is the standard deviation of the sample and n is the sample size.

  So... you can claim with 95% confidence that the mean of the full population is in the interval $\bar{x} \pm 2SE_{\bar{x}}$.

  Again, the standard error depends on the sample size; so the confidence interval can be made smaller by increasing the sample size.

- The estimate of the mean given above is only good for large sample sizes. For smaller sample sizes, you have to admit that you know less by reporting a larger interval, even for the same 95% confidence level. Instead of using twice the standard error, you use a factor defined by the t-distribution. To find that factor, you have to look it up on a table or use a software package. The factor depends only on the desired confidence level and the degrees of freedom of the sample which is $n-1$, one less than the sample size.

  For an example, look at the table at the bottom of p. 132 in the text. The first row of the table is the confidence level. So for 95% confidence, you would use the second-to-last column. If you had a sample size of only 11, you would need to use a factor of 2.23 instead of the factor of 2 used for the usual 95% confidence interval. So the range you would report would be $\bar{x} \pm 2.23SE_{\bar{x}}$.

That's it! Again, the ideas are very simple. The complexity in the text is mostly about using z and t tables to look up factors for different confidence levels. If you are confident about the ideas discussed above, you can push yourself further by reading the chapter. Otherwise, just read pp. 110-113 and stick with the ideas above.

7.3 Exercises

Exercise 7.1: You take random sample of 100 people and find that 30% of them are left-handed. What can you report with 95% confidence about the population as a whole? How would what you report change if you got the same data from a sample 0f 10,000 people?

Exercise 7.2: You take a random sample of 100 women and find that the data for height of the women have a mean of 66 inches and a standard deviation of 3 inches. What can you report with 95% confidence about the population as a whole? How would what you report change if you got the same data from a sample 0f 10,000 people? What if the sample size were only around 10?

Notice that these exercises do not start with "Assume that it is known...." These are the types of questions you will use in the real world! You take the sample, measure what you want to measure, and report a claim about the population as a whole.