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Poincaré's Philosophy

From Conventionalism to Phenomenology

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OPEN COURT
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To Helen, Isabelle, and George†,
who often spoke about this book

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Introduction

Speaking about Kant, Schopenhauer rightly claimed that it is far easier to criticize a genius than to praise him: his errors, always finite in number, can be easily circumscribed while his contribution to truth remains both inexhaustible and unfathomable (Schopenhauer 1958, Vol.1, Appendix). If I say this, it is by way of an excuse for the criticisms which will be levelled at some paradoxical aspects of Henri Poincaré's philosophy. However, my aim, which is a constructive one, is that of clarifying and of then reconciling Poincaré's various theses about the foundations of mathematics and of the natural sciences.

With Hilbert, Poincaré was the greatest mathematician of his time and one of the greatest ever. In E.T. Bell's words, he was the last universalist in that he looked upon every branch of mathematics and of theoretical physics as part of his domain (Bell 1937, Chapter 28). Poincaré had a prodigious knowledge of the field of complex analysis on which he left an indelible mark; he founded the discipline of *analysis situs* or *topology*; he studied the theory of groups, the probability calculus, classical mechanics, optics, electromagnetism, and astronomy; he attacked the three-body problem and addressed the question of planetary stability, thereby creating chaos theory. In this context, he established a result which proved essential to the second law of thermodynamics. Together with Lorentz and simultaneously with Einstein, he discovered Special Relativity. We owe him the Principle of Lorentz-covariance and one of the first applications of a symmetry requirement to physical laws; also the concept of an integrated four-dimensional physical continuum which was later attributed exclusively—and hence wrongly—to Minkowski. He determined the relativistic transformation equations for the electromagnetic field and for the charge-density, thereby correcting a serious mistake made by Lorentz in his 1904 paper. Poincaré finally developed a Lorentz-covariant gravitational theory which yielded Newton's law as a limiting case for small velocities. It is worth adding that he achieved these breakthroughs concurrently with, but independently of Einstein (see Chapter 4 below). In the history of French mathematics, Poincaré's phenomenal output was superseded only by Cauchy's.

Poincaré wrote several articles and books on epistemology and on the psychology of discovery. Like Einstein, he offers the rare example of a creative scientist whose work is informed by an explicit methodological standpoint. Poincaré was described—by Popper—as the greatest philosopher of science ever (see Bouveresse 1998, p. 191). He was however no systematic philosopher, or rather: he did not have enough time to become one. As a result, his philosophical pieces, written at different times and in different circumstances, contain several incoherencies while his superbly lucid style conceals the complexity and depth of his thought. These apparent paradoxes prevented Poincaré's contemporaries from appreciating his philosophical insights at their true value. His theses nonetheless helped to shape the philosophy which was to guide the development of science throughout the twentieth century; for it can be maintained that the Neopositivists—the members of the Vienna and Berlin Circles—did no more than 'precisify' Poincaré's (and Duhem's) insights into the problems of induction, of the empirical basis, and of the mathematical foundations of physics.

Going back to the purpose of this book—a purpose which might make its reading worthwhile to philosophers, to historians and even to working scientists—it consists of an exposé followed by a rational reconstruction of Poincaré's position. The book can also be regarded, from a non-exegetic standpoint, as a treatise on modern epistemology.

The following passages, both taken from the Introduction to *The Value of Science,* demonstrate the necessity of reinterpreting Poincaré's philosophy; for these quotations, which expose his conflicting epistemological views in their most radical form, illustrate the 'Problematik' underlying the whole of the present work:

> The search for truth should be the goal of our activities. It is the sole end worthy of them. . . .
>
> Some people have exaggerated the role of convention in science, they have gone so far as to say that law, that scientific fact itself, was created by the scientist. This is going much too far in the direction of nominalism. No, scientific laws are not artificial creations; we have no reason to regard them as accidental, though it be impossible to prove they are not . . .
>
> Does the harmony the human intelligence thinks it discovers in nature exist outside of this intelligence? No, beyond doubt, a reality completely independent of the mind which conceives it, sees or feels it, is an impossibility. (Poincaré 1906a, pp. 11, 14)

Let me now give a few details about the contents of the present book. I propose first to examine two areas in which Poincaré's philosophy of physics came into close contact with his theses about the foundations of mathematics. By not strictly distinguishing between physical space and the mathematical continuum, he was led to a neo-Kantian conception of geometry; and because of his reliance on Kant's views concerning our intuition of time, he assessed the status of certain mathematical postulates, more particularly that of mathematical induction, in a manner which remains highly questionable (see Chapters 3, 5 below).

Poincaré took the mathematical and the empirical logics of discovery to be essentially the same. He stressed the universal role of intuition by which we *guess* the solution to a problem *before* trying either to establish or to refute our conjecture. Prima facie, this approach presupposes a belief in the mind-independent existence *both* of a physical *and* of a mathematical reality; but despite his structural-realist stance in the philosophy of physics, Poincaré insisted not only that *mathematical* entities must in principle be constructible, but also that they ought actually to have been constructed before we can meaningfully talk about them (see Chapters 2, 5 below). In Kantian fashion, he held that infinite classes are not completed entities about which we are entitled to reason; for they remain open-ended collections to which new members are continually added by discrete acts of consciousness. Thus Poincaré defines *all* aggregates in strictly *temporal* terms: we cannot legitimately refer to any infinite classes, for these are continually altered by the introduction of new elements; and this constant change takes place because no *mathematical* entity exists prior to its construction in pure time. Mathematical objects are thus thrown into existence by the activity of the human mind; they do not antedate its operations. If we want mathematical statements to retain some objective validity, Poincaré seems to leave us no choice but to postulate a set of universal rules to which everybody's mind conforms; a thesis which was in fact refuted by Poincaré's own controversies with both Russell and Zermelo.

Poincaré thus held a quasi-Kantian view of mathematics. He must nonetheless have known that in classical analysis, a field to which he made fundamental contributions, the real numbers are defined by means of *infinite* sets of fractions or of *infinite* classes of Cauchy sequences. His practice was therefore in open conflict with his philosophical position; and his constructivism appears philosophically more dubious than Platonist realism which postulates a realm of mind-independent mathematical entities (see Chapter 6 below). From Poincaré's own viewpoint, Platonism would moreover have had the merit of treating the logics of scientific and of mathematical discovery on a par: in both cases, the mind would be trying to capture a reality existing prior to its operations. Moreover, only Platonism seems capable of making disagreements between mathematicians a matter for rational discussion rather than for a mere recognition of irreducible differences between personal preferences. Needless to say, there remains one important contrast between physics and mathematics: scientific hypotheses can be subjected to empirical tests and hence to the verdict of a jury over whose decisions we have no complete control. Such tests are unavailable to mathematicians; apart from a minimal condition of consistency, the intuitive self-evidence of the theorems must therefore continue to play a central role in the progress of mathematics.

Another object of this book is to isolate some seemingly incompatible strands in Poincaré's philosophy of physics. I shall distinguish between his so-called conventionalism on the one hand, and a marked tendency towards an inductivist form of empiricism on the other. It can be shown that most of these conflicts can be resolved, that Poincaré was a structural realist with strong empiricist leanings, and that the more carefully one analyses his conventionalism, the more it assumes the form of a *'pétition de principe'*. In certain contexts, Poincaré held that high-level theories are neither true nor false but more or less convenient *and*—in a different context—that these selfsame hypotheses prove convenient only to the extent that they mirror reality. Thus he treated the degree of convenience of physical principles, their level of aesthetic-cum-mathematical perfection, as an index of their verisimilitude. (It should be immediately added that he treated 'verisimilitude' as an intuitive concept unrelated to the correspondence theory of truth). Though his conventionalism seemed bound to lead to epistemological relativism, Poincaré remained a structural realist through and through (see Chapter 2). He insisted on rejecting *as false* all experimentally undermined laws. As for his principle of convenience *(commodité)*, it enjoined him to accept unified scientific systems only as long as they remained strongly supported by *crude* facts (see Chapter 1).

Poincaré's views will often be compared and contrasted with those of Duhem (see Chapter 2). This is why it seems appropriate that I should end this introduction by saying something about the different meanings given by these two influential authors to the word 'conventionalism'. In a nutshell, Duhem's thesis is that the human mind is intrinsically incapable of grasping reality as ~~she~~ really is, as created by God; for the divine mind transcends all human understanding. The most that a scientist can hope to achieve is some *natural classification* of the phenomena, a scheme which simulates the ontological order without actually referring to it. According to

Duhem, knowledge is definitely not an explanation but a coherent pigeon-holing of appearances. We shall see that this attitude contrasts with Poincaré's position—as rationally reconstructed. Unlike Duhem who tried to make the world safe for a specific theology, Poincaré had *a priori* no *metaphysical* axe to grind; but right from the start, one important aim of his philosophy was to account for the creativity of man's scientific theorizing. In this respect, he can again be compared to Kant who tackled the problem posed by the presumed infallibility of certain components of human knowledge. Kant's well-known answer was that man's *spontaneous* or *creative* mind prescribes its own laws to physical nature. Poincaré was similarly concerned with the—admittedly relative—autonomy of thought vis-à-vis the world of sense-experience. In the last analysis, his Conventionalism merely serves to emphasize the mind's active and largely independent role in scientific discovery. 'Conventionalism' actually turns out to be a misnomer which ought to be replaced by something like a 'Principle of Mathematical Coherence'.

Wittgenstein? The Unreasonable Effectiveness of Mathematics

4

Poincaré's Relativity Programme

To my knowledge, Edmund Whittaker is the only historian to have attributed the discovery of Special Relativity (STR) exclusively to Lorentz and Poincaré. He does this in a chapter of his *History of the Theories of Aether and Electricity*, in which Einstein is barely mentioned. Though unjust towards Einstein, Whittaker's *positive* account of Poincaré's achievement contains much more than a grain of truth (Whittaker 1953, Chapter 2). In what follows I propose to defend a thesis as forthright as Whittaker's, namely the view that Poincaré did discover STR, also that his structural realism and his 'conventionalist' epistemology provided him with essential heuristic guidelines, where, as shown in the previous chapter, the word 'conventionalism' should be regarded as a misnomer. This account in no way tarnishes Einstein's merit: following a path parallel to but independent of Poincaré's, Einstein finally transcended STR by constructing a generally covariant theory of gravitation.

Going back to Poincaré: we shall see that in 1900 he had already given the operational definition of clock-synchronisation which has since been attributed exclusively to Einstein; and that by 1905, he had gone far beyond the theoretical results obtained by all the other Relativists. Long before Minkowski appeared on the scene, Poincaré had determined the structure of the Lorentz-group and founded his Relativity Programme on the notion of Lorentz-covariance in a 4-dimensional manifold. He was the first physicist to have consciously based his heuristics on a symmetry requirement conceived as a condition of form-invariance under a given set of transformations. The latter was used in order to correct a major error in Lorentz's law for the electric density and to find the transformation equations for the electromagnetic field, which is why it seemed appropriate that I should devote a whole section to a general study of the problems posed by symmetry principles. Poincaré furthermore reflected on the parallel between gravity and inertia as well as on the possibility of a dependence of the gravitational attraction on the velocity of a particle. He proposed a Lorentz-covariant theory of gravitation which yields Newton's inverse-square law as a limiting case for small speeds. He had thus made use not only of one specific symmetry requirement but also of the general Principle of Correspondence, which was later attributed to Niels Bohr. Given the heuristic importance of this principle, another section will be devoted to examining some of its presuppositions, more especially the assumption of the uniform continuity of certain functions. Poincaré had thus pushed STR to the utmost limits of which it seemed capable.

With regard to Lorentz, Poincaré, and Einstein, historians therefore have an important duty; namely that of not only acknowledging Einstein's unsurpassed contributions to Relativity, but also of rectifying a certain imbalance. After Poincaré's death in 1912, Einstein saw the necessity of going beyond STR towards a generally covariant theory of gravitation. Because of the great beauty of the general theory, there has been an unfortunate tendency to forget Poincaré's achievement and to regard Einstein as the sole founder of the Relativity programme, not only in its general, but also in its special phase. The present chapter is, in part, intended to redress this injustice.

4.1 First Steps towards Relativity

With regard to conventions, Poincaré put forward two theses and made two prognoses. He maintained that we shall always be in a position to hold on to Classical Mechanics *and* to Euclidean Geometry, provided we be prepared to alter—perhaps in a drastic way—the rest of physics. We have already seen that these two claims, and more particularly the second one, have a logical character and must consequently be considered valid, but Poincaré also predicted that Classical Mechanics *and* Euclidean Geometry would always turn out to be the most convenient systems and therefore have nothing to fear from fresh experiments (see above, Chapters 1 and 3). It is ironical that through criticising Lorentz's hypotheses and then launching the Relativity Programme, Poincaré was destined to refute the first prognosis. He unfortunately did not live long enough to accept—as he most certainly would have done—Einstein's refutation of his second prediction.

Let me present a survey of the most important results achieved by Lorentz and then reconstruct Poincaré's views from his criticisms of Lorentz's position. In the absolute or ether frame, the Maxwell-Lorentz equations can be written in the form:

(1) $\nabla \cdot \underline{E} = \rho; \nabla \cdot \underline{H} = 0; \nabla \times \underline{E} = -[\partial \underline{H}/\partial t]/c; \nabla \times \underline{H} = [(\partial \underline{E}/\partial t) + \rho \underline{v}]/c;$