E/Z
25 Jan 05

# Calculus Ch 3.1 p. 112

#46 $f = x^3 - 6x^2 - 15x + 20$

$f' = 3x^2 - 12x - 15 = 0$ when

$0 = x^2 - 4x - 5 = ax^2 + bx + c$

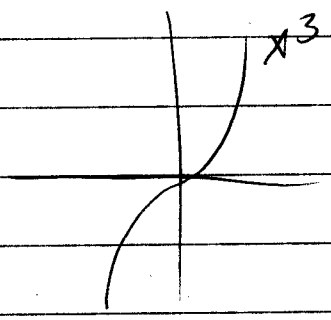
$a = 1 \quad b = -4 \quad c = -5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 + 20}}{2} = \frac{4 \pm \sqrt{36}}{2} = \frac{4 \pm 6}{2}$

$x = \left(\frac{10}{2}, -\frac{2}{2}\right) = (5, -1)$

$\frac{f'}{3} = x^2 - 4x - 5 = (x-5)(x+1) = 0$ when

$x = 5$ or $x = -1$

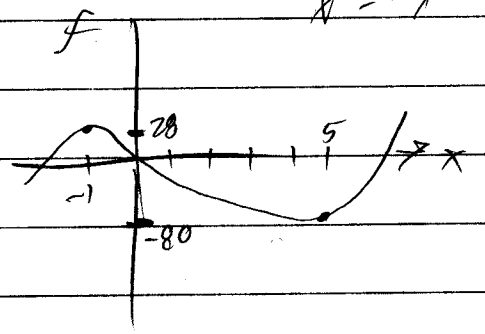



gives the general shape for large $|x|$

$f$ flattens out ($f' = 0$) when

$x = 5 : f(5) = 5^3 - 6 \cdot 5^2 - 15 \cdot 5 + 20 = -80$

$x = -1 : f(-1) = -1 - 6 + 15 + 20 = 28$




#54: Find $n$ in $y = x^n$ to satisfy $13x \frac{dy}{dx} = y$

$\frac{dy}{dx} = \frac{y}{13x} = \frac{x^n}{13x} = \frac{x^{n-1}}{13}$

Try $y = x^{1/13}$. Then $\frac{dy}{dx} = \frac{1}{13} x^{1/13 - 1} = \frac{x^{1/13}}{13x} = \frac{y}{13x}$ ✓ $n = \frac{1}{13}$

Calculus Ch 3,2 p.117

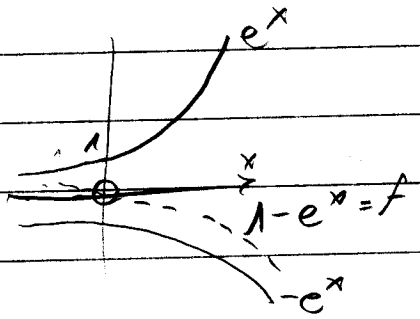
#42 $f(x) = 1 - e^x$, $\frac{df}{dx} = -e^x$

f crosses x axis at $f = 0 = 1 - e^x$

$1 = e^x$

$\ln 1 = \ln e^x = x \ln e = x$

f passes x axis at $x = \ln 1 = 0$ / $(e^0 = 1)$



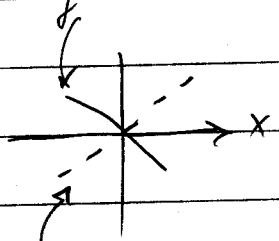

Slope of f at $x = 0$ is

$\left.\frac{df}{dx}\right|_{x=0} = -e^0 = -1$

(b) Equation of the tangent line to f at $x = 0$:

$(y - y_0) = \text{slope}\,(x - x_0)$ $\quad y_0 = 0,\ x_0 = 0$

$y = -1\,x$




(c) Normal line has perpendicular slope $= -\frac{1}{f'} = \frac{-1}{-1} = 1$

Equation of normal line:

$(y - a) = +1\,(x - 0)$

$y = x$