Modeling Motion
Orbital Motion Lab

This lab will not be assessed on any of the lab assessment criteria, but you should include answers to all questions and all graphs in your lab notebook.

Part I

Introduction:

One day you accidentally fall into a wormhole and by some freak of nature emerge alive and unscathed. You find yourself marooned on a tiny dead star. Rather than being scared of the unknown and worrying about the how you will ever return to Earth you jump at the chance to study the star and its surroundings.

Procedure: Part I

After initial observations of the star you determine that its radius is 10.00 m and that the acceleration due to gravity at its surface is 1.00 m/s^2. You quickly realise that on this star you should be very careful when jumping!

- Find the mass and density of this star. ("What is it made of?" you ask yourself. You are encouraged to ask someone else this question!)
- What is the acceleration due to gravity at 2.00 m above the surface of the star?
- What is the escape velocity at the surface of the star?

As you gaze with awe at the beautiful vista a low orbit planet passes by 2.00 m above the ground travelling at 2.0 m/s in a direction tangential to the surface. Thankfully it misses you and continues to orbit the star. You realise this is the perfect opportunity to study orbital mechanics and decide to study the motion of this planet. First you wonder if the orbit is circular (Believe me you do!).

- What would be the speed of a planet in circular orbit 2.00 m above the surface of this star?

You realise immediately that it is not a circular orbit and so expect, according to Kepler's 1st law, that the motion is elliptical. In order to make accurate predictions about the behaviour of the planet you model the motion using the above information and your computer. Write a program using Euler's method to answer the following questions.

1. Plot an x-y graph showing the orbit of the planet. Is the orbit elliptical? (Make sure you have your axes set up so that they are scaled equally.)
2. Plot a radius vs. time graph and determine the period of the orbit of the planet.
3. Plot a speed vs. radius graph and determine the speed and radius of the planet at its apogee and perigee. Do these results satisfy Kepler's 2nd Law in the form vr=constant?

In order to investigate Kepler's third law you search for other planets, but sadly there are none. Then you think of a wonderfully horrible idea and in a callous act of environmental insensitivity you catch the planet. (You think to yourself, "I can always put it back when I am finished".) You decide to throw the planet tangentially starting at a radius of 2.00 m above the surface of the star with a variety of different initial speeds.

Please plan this experiment carefully to avoid damaging the planet unnecessarily. Before throwing the planet establish the range of allowable initial speeds.

- What is the maximum speed above which the planet would disappear forever into outer space?
- What is the minimum speed below which the planet would smash into smithereens on the star's surface? (This one you may have to determine experimentally – but please catch the planet before it hits the star)

Choose about 5 different initial speeds and for each resulting orbit find the period, T, and the length of of the perigee, P, the length of the apogee, A and then the average radius R=(P+A)/2

Make a plot of T^2 vs. R^3 and verify Kepler's third law. Put the planet back when you are finished.

Part II:

Introduction:

Thankfully you find a way to return to earth. Armed with your knowledge of celestial mechanics you are selected to complete a mission to mars. To practice you first first attempt a mission to the moon.

Procedure:

You start your journey in a circular orbit 630 km above the Earth's surface.

- What is your orbital radius? What is your orbital speed? What your period?

The most energy efficient way to transfer your rocket to the moon is to boost your orbital velocity in the direction of motion by an amount sufficient to put you in an elliptical orbit where the perigee is equal to your current orbital radius and the apogee is the orbital radius of the moon. Your task is to find out by how much you need to boost the velocity to achieve this. Second you must make sure that the moon is in the right position when you boost your speed so that it is in the right place when your rocket reaches its apogee. To address the second problem answer the following questions.

- According to Kepler's Third Law what would be the period of the elliptical orbit described above?
- How long would it take to go from the perigee to apogee in this orbit.(give your answer in days).
- What is the period of the moon in its orbit around the earth? Through what angle does it travel in the time it takes your rocket to go from the perigee to the apogee?

From your responses above you now know what the position of the moon should be when you give your rocket its boost. We will find the required boost velocity experimentally.

- Modify your code from Part I so that the masses, initial velocity and initial position match the values you have calculated in this part.
- Run your program varying the initial velocity of your rocket until you find the correct initial tangential velocity that will result in an elliptical orbit with apogee equal to the moon radius.
- Verify that the time to complete the trip from perigee to apogee is what you calculated previously.

You are almost ready for you mission to the moon. First you need to add the moon to your system.

- Add a moon object to your code with the correct initial velocity and position for a circular orbit around the Earth. You obviously must account for the gravitational attraction between the moon and Earth when you do this, but for now do not include the gravitational attraction between the moon and the rocket.
- Run your program with an x-y plot showing the of the orbits of the moon and the rocket around Earth. (different colours would be nice).

- Adjust the initial angular position of the moon so that when the the rocket is at its apogee the moon is there to greet it. Verify that this initial position fits with what you calculated previously.

Now you need to turn on the gravitational attraction between the moon and the rocket to see if you can get the rocket to go into orbit around the moon when when they meet. To do this just include an extra term in your calculation of the rocket's acceleration to account for the gravitatinal attraction to the moon.

- Run your program. Did the rocket go into orbit around the moon? Describe what happens.
- Adjust your initial rocket speed by tiny amounts to see how this affects the interaction between the moon and the rocket. Can you adjust the initial velocity in such a way as to achieve an orbit around the moon?

Extension:

1. Modify your code to get an Earth based rocket to go from Earth to Mars. In this case all the objects orbit the Sun (including the rocket). And you need to consider the attraction of the rocket to Earth, Mars and the Sun. You can probably start the rocket out on the surface of the Earth in this problem, ignore the spin of the Earth and have the rocket pointing initially in the direction of the Earth's orbit around the Sun so that your rocket gains the benefit of the initial obital velocity around the Sun..