Part I

1. A xenon arc lamp is covered with an interference filter that only transmits light of 400 nm wavelength. When the transmitted light strikes a metal surface, a stream of electrons emerges from the metal. If the intensity of the light striking the surface is doubled,

   (a) more electrons are emitted in a given time interval.

   (b) the electrons that are emitted are more energetic.

   (c) both of the above.

   (d) neither of the above.

   Answer (a): Higher light intensity means more photons, but each photon still has the same energy

2. In the photo electric effect the work function of a metal depends on

   (a) the frequency of the incident light.

   (b) the intensity of the incident light.

   (c) the maximum kinetic energy of the emitted electrons.

   (d) none of the above.

   Answer (d) The work function depends only on the metal and not on the incident light. However, the work function will determine the cut off frequency and hence will limit the kinetic energy of the emitted electrons.

Part II

1. The minimum frequency of light that will cause photoemission from a lithium surface is 5.5×10^{14} Hz.

   (a) Would red light of wave length 650 nm cause photoemission? Explain.

   The cut off frequency for photoemission is 5.5×10^{14} Hz which corresponds to a wavelength of $c/f = 545nm$. Since the red light has a longer wave length it will not cause photo emission.

   (b) Calculate the work function of lithium in electron volts.

   The energy conservation equation for photoemission is $hf = \text{KE} + \phi$ where ϕ is the work function. When f is at the cut off frequency the kinetic energy of emitted photons is zero. Hence $\phi = hf_{\text{min}} = (6.6\times10^{-34})(5.5\times10^{14}) = 3.6\times10^{-19}$ J. or $(3.6\times10^{-19})/(1.6\times10^{-19}) =$ 2.25 ev.

   (c) If the surface is illuminated by light of frequency 6.5×10^{14} Hz, find the maximum energy of the emitted photoelectrons in electron volts.

   From energy conservation $\text{KE} = \text{hf} - \phi = \text{h}(\text{f} - \text{f}_{\text{min}}) = (6.6 \times 10^{-34})(1.0 \times 10^{14}) =$ 6.6×10^{-20} J, or 0.412 ev.

2. The wavelength of the yellow sodium doublet in the emission spectrum of sodium is close to 590 nm. What is the energy of one photon of this light. A sodium vapour street light is rated at 300 W, 30% of which is is emitted as light. How many photons of light does it emit per second.

   First the energy of one photon is $E = hf = hc/\lambda = (6.6 \times 10^{-34})(3.0 \times 10^{8})/(590 \times 10^{-9}) = 3.4 \times 10^{-19}$ Joules. The power of the light emitted as light is $300 \times 30/100 = 90$ W or 90 J/s. Therefore the number of photons per second is $90/(3.4 \times 10^{-19}) = 2.7 \times 10^{20}$ photons per second.

3. (a) What is the wavelength of an electron in the ground state of the hydrogen atom? (You may use the fact that the ground state radius for the hydrogen atom is 0.529 Å.)

   In the ground state the circumference of the orbit $2\pi r$ is one wave length so $\lambda = 2\pi r = 3.3\ \mathring{A} = 0.33$ nm.

   (b) Find the speed of the electron in the ground state, expressing your answer as a fraction of the speed of light, c.

   According to de Broglie $\lambda = h/mv \;\Rightarrow\; v = h/m\lambda = 2.20 \times 10^{6}$ m/s or $0.073c$

   (c) What is the wavelength of photons emitted in a transition from the $n = 2$ so the $n = 1$ energy level. What part of the electromagnetic spectrum do these photons come from?

   From the Rydberg formula $\lambda^{-1} = 1.097 \times 10^{7}(1/1^{2} - 1/2^{2}) = 0.82 \times 10^{7}$ m^{-1} so $\lambda = 122$ nm

   (d) Use the Bohr Model to predict the ionization energy of the He^{+} ion in the ground state. Would a prediction of the ionization energy of the He atom be accurate? If not, what would be the source of the error?

   The ionization energy of hydrogen is 13.6 ev. Since the He nucleus has twice the charge it will have four times the ionization energy (see the problem below for the details). So the ionization energy of He^{+} is 54.4 ev. We cannot use the Bohr model to predict the ionization energy of the helium atom since it is a two electron system,

4. Starting with the Bohr postulate $mvr = n\hbar$

(a) use Newton's second law for circular motion to derive an expression for the radius of the nth orbital in the hydrogen atom as a function of n.

For the sake of generality lets do this calculation for the case of an atom whose nucleus has charge Ze. That is we will do it for an atom with Z protons. For hydrogen we would take $Z = 1$ In this case Newton's second law for circular motion states $F_{\text{net}} = ma \Rightarrow kQq/r^2 = mv^2/r$ and $Q = Ze$ $q = e$ and from the Bohr postulate $v = n\hbar/mr$ so that $kZe^2/r^2 = m(n\hbar/mr)^2/r = n^2\hbar^2/mr^3$ So that

$$r = \frac{n^2\hbar^2}{kZe^2m}$$

(b) Use the above result and the expression for total energy $E_{\text{tot}} = KE + PE$ to derive an expression for the total energy of the nth orbital.

$E_{\text{tot}} = KE + PE = \frac{1}{2}mv^2 - kQq/r$. But since from Newton's second law $mv^2/r = kQq/r^2$ then $\frac{1}{2}mv^2 = \frac{1}{2}kQq/r$ so that $E_{\text{tot}} = \frac{1}{2}kQq/r - kQq/r = \frac{1}{2} - kQq/r = -ke^2Z/2r$. Using the formula for r above we get

$$E_{\text{tot}} = -\frac{ke^2Z}{(2n^2\hbar^2/kZe^2m)} = -\frac{k^2Z^2e^4m}{2n^2\hbar^2}$$

(c) When an electron moves from orbital m to orbital n it does so by emitting or absorbing a photon of energy $E = |E_n - E_m|$. Use this fact and the expression for the energy of a photon $E = hf$ to derive Rydberg's formula for the energy spectrum of hydrogen.

The energy difference ΔE is given by

$$\Delta E = E_n - E_m = \frac{k^2Z^2e^4m_e}{2\hbar^2}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) .$$

Now this energy is carried away by a photon with wavelength given by $\Delta E = hf = hc/\lambda$ So that

$$\frac{hc}{\lambda} = \frac{k^2Z^2e^4m_e}{2\hbar^2}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \Rightarrow \frac{1}{\lambda} = \frac{k^2Z^2e^4m_e}{2\hbar^2hc}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

The rather impressive coefficient

$$\frac{k^2Z^2e^4m_e}{2\hbar^2hc}$$

should be equal to Rydbergs constant. Substituting the appropriate values for hydrogen

$$R_H = \frac{(9\times10^9)^2(1.6\times10^{-19})^4(9.1\times10^{-31})}{2(6.63\times10^{-34}/2\pi)^2(6.63\times10^{-34})(3\times10^8)} = 1.09\times10^7\text{m}^{-1}$$

which, remarkably, is equal to the experimental result.