

2. Although we have been using Euler's method to provide numerical solutions to the logistic equation

$$\frac{dP}{dt} = P\left(1 - \frac{P}{100}\right)$$

it turns out that there is analytical solution given by the formula

$$P(t) = \frac{100e^t}{1 + e^t}$$

- (a) Use algebra and the rules for differentiation to verify that this formula is in fact a solution to the logistic equation.

- (b) Show that the following modified version of the formula,

$$P(t) = \frac{100Ae^t}{1 + Ae^t}$$

with the extra parameter A is *also* a solution to the logistic equation for any value of A .

- (c) Write a computer program to plot a graph of this solution with the parameter $A = 1/4$ on the interval $0 \leq t < 10$. Include the program and plot with your solution.
- (d) Write a computer program using Euler's method to graph an approximate solution on the interval $0 \leq t < 10$ with an initial value of $P = 20$ and a stepsize $\Delta t = .005$. Include your program and plot with your solution.
- (e) Show mathematically that using an initial value $P = 20$ corresponds to choosing $A = 1/4$.

Part II

The following questions may look familiar. They should. A brief scan of the in class test suggests that some of you did not have much time to complete these questions. Now is your chance to shine.

1. Differentiate the following functions. Show your work and simplify as far as possible

(a) $f(x) = 3x^4 - 2 \ln x - \frac{1}{x^2}$

(b) $f(x) = x^2 e^{3x}$

(c) $f(x) = \frac{x^2 - 1}{1 + x}$

(d) $f(x) = \sqrt{1 - \sin^2 x}$

2. For the following rate equation find y as a function of x if $y' = \sin x - \cos x$ and $y = 2$ when $x = 0$.

3. Find the area under the curve $y = 12x - 3x^2$ between $x = 0$ and $x = 4$.