Chapter 8

Hypothesis Testing

While chapter 7 was one chapter of *The Cartoon Guide to Statistics* worth skipping, chapter 8 is a very good chapter. It presents motivations, examples, and good extra intormation. Take the time to read the text. Read it more than once!

8.1 Vocabulary

Again, much of the vocabulary of the chapter is based on that of previous chapters. The new terms which need to be mastered are

- Null hypothesis
- Alternate hypothesis
- Test statistic
- p-value
- Significance level
- Type I error
- Type II error

8.2 Equations, specific rules, etc.

The key to understanding this chapter is to remember the z-score discussed in chapter 2. This was defined as $z = \frac{x-\bar{x}}{s}$. We are now very familiar with $\bar{x}$ and s, and we know that the standard deviation s is often replaced by the standard error SE in cases where we are taking random samples as a way of estimating some characteristic of a population. So it should not be surprising to see, in this chapter, a z-score-like thing where the standard error is in the denominator.

The rules in this chapter follow the themes set up by chapters 6 and 7. Again, there are two types

of questions – frequency questions and numerical measurements. Frequency questions are assessed by using $\hat{p}$ and its associated standard error. Numerical measurements are assessed by using $\bar{x}$ and its associated standard error. As in chapters 6 and 7, if a small sample size is used, the t-statistic is used instead of the z-statistic.

As a summary, here are the test statistics you should use in each case

- For estimates of frequency or proportion:

$$z = \frac{\hat{p} - p_0}{SE_{\hat{p}}} \qquad \text{where } SE_{\hat{p}} = \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$$

- For estimates of means where you have a large sample size:

$$z = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} \qquad \text{where } SE_{\bar{x}} = \frac{s}{\sqrt{n}}$$

- For estimates of means where you have a small sample size, use the t distribution rather than the z distribution to determine the p-value.

8.3 Specific notes

- pp. 137-139: These pages give an example and provide some motivation for the types of questions you may want to answer using the tools of this chapter.
- pp. 140-142: These pages give a good summary of the general way to formulate questions about hypotheses. The method of identifying a null hypothesis and an alternative hypothesis may seem tedious and unnecessary, but the clarification provided by taking this step will help prevent silly mistakes in your analyses.
- pp. 142-145: These pages take the general method of the chapter and apply it to specific questions about frequency or proportion.
- p. 144: The formula in Step 2 on this page is just a z-score with the standard error in the denominator. Once you have a z-score, you can asses whether the value you observed ($\hat{p}$) is just an expected variation of a measurement of your null hypothesis p_0 or whether it is different enough to support the claim that p_0 is incorrect.
- pp. 146-148: Again, the general plan is applied to a specific situation. This time the hypotheses in question are about the average value of some characteristic of the population, so the mean of the sample is used as the estimator. Again, a z-score is calculated comparing the observed mean of the sample to the hypothesized mean of the population.
- pp. 149-150: These pages emphasize that the t-distribution should be used when your sample size is small. The only effect this has is in the numerical reporting of the probability (the p-value) for the observed mean.

- pp. 151-156: Although the information in these pages may seem extraneous, it is very important. It is also clearly presented. Read it over a couple times.

8.4 Exercises

Exercise 8.1: A group composed of 100 men and 100 women wants to create a committee of eight randomly-selected members to study some issue. The committee selected ends up with 6 men and 2 women, and certain vocal members of the group begin to argue that there is some bias in the selection of the committee. You are assigned the task of assessing the claim that there is bias. To complete your assignment, you should answer the following questions:

(a) If the selection process is random, what is the expected value for the number of men on the committee? (This is your null hypothesis.)

(b) What is the z-statistic for comparing the current committee distribution to the expected value?

(c) If you were asked to report a p-value for this situation, should you use the z-distribution or the t-distribution?

(d) What is your assessment? Was the selection process random?

Exercise 8.2: In Oregon, a random sample of 1000 people between the ages of 20 and 25 are asked whether they graduated from high school, and 78% claim they have. A similar survey in Washington found that 73% claimed to have graduated from high school. You are asked to write an article in which you discuss whether graduation rates are different in Washington and Oregon. To complete your assignment, you should answer the following questions:

(a) What is your null hypothesis?

(b) What is the z-statistic for comparing the new survey to the old one?

(c) If you were asked to report a p-value for this situation, should you use the z-distribution or the t-distribution?

(d) What is your assessment? Are the graduation rates different?
