Zita's solutions

This is an in-class, closed-book exam. Use only your homework and quizzes, with no help from people, computers, calculators, texts, or solutions.
This is designed as a 1.5-hour exam. You have 2 hours to do it.

Estimate or leave answers in simplest exact form whenever feasible.

SHOW YOUR WORK as neatly and concisely as possible.

Hints for completing the exam efficiently:

- skim through the whole exam before you start it
- pace yourself – don't spend too much time on any one part
- take a break or move on if you find yourself getting stuck or spinning your wheels
- do the easiest questions first
- sketch your strategy for harder problems first, before diving into math
- think about relationships, sketch, set up equations, and check units, before you plug in any numbers
- show your work
- use words to CONCISELY explain your reasoning and results
- do not waste time with a calculator or unphysical precision

Possibly useless information:

$R_{sun} = 7 \times 10^{10}$ cm

$M_{sun} = 2 \times 10^{33}$ g

$L_{sun} = 3.826 \times 10^{33}$ erg/s $= 3.826 \times 10^{26}$ J/s

$F_{sun} = 1.36 \times 10^{6}$ erg/s cm^2 $\left|\frac{J}{10^7 erg}\right|\left|\frac{10^2 cm}{m}\right|^2 = 1.36 \times 10^3$ W/m^2

Absolute mag $_{sun}$ = 4.76

Density $_{photosphere}$ ~2.5×10^{-7} g/cm^3

$\sigma = 5.67 \times 10^{-5}$ erg /(s cm^2 K^4)

$c = 3 \times 10^{10}$ cm/s

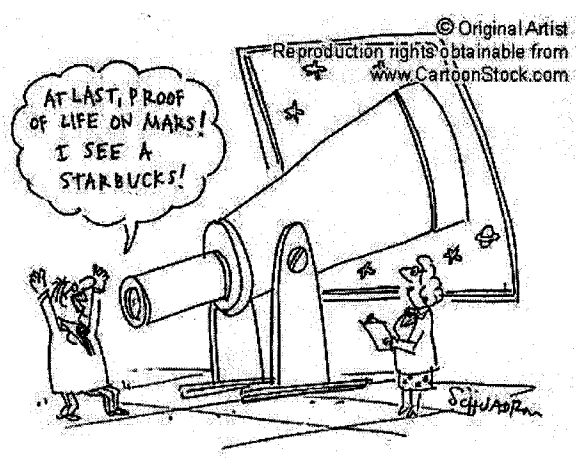
pc = 3.08×10^{18} cm

$m_e = 9.1 \times 10^{-28}$ g

$m_p = 1.67 \times 10^{-24}$ g

$e = 4.8 \times 10^{-10}$ esu $= 1.6 \times 10^{-19}$ C

$k = 1.38 \times 10^{-16}$ erg/K



(1.a) **How do you feel about your work in this program?**

(1.b) How prepared do you feel for this midterm?

(2.a) **What can you find out about a STAR by analyzing its light?** List all you can think of.

Temperature, atmospheric composition, radiant flux, Luminosity, radius, Type
proper motion
Magnetic fields, oscillations, internal rotation (esp. of Sun), spots, companions; can infer mass, distance, age

(2.b) **What can you find out about a GALAXY by analyzing its light?** List all you can think of.

Flux, luminosity, composition, star types, mass (from Doppler shifts of stellar orbits), distance, age mass distribution, proper motion

(2.c) What are some things you can**not** tell about a star or galaxy from its light?

Internal composition can only be inferred (Need an orbiting companion to determine masses e.g. of planets)

(3) Luminosity, brightness, and temperature of stars:

(a) What are the units of radiant flux? *Intensity = power/area*

(b) What are the units of luminosity? *Power = energy / time*

(c) How is radiant flux related to a star's luminosity? *Flux = L/area*

(d) How is radiant flux related to a star's temperature? $F = \sigma T^4$

(e) Use your last two equations to derive a relation between luminosity, radius, and temperature.

$F = \sigma T^4 = L/area$ *therefore* $L = \sigma T^4 * area = \sigma T^4 4 \pi R^2$

(f) What assumptions are inherent in the relationship you derived?

The star radiates as a perfect blackbody.

(4) Comparing stars:

$R_A = 2R_B$ $T_B = 2T_A$

(a) Compare the luminosity of two stars. Star A is twice as big (R) as star B. Star B is twice as hot (T) as star A. Which produces more power? By what factor?

$$\frac{L_B}{L_A} = \frac{4\pi\sigma R_B^2 T_B^4}{4\pi\sigma R_A^2 T_A^4} = \left(\frac{R_B}{R_A}\right)^2\left(\frac{T_B}{T_A}\right)^4 = \left(\frac{1}{2}\right)^2 2^4 = 2^2 = 4 \times \text{ power as Star A}$$

(b) Due in part to their different distances, star A appears 100 times brighter than star B. Find their apparent magnitude difference: $b_A = 100\, b_B$

$$m_A - m_B = \frac{5}{2}\log_{10}\left(\frac{b_B}{b_A}\right)$$

$$= \frac{5}{2}\log_{10}(10^{-2}) = \frac{5}{2}(-2) = -5$$

(c) Which star is closer, A or B? Explain. (Do not calculate actual distances.)

$$\frac{b_A}{b_B} = \frac{F_A}{F_B} = \frac{L_A/4\pi d_A^2}{L_B/4\pi d_B^2} = \frac{1}{4}\left(\frac{d_B}{d_A}\right)^2 = 100 \rightarrow \frac{d_B}{d_A} = \sqrt{400} = 20$$

B is 20× further than A

(5) Consider a model of a star consisting of a spherical blackbody with a surface temperature of 10,000 K and a radius of 3 R_{sun}. Let this model star be located at a distance of 200 pc from Earth. Determine the following for this star. See #3.8 (Ch 0)

(a) Radius $= 3R_\odot = 21\times10^{10}$ cm
(b) Luminosity $L = 4\pi\sigma R^2 T^4 = 4\pi R_L^2 F$
(c) Radiant flux at the star's surface $F = \frac{L}{4\pi R^2} = \sigma T^4$
(d) Radiant flux at Earth's surface (compare this with the solar constant) $F = \frac{L}{4\pi d^2}$
(e) Peak wavelength $\lambda(m) = 3\times10^{-3}/T(K)$
(f) What kind of star would this be?

(b) $L = 4\pi(21\times10^8 m)^2 (F = 5.67\times10^8 \frac{W}{m^2})$ 3.1×10^{28} $W = 82 L_\odot$

(c) $F_\star = \frac{L}{4\pi R^2} = \sigma T^4 = 5.67\times10^{-8}\frac{W}{m^2K^4}(10^4 K)^4 = 5.67\times10^8 \frac{W}{m^2}$
$= 4.2\times10^5 F_\odot$

(d) $F_\oplus = \frac{L}{4\pi d^2} = \frac{L}{4\pi R^2}\frac{R^2}{d^2} = F_\star\left(\frac{R}{d}\right)^2$ where $d = 200\,pc\left|\frac{3.08\times10^{18}\,cm}{pc}\right| = 6.2\times10^{20}$ cm

$F_\oplus = 5.67\times10^8 W\left(\frac{21\times10^{10}\,cm}{6.2\times10^{20}\,cm}\right)^2 = 6.6\times10^{-11}\frac{W}{m^2} = 4.8\times10^{-14} F_\odot$

$F_\odot = 1.36\times10^3 \frac{W}{m^2}$

(e) $\lambda = \frac{3\times10^{-3}}{T} = \frac{3\times10^{-3}}{10^4} = 3\times10^{-7} m\left|\frac{nm}{10^{-9} m}\right| = 300$ nm UV (blue-white)

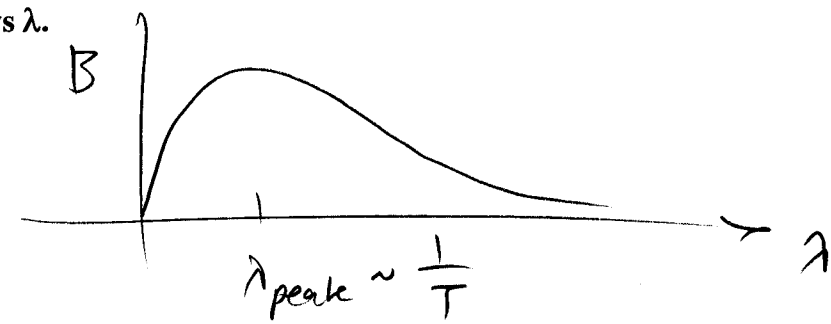
(f) This is a B Star.

For $T = 10^3$ K, $\frac{hc}{kT} \sim \frac{7\times10^{-27}\ \text{erg}\cdot\text{s} \times 3\times10^{10}\ \frac{\text{cm}}{\text{s}}}{10^{-16}\ \frac{\text{erg}}{\text{K}} \times 10^3\ \text{K}} \sim \frac{20\times10^{-17}}{10^{-13}} \sim 2\times10^{-3}$

(6a) The Planck function is $B = \frac{a/\lambda^5}{e^{b/\lambda}-1}$, where a=2hc^2 and b=hc/kT. **What physical situation does this apply to? What does B represent physically, in words?**

B = intensity of radiation (at each wavelength) for a black body at temperature T.

(b) Sketch B vs λ.



(c) Find an expression for the frequency λ_{peak} at which the Planck function B attains its maximum value.

$B = \frac{N}{D}$ where $N = \frac{a}{\lambda^5}$ and $D = e^{b/\lambda} - 1$. B peaks where $\frac{dB}{d\lambda} = 0$.

$$\frac{dN}{d\lambda} = \frac{-5a}{\lambda^6} \qquad \frac{dD}{d\lambda} = -\frac{b}{\lambda^2} e^{b/\lambda}$$

$$\frac{dB}{d\lambda} = \frac{D\frac{dN}{d\lambda} - N\frac{dD}{d\lambda}}{D^2} = 0 \text{ when } D\frac{dN}{d\lambda} = N\frac{dD}{d\lambda}$$

$$D\frac{dN}{d\lambda} = (e^{b/\lambda}-1)\left(\frac{-5a}{\lambda^6}\right) = N\frac{dD}{d\lambda} = \frac{a}{\lambda^5}\left(-\frac{b}{\lambda^2}e^{b/\lambda}\right)$$

$$(e^{b/\lambda}-1)\frac{5}{\lambda} = \frac{b}{\lambda^2}e^{b/\lambda}$$

$$\frac{5}{b}\lambda = \frac{e^{b/\lambda}}{e^{b/\lambda}-1} = \frac{1}{1-e^{-b/\lambda}}$$

Could solve this graphically or numerically

(d) What is physically significant about λ_{peak}? What can it tell you about a star?

$\lambda_{peak}(\text{m}) \approx 3\times10^{-3}\ T(\text{K})$: Wien's law

λ_{peak} is the brightest color in the spectrum.
It tells you the blackbody's temperature.

EXTRA

Compare $B_\lambda = \frac{dB}{d\lambda}$ to $B_\nu = \frac{dB}{d\nu} = \frac{2h\nu^3/c^2}{e^{h\nu/kT}-1}$ and $E = \frac{hc}{\lambda} = h\nu$

$$\frac{c}{\lambda} = \nu$$

$$-\frac{c}{\lambda^2}d\lambda = d\nu$$

$$B_\nu = \frac{2h}{c^2}\left(\nu^3 = \frac{c^3}{\lambda^3}\right) \Bigg/ \left(e^{hc/\lambda kT}-1\right) = \frac{2hc/\lambda^3}{e^{hc/\lambda kT}-1} = \frac{dB}{d\nu}$$

$$B_\lambda = \frac{dB}{d\lambda} = \frac{dB}{d\nu}\frac{d\nu}{d\lambda} = B_\nu\left(-\frac{c}{\lambda^2}\right) = \frac{-2hc^2/\lambda^5}{e^{hc/\lambda kT}-1} \quad \checkmark$$

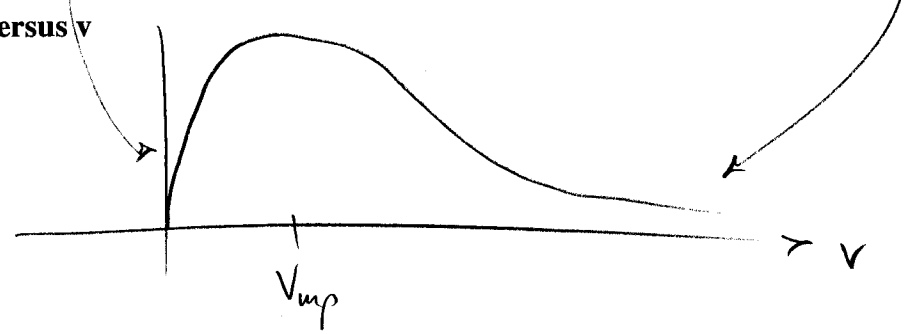
$n_v = a v^2 e^{-bv^2}$ where $a = 4\pi n$, $b = +\frac{m}{2kT}$

(7a) The Maxwell-Boltzmann distribution is $n_v = 4\pi n c v^2 e^{-mv^2/2kT}$. What physical situation does this apply to? . What does n_v represent physically, in words?

For a gas of temperature T, it describes the number of particles in each velocity interval.

($v \sim 0$): There are relatively few very slow (cold) particles.
Middle: There are many more particles with speeds near the peak
($v \to \infty$) There are relatively few very fast (hot) particles, out on the "tail" of the velocity distribution.

(b) **Sketch n_v versus v**



(c) **Derive the most probable speed of the Maxwell-Boltzmann distribution**

The function peaks where its slope vanishes. Differentiate n and find where dn/dv=0.

$$\frac{dn_v}{dv} = \frac{d}{dv} a v^2 e^{-bv^2} = a\left[v^2 \frac{d}{dv}\left(e^{-bv^2}\right) + e^{-bv^2}\frac{d}{dv}(v^2)\right]$$

$$= a\left[v^2\left(-2bv e^{-bv^2}\right) + e^{-bv^2}(2v)\right]$$

$\frac{dn_v}{dv} = 0$ where $v^2\left(2bve^{-bv^2}\right) = e^{-bv^2}\, 2v$

$$v^2 b = 1$$

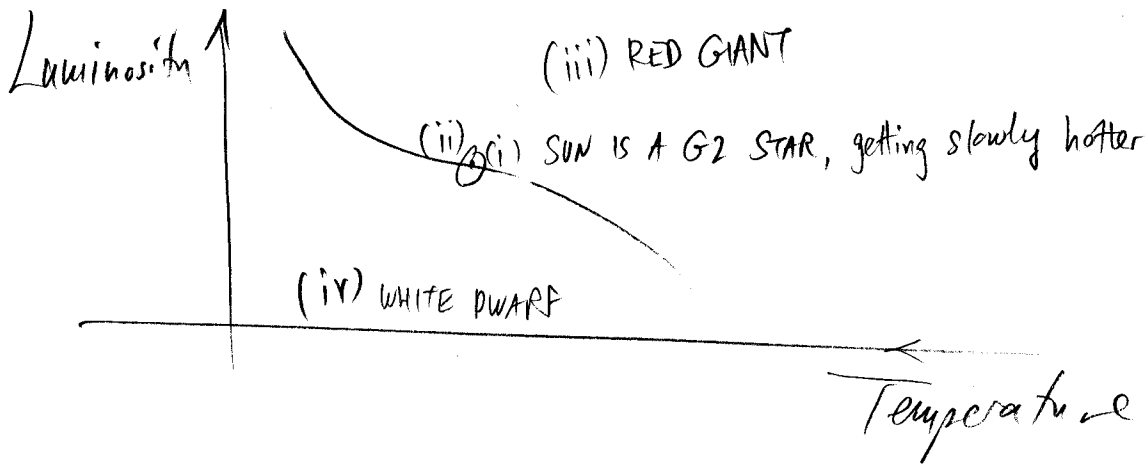
$$v^2 = \frac{1}{b}$$

$$v = \sqrt{\frac{1}{b}} = \sqrt{\frac{2kT}{m}} = V_{mp}$$

(d) **Give an example of how you can use the Maxwell-Boltzmann distribution function to find out something interesting about stars.**

You can find the fraction of particles with high enough speed V to overcome to Coulomb repulsion, and fuse.

(8) Hertzsprung-Russell diagram:
(a) Sketch it, label the axes, and indicate the main sequence.



(b) What does the main sequence represent, physically?

The H- burning phase where most stars spend most of their active life in hydrostatic equilibrium

(c) Indicate the **location** of the Sun on the HR diagram at four points in time (i-iv below), and **label** what kind of star it is at each point in time

(d) Give a **ballpark estimate** of the luminosity, temperature, and size of the Sun at each point in time. Don't calculate much. See Universe Ch 18, 21

	TIME:	LUMINOSITY/L_{sun}	TEMP (K)	Radius/R_{sun}
(i)	4 billion years ago	*about 25% less* -40%	*<5700*	*about 10% less* 6-
(ii)	now	*1*	*~ 5800*	*1*
(iii)	3 billion years from now	*a little less*	*less – red giant* 3000 K	*about 100x*
(iv)	6 billion years from now	*much less – WD*	*more ~ 10,000*	*about 1/100* ~R_{earth}

(9) **If gravitational contraction supplies Sun's energy, you can find out how long the Sun could burn.**

(a) How could gravitational potential energy make the Sun hot (conceptually)?
As a star slowly collapses, its U_{grav} decreases and is converted to heat (and a little motion)

(b) How can you find the lifetime τ of an object with luminosity L and total available energy E? Explain.
L = power = energy / time so τ=E/L

(c) Derive an equation for the gravitational potential energy of an object with mass M and size R, from first principles.

F=-GmM/r2 = -dU/dr. Integrate (and set U=0 at r=infinity) to find U = -GMm/r

(d) ***Estimate*** the gravitational potential energy of the Sun.

$U \sim aGM^2/R$ where a is some constant of order 1. If a=1 (it's closer to 3/10: see p.330), then

$U \sim (6.67 \times 10^{-8}$ dyne $cm^2/g^2)\ (2 \times 10^{33}\ g)^2 / (7 \times 10^{10}\ cm) \sim 3 \times 10^{48}$ ergs

(e) The solar luminosity L_{sun} is given on the front page. Use this to find the lifetime of the Sun, if its power is supplied by gravitational contraction.

$\tau = E/L \sim 3 \times 10^{48}$ ergs / 3.826×10^{33} erg/s $\sim 10^{15}$ s / 3×10^7 s/yr $\sim 1/3 \times 10^8$ years $\sim 10^7$ years

Sun would be on the order of 10 million years old

(f) What does this result imply about radioisotope and fossil evidence that suggests the Earth is billions of years old?

When Kelvin derived this age for the Sun, he told Darwin he was wrong: the Earth couldn't possibly be billions of years old.

(g) What is our current understanding about the energy supply of the Sun?
Fusion and $E=mc^2$: 4 H -> He + 2e + 2ν_e energy

(h) What are problems with this model? *Recall problem* 10,4 with $v = 10\, v_{rms}$

1. At core temperatures around 10^7 K, there are very few protons fast enough to fuse (they need 10^9 K) ($10^{-63} = \frac{h\nu}{h_{rms}}$)
and even though each fusion reaction releases a lot of energy, for a total of (10^{-11} *ergs)*
that would sustain the Sun for only 10^{-44} s

2. Plus the solar neutrino problem: we only see 1/3 the electron neutrinos expected.

(i) What solution(s) could address those problems?

1. *QM tunneling permits cooler protons to overcome the Coulomb barrier and fuse*
2. *Neutrinos have mass, therefore some ν_e convert to ν_μ and ν_τ .*