IS NATURE PROBABLE OR CAPRICIOUS?

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Probably the most important event in the history of modern science, although it is not always recognized as such, was the publication in 1637 of Descartes *Discourses*, especially of Part V, wherein he described the metaphor of the animal as a machine. Soon to be followed by La Méttrie's "*homme machine*," the "*bête machine*" of Descartes was the foundation of a world view that has colored science ever since and truly makes modern knowledge a Cartesian system. The concept of the universe as a machine is really the notion that for every cause there is an effect, that the universe is deterministic. Put in another way, it asserts that given the state of all the relevant variables at some time t_0, it is possible to predict exactly what the state of the universe will be at some future time t_1; that is, the universe can be described as a solution to a large set of simultaneous differential equations. This world view is responsible for vast progress in physics, chemistry, and physiological biology, and is, moreover, the starting point for work in any new science such as psychology. We always begin with the hope and assumption that rigorous laws leading to exact predictions are possible.

It is ironic that at the time of Descartes death in 1650 there was being laid the foundation of an alternate world view, one that would not have its full impact for more than 200 years. In the *De alea geometriae*, Pascal, Fermat, and Huyghens created an entirely new metaphor, that of the universe as a gaming table in which the cast of dice determined the outcome. The idea that human affairs were the outcome of chance events was, of course, a very old one, and Caesar at the Rubicon was not the first to say "*Alea jacta est*" at a critical moment. Nevertheless, we owe the explicit systemization of chance as an exact doctrine to Pascal and Fermat and its rigorous mathematization to Bernoulli, and much later to Laplace.

So great was the influence of Cartesianism on science, however, that even the founders of the probability theory were not willing to accept its antideterministic implications. Laplace in his *Essai philosophique sur les probabilitiés* reconciled chance events with a deterministic universe by the principle of ignorance. Events *seem* to be governed by chance because the immense complexity of the universe makes it impossible to know all the relevant facts about the state of the system. If there were a demon capable of comprehending all the important information about a die, including the exact forces involved in throwing it, its shape, the properties of the gaming table, and so on, then that demon could foretell exactly the outcome of a cast. For Laplace, then, the universe was still one of cause and effect, but human ignorance and fallibility made exact prediction impossible.

The first important physical applications of probabilistic notions came in the middle of the 19th century with the development by Maxwell and Boltzman of the kinetic theory of gases. But the influence of Laplace and Cartesianism was so great that even at the level of molecular events it was assumed that there was strict determination. Only our inability to distinguish among molecules made, in practice, a probabilistic system out of a deterministic one. There is a great similarity between Maxwell's demon and Laplace's.

It is only in the 20th century that physical science has accepted the full metaphysical implications of the notion of chance. Quantum mechanics is profoundly antimaterialistic and anti-Cartesian. It states, as a fundamental property of the universe, the probabilistic nature of events. No matter how much information there is about the past history of a given unstable nucleus, it is, in principle, impossible to decide when it will decay; that is, up until the instant of decay there is no difference between the nucleus that decays and its neighbor that does not. All that can be specified is the proportion of an ensemble that will decay in a given interval. Laplace and Maxwell postulate demons that do not exist in fact; Schroedinger and Heisenberg deny the possibility of their existence.

I have discussed the history of determinism and its eventual rejection as a universal because I would like to carry the process one step further and ask whether even the statements of a probabilistic universe are too certain for some phenomena. But this is a paradox. How can anything be less certain than chance? To understand this we must look at a fundamental axiom of probability theory, the Law of Large Numbers, and at the notion of statistical information.

The Law of Large Numbers states, in one of its many forms, that the average value of some variate will tend to lie in an interval around the true mean of that variate, which interval gets smaller and smaller as the number of observed values grows larger. As the sample size grows larger and larger, the observed mean is surer and surer to be closer and closer to the true value. This law holds for a very wide variety of cases, including mixtures of different distributions, and can be said to be the most fundamental axiom in probability theory. An example of its operation is shown in Figure 1. Fifty values were chosen from a table of random numbers and the mean of these numbers was taken using only the first number, then the first and second, then the first, second, and third, and so on up to a cumulative mean of all 50. These means are plotted against the sample size. The two solid lines represent the same group of random numbers taken in two different orders, while the dashed lines represent a completely different set of numbers also taken in two different orders. We see that for small sample sizes the means tend to deviate widely from the true mean of 5, but that as the samples grow larger and larger the means get closer and closer to the true value. Also evident is the tendency for the cumulative mean to remain on the same side of the true value for long periods. Finally, we see that the order in which the numbers are taken makes